# COMPUTATIONAL REVERSE DIVE PROFILES AND STAGING CONTRASTS
## RGBM Technical Series 2

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## INTRODUCTION

### Overview

Although the manifestations of DCI are statistically distributed, tables and meters employ deterministic models to stage reverse dive profiles RPs, with models broadly categorized as Haldane (dissolved phase) or as bubble (combination of dissolved and free phases). A summary of models and their underpinnings, correlations with data, and contrasts for the 100/60 and 60/100 RPs applying variable surface intervals are given. Suggestions for experiments are tendered, and in related vein, extreme statistics on RPs gathered at Nuclear Emergency Strategy Team (NEST) exercises on various gas mixtures are sketched.

We first discuss DCI risk and coupled statistics, return to broad base description of gas transfer models used in decompression applications, apply these models to the RPs, contrast staging regimens, denote differences, suggest testing, and then summarize experience with NEST RPs on mixed gases.

### Decompression Algorithms

Diving models address the coupled issues of gas uptake and elimination, bubbles, and pressure changes in different computational frameworks. Application of a computational model to staging divers is called a diving algorithm. Consider the computational models and staging regimens for six popular algorithms, namely, the perfusion limited, diffusion limited, thermodynamic, varying permeablity, reduced gradient bubble, and tissue bubble diffusion algorithms The first two are Haldane models (workhorse algorithms in most tables and meters), while the remaining four are bubble models in the generic sense (coming online in tables and meters, often driven by tech diving). The first two track just dissolved gas transfer, using *critical tissue tensions* as limit points, while the latter four treat both dissolved and free phase transfer, using *free phase volumes* as limit points.

Though the systematics of gas exchange, nucleation, bubble growth or collapse, and decompression are so complicated that theories only reflect pieces of the DCI puzzle, the risk and statistics of decompressing divers are fairly straightforward. And the folding of DCI risk and statistics over data and model assumptions is perhaps the most satisfying means to safety and model closure. Some Workshop papers and presentations center on probabilistic decompression, risk, and parameter fitting, using variants of the models discussed.

## PROBABILITIES AND STATISTICS

### Decompression Risk

Computational algorithms, tables, and manned testing are requisite across a spectrum of activities. And the potential of electronic devices to process tables of information or detailed equations underwater is near maturity, with virtually any algorithm or model amenable to digital implementation. Pressures for even more sophisticated algorithms are expected to grow.

Still computational models enjoy varying degrees of success or failure. More complex models address a greater number of issues, but are harder to codify in decompression tables. Simpler models are easier to codify, but are less comprehensive. Some models are based on first principles, but many are not. Application of models can be subjective in the absence of definitive data, the acquisition of which is tedious, sometimes controversial, and often ambiguous. If deterministic models are abandoned, statistical analysis can address the variability of outcome inherent to random occurrences, but only in manner indifferent to specification of controlling mechanisms. The so called dose-reponse characteristics of statistical analysis are very attractive in the formulation of risk tables. Applied to decompression sickness incidence, tables of comparative risk offer a means of weighing contributing factors and exposure alternatives. At the basis of statistical and probabilistic analyses of decompression sickness is the binomial distribution. The binomial distribution is the fundamental frequency distribution governing random events:

1. Binomial Distribution

Decompression sickness is a hit, or no hit, situation. Statistics are binary, as in coin tossing. Probabilities of occurrence are determined from the binomial distribution, which measures the numbers of possibilities of occurrence and nonoccurrence in any number of events, given the incidence rate. Specifically, the probability, $P$, in a random sample of size, $N$, for $n$ occurrences of decompression sickness and $m$ nonoccurrences, takes the form,

$$P(n) = \frac{N!}{n!\ m!}\ p^n q^m \quad , \tag{1}$$

with,

$$n + m = N \quad , \tag{2}$$

$p$ the underlying incidence rate (average number of cases of decompression sickness), and $q$,

$$q = 1 - p \quad , \tag{3}$$

the underlying nonincidence. The discrete probability distributions, $P$, are the individual terms of the binomial expansion of $(p+q)^N$,

$$(p+q)^N = \sum_{n=0}^{N} P(n) = 1 \quad . \tag{4}$$

In risk analysis, $p$ and $q$ are also the failure and success rates, gleaned, for instance, from random or strategic sampling of arbitrary lot sizes. Obviously, the larger the sample size, the better are the estimates of $p$ or $q$. Once $p$ or $q$ is determined, the binomial statistics and probabilities are also fixed. The statistical mean, $M$, and variance, $s$, are given by,

$$M = \sum_{n=1}^{N} nP(n) = pN \quad , \tag{5}$$

$$s = \sum_{n=1}^{N} (n - M)^2\ P(n) = pqN \quad , \tag{6}$$

the usual measures of a statistical distribution. The square root of the variance is the standard deviation. The cumulative probability for more than $n$ cases of decompression sickness, $P_>(n)$, is written,

$$P_>(n) = \sum_{j=n+1}^{N} P(j) = 1 - \sum_{j=0}^{n} P(j) \quad , \tag{7}$$

and the probability of less than $n$ cases, $P_<(n)$, is similarly,

$$P_<(n) = \sum_{j=0}^{n-1} P(j) = 1 - \sum_{j=n}^{N} P(j) \quad . \tag{8}$$

The probability of nonoccurrence in any set of $N$ trials is simply,

$$P(0) = q^N \quad , \tag{9}$$

while the probability of total occurrence in the same number, $N$, of trials is given by,

$$P(N) = p^N \quad . \tag{10}$$

The binomial distribution is a special case of the multinomial distribution describing processes in which several results having fixed probabilities, $p_l$, $q_l$, for $l = 1,\ L$, are possible. Separate probabilities are given by the individual terms in the general multinomial expansion,

$$(p_1 + q_1 + ... + p_L + q_L)^N = \sum_{n_1,...,n_{L-1}=0}^{N} P(n_1, ..., n_{L-1}) = 1 \ , \tag{11}$$

as in the binomial case. The normal distribution is a special case of the binomial distribution when $N$ is very large and variables are not necessarily confined to integer values. The Poisson distribution is another special case of the binomial distribution when the number of events, $N$, is also large, but the incidence, $p$, is small.

2. Normal Distribution

The normal distribution is an analytic approximation to the binomial distribution when $N$ is very large, and $n$, the observed value (success or failure rate), is not confined to integer values, but ranges continuously,

$$-\infty \leq n \leq \infty \ . \tag{12}$$

Normal distributions thus apply to continuous observables, while binomial and Poisson distributions apply to discontinuous observables. Statistical theories of errors are ordinarily based on normal distributions.

For the same mean, $M = pN$, and variance, $s = pqN$, the normal distribution, $P$, written as a continuously varying function of $n$,

$$P(n) = \frac{1}{(2\pi s)^{1/2}} \, exp \, [-\ (n-M)^2/2s] \ , \tag{13}$$

is a good approximation to the binomial distribution in the range,

$$\frac{1}{N+1} < p < \frac{N}{N+1} \ , \tag{14}$$

and within three standard deviations of the mean,

$$pN - 3\ (pqN)^{1/2} \leq n \leq pN + 3\ (pqN)^{1/2} \ . \tag{15}$$

The distribution is normalized to one over the real infinite interval,

$$\int_{-\infty}^{\infty} P dn = 1 \ . \tag{16}$$

The probability that a normally distributed variable, $n$, is less than or equal to $b$ is,

$$P_<(b) = \int_{-\infty}^{b} P dn \ , \tag{17}$$

while the corresponding probability that $n$ is greater than or equal to $b$ is,

$$P_>(b) = \int_{b}^{\infty} P dn \ . \tag{18}$$

The normal distribution is extremely important in statistical theories of random variables. By the central limit theorem, the distribution of sample means of identically distributed random variables is approximately normal, regardless of the actual distribution of the individual variables.

3. Poisson Distribution

The Poisson distribution is a special case of the binomial distribution when $N$ becomes large, and $p$ is small, and certainly describes all discrete random processes whose probability of occurrence is small and constant. The Poisson distribution applies substantially to all observations made concerning the incidence of decompression

sickness in diving, that is, $p << 1$ as the desired norm. The reduction of the binomial distribution to the Poisson distribution follows from limiting forms of terms in the binomial expansion, that is, $P(n)$.

In the limit as $N$ becomes large, and $p$ is much smaller than one, we have,

$$\frac{N!}{(N-n)!} = N^n \quad , \tag{19}$$

$$q^m = (1-p)^{N-n} = exp\ (-pN) \quad , \tag{20}$$

and therefore the binomial probability reduces to,

$$P(n) = \frac{N^n p^n}{n!}\ exp\ (-pN) = \frac{M^n}{n!}\ exp\ (-M) \quad , \tag{21}$$

which is the discrete Poisson distribution. The mean, $M$, is given as before,

$$M = pN \tag{22}$$

and the variance, $s$, has the same value,

$$s = pN \quad , \tag{23}$$

because $q$ is approximately one. The cumulative probabilities, $P_>(n)$ and $P_<(n)$, are the same as those defined in the binomial case, a summation over discrete variable, $n$. It is appropriate to employ the Poisson approximation when $p \leq 0.10$, and $N \geq 10$ in trials. Certainly, from a numerical point of view, the Poisson distribution is easier to use than than binomial distribution. Computation of factorials is a lesser task, and bookkeeping is minimal for the Poisson case.

In addition to the incidence of decompression sickness, the Poisson distribution describes the statistical fluctuations in such random processes as the number of cavalry soldiers kicked and killed by horses, the disintegration of atomic nuclei, the emission of light quanta by excited atoms, and the appearance of cosmic ray bursts. It also applies to most rare diseases.

## PROBABILISTIC DECOMPRESSION

### Binomial Probabilities

Table 1 lists corresponding binomial decompression probabilities, $P(n)$, for 1% and 10% underlying incidence (99% and 90% nonincidence), yielding 0, 1, and 2 or more cases of decompression sickness. The underlying incidence, $p$, is the (fractional) average of hits.

As the number of trials increases, the probability of 0 or 1 occurrences drops, while the probability of 2 or more occurences increases. In the case of 5 dives, the probability might be as low as 5%, while in the case of 50 dives, the probability could be 39%, both for $p = 0.01$. Clearly, odds even percentages would require testing beyond 50 cases for an underlying incidence near 1%. Only by increasing the number of trials for fixed incidences can the probabilities be increased. Turning that around, a rejection procedure for 1 or more cases of decompression sickness at the 10% probability level requires many more than 50 dives. If we are willing to lower the confidence of the acceptance, or rejection, procedure, of course, the number of requisite trials drops. Table 1 also shows that the test practice of accepting an exposure schedule following 10 trials without incidence of decompression sickness is suspect, merely because the relative probability of nonincidence is high, near 35%.

Questions as to how safe are decompression schedules have almost never been answered satisfactorily. As seen, large numbers of binary events are required to reliably estimate the underlying incidence. One case of decompression sickness in 30 trials could result from an underlying incidence, $p$, bounded by .02 and .16 roughly. Tens more of trials are necessary to shrink those bounds.

Table 1. Probabilities Of Decompression Sickness For Underlying Incidences

| $N$ (dives) | $n$ (hits) | $P(n)$ $p=.01$ $q=.99$ | $P(n)$ $p=.10$ $q=.90$ |
|---|---|---|---|
| 5 | 0 | .95 | .59 |
| | 1 | .04 | .33 |
| | 2 or more | .01 | .08 |
| 10 | 0 | .90 | .35 |
| | 1 | .09 | .39 |
| | 2 or more | .01 | .26 |
| 20 | 0 | .82 | .12 |
| | 1 | .16 | .27 |
| | 2 or more | .02 | .61 |
| 50 | 0 | .61 | .01 |
| | 1 | .31 | .03 |
| | 2 or more | .08 | .96 |

Biological processes are highly variable in outcome. Formal correlations with outcome statistics are then generally requisite to validate models against data. Often, this correlation is difficult to firmly establish (couple of percent) with fewer than 1,000 trial observations, while ten percent correlations can be obtained with 30 trials, assuming binomial distributed probabilities. For decompression analysis, this works as a disadvantage, because often the trial space of dives is small. Not discounting the possibly small trial space, a probabilistic approach to the occurrence of decompression sickness is useful and necessary. One very successful approach, developed and tuned by Weathersby, and others for decompression sickness in diving, called maximum likelihood, applies theory or models to diving data and adjusts the parameters until theoretical prediction and experimental data are in as close agreement as possible.

Validation procedures require decisions about uncertainty. When a given decompression procedure is repeated with different subjects, or the same subjects on different occasions, the outcome is not constant. The uncertainty about the occurrence of decompression sickness can be quantified with statistical statements, though, suggesting limits to the validation procedure. For instance, after analyzing decompression incidence statistics for a set of procedures, a table designer may report that the procedure will offer an incidence rate below 5%, with 90% confidence in the statement. Alternatively, the table designer can compute the probability of rejecting a procedure using any number of dive trials, with the rejection criteria any arbitrary number of incidences. As the number of trials increases, the probability of rejecting a procedure increases for fixed incidence criteria. In this way, relatively simple statistical procedures can provide vital information as to the number of trials necessary to validate a procedure with any level of acceptable risk, or the maximum risk associated with any number of incidences and trials.

One constraint usually facing the statistical table designer is a paucity of data, that is, number of trials of a procedure. Data on hundreds of repetitions of a dive profile are virtually nonexistent, excepting bounce diving perhaps. As seen, some 30-50 trials are requisite to ascertain procedure safety at the 10% level. But 30-50 trials is probably asking too much, is too expensive, or generally prohibitive. In that case, the designer may try to employ global statistical measures linked to models in a more complex trial space, rather than a single profile trial space. Integrals of risk parameters, such as bubble number, supersaturation, separated phase, etc., over exposures in time, can be defined as probability measures for incidence of decompression sickness, and the maximum likelihood method then used to extract appropriate constants:

1. Maximum Likelihood

   We can never measure any physical variable exactly, that is, without error. Progressively more elaborate experimental or theoretical efforts only reduce the possible error in the determination. In extracting parameter estimates from data sets, it is necessary to also try to minimize the error (or data scatter) in the extraction process. A number of techniques are available to the analyst, including the well known maximum likelihood approach.

   The measure of any random occurrence, $p$, can be a complicated function of many parameters, $x = (x_k, k = 1, K)$, with the only constraint,

$$0 \leq p(x) \leq 1 \ , \tag{24}$$

for appropriate values of the set, $x$. The measure of nonoccurence, $q$, is then by conservation of probability,

$$q(x) = 1 - p(x) \ , \tag{25}$$

over the same range,

$$0 \leq q(x) \leq 1 \ . \tag{26}$$

Multivalued functions, $p(x)$, are often constructed, with specific form dictated by theory or observation over many trials or tests. In decompression applications, the parameters, $x$, may well be the bubble-nucleation rate, number of venous gas emboli, degree of supersaturation, amount of pressure reduction, volume of separated gas, ascent rate, or combinations thereof. Parameters may also be integrated in time in any sequence of events, as a global measure, though such measures are more difficult to analyze over arbitrary trial numbers.

The likelihood of any outcome, $\Phi$, of $N$ trials is the product of individual measures of the form,

$$\Phi(n) = p^n q^m = p^n (1-p)^m \ , \tag{27}$$

given $n$ cases of decompression sickness and $m$ cases without decompression sickness, and,

$$n + m = N \ . \tag{28}$$

The natural logarithm of the likelihood, $\Psi$, is easier to use in applications, and takes the form,

$$\Psi = \ln \ \Phi = n \ \ln \ p + m \ \ln \ (1-p) \ , \tag{29}$$

and is maximized when,

$$\frac{\partial \Psi}{\partial p} = 0 \ . \tag{30}$$

In terms of the above, we then must have,

$$\frac{n}{p} - \frac{m}{1-p} = 0 \ , \tag{31}$$

trivially requiring,

$$p = \frac{n}{n+m} = \frac{n}{N} \ , \tag{32}$$

$$1 - p = q = \frac{m}{n+m} = \frac{m}{N} \ . \tag{33}$$

Thus, the likelihood function is maximized when $p$ is the actual incidence rate, and $q$ is the actual nonincidence rate. The multivalued probability functions, $p(x)$, generalize in the maximization process according to,

$$\frac{\partial \Psi}{\partial p} = \sum_{k=1}^{K} \frac{\partial \Psi}{\partial x_k} \frac{\partial x_k}{\partial p} = 0 \ , \tag{34}$$

satisfied when,

$$\frac{\partial \Psi}{\partial x_k} = 0 \ \ for \ k = 1, \ K \ . \tag{35}$$

In application, such constraints are most easily solved on computers, with analytical or numerical methods.

In dealing with a large number of decompression procedures, spanning significant range in depth, time, and environmental factors, an integrated approach to maximum likelihood and risk is necessary. Integral measures, $p(x,t)$ and $q(x,t)$, can be defined over assumed decompression risk, $\zeta(x,t)$,

$$p(x,t) = 1 - exp \ \left[ - \int_0^t \ \zeta(x,t') dt' \right] \ , \tag{36}$$

$$q(x,t) = exp\ \left[ -\int_0^t\ \zeta(x,t')dt' \right]\ , \tag{37}$$

with $t'$ any convenient time scale, and $\zeta$ any assumed risk, such as bubble number, saturation, venous emboli count, etc. as mentioned. Employing $p(x,t)$ and $q(x,t)$ in the likelihood function, and then maximizing according to the data, permits maximum likelihood estimation of $\zeta(x,t)$. Such an approach can be employed in decompression table fabrication, yielding good statistical estimates on incidence rates as a function of exposure factors.

2. Saturation Bends Probability

   Many factors contribute to bends susceptibility. Age, obesity, temperature, physical condition, alcohol, and cigarettes are a few. Whatever the contributing factors, the distribution of bends depths for saturation exposures has been characterized in terms of the saturation tension, $Q$, and ambient pressure, $P$, by Hills. This characterization is not only of academic interest, but is also useful in assigning formal risk to decompression formats.

   The distribution of saturation bends depths, $\chi$, fits a Weibull function. This is true for all breathing mixtures, nitrox, heliox, trimix, etc. If cumulative fraction of air bends cases up to $G$ is $\chi$, the survivor fraction, $1-\chi$, satifies,

$$\ln\ (1-\chi) = -\left[ \frac{G-14.3}{25.1} \right]^{4.73} \tag{38}$$

   for cumulative bends probability, $\chi$, the usual integral over bends risk, $\zeta$, as a function of gradient, $G$,

$$\chi = \int_0^G \zeta(G')dG' \tag{39}$$

   with saturation bends gradient, $G$, measured in $fsw$,

$$G = Q - P \tag{40}$$

   As the gradient grows, the survivor function approaches zero exponentially. The smallest bends gradient is 14.3 $fsw$, which can be contrasted with the average value of 26.5 $fsw$. The root mean square gradient is 27.5 $fsw$. At 27 $fsw$, the survivor fraction is 0.96, while 67% of survivors fall in the range, $26.5 \pm 7.6$ $fsw$, with 7.6 $fsw$ the standard deviation. For gas mixtures other than air, the general form is given by,

$$\ln\ (1-\chi) = -\varepsilon \left[ \frac{(P_f - 20.5)}{(P_i - 33.0)} - \frac{1}{f_i} \right]^{\delta} \tag{41}$$

   where $f_i$ is the total volume fraction of inert breathing gases, for $G = P_f - P_i$, and with $\varepsilon$, $\delta$ constants.

   The efficiency of the Weibull distribution in providing a good fit to the saturation data is not surprising. The Weibull distribution enjoys success in reliability studies involving multiplicities of fault factors. It obviously extends to any set of hyperbaric or hypobaric exposure data, using any of the many parameter risk variables described above.

3. Table And Profile Risks

   A global statistical approach to table fabrication consists of following a risk measure, or factor $p$, throughout and after sets of exposures, tallying the incidence of DCI, and then applying maximum likelihood to the risk integral in time, extracting any set of risk constants optimally over all dives in the maximization procedure. In analyzing air and helium data, Weathersby assigned risk as the difference between tissue tension and ambient pressure divided by ambient pressure. One tissue was assumed, with time constant ultimately fixed by the data in ensuing maximum likelihood analysis. The measure of nonincidence, $q$, was taken to be the exponential of risk integrated over all exposure time,

$$q(\kappa,\tau) = exp\ \left[ -\int_0^\infty\ \zeta(\kappa,\tau,t')dt' \right]\ , \tag{42}$$

$$\zeta(\kappa,\tau,t') = \kappa\,\frac{p(t') - p_a}{p_a} \quad , \tag{43}$$

with $\kappa$ a constant determined in the likelihood maximization, $p_a$ ambient pressure, and $p(t')$ the instantaneous Haldane tension for tissue with halftime, $\tau$, also determined in the maximization process, corresponding to arbitrary tissue compartments for the exposure data. Other more complex likelihood functions can also employed, for instance, the separated phase volume according to the varying permeability and reduced gradient bubble models,

$$\zeta(\kappa,\xi,\tau,t') = \kappa\Lambda(t')G(t') \quad , \tag{44}$$

$$\Lambda(t') \;= \left[1 - \frac{r(t')}{\xi}\right] \quad , \tag{45}$$

with $\Lambda$ the permissible bubble excess, $r$ the bubble radius, $G$ the bubble diffusion gradient (dissolved-free gas), and $\kappa$ and $\xi$ constants determined in the fit maximization of the data. Another risk possibility is the tissue ratio,

$$\zeta(\kappa,\tau,t') = \kappa\,\frac{p(t')}{p_a} \quad , \tag{46}$$

a measure of interest in altitude diving applications.

Hundreds of air dives were analyzed using this procedure, permitting construction of decompression schedules with 95% and 99% confidence (5% and 1% bends probability). These tables were published by US Navy investigators, and Table 2 tabulates the corresponding nonstop time limits ($p = 0.05, 0.01$), and also includes the standard US Navy (Workman) limits for comparison. Later re-evaluations of the standard set of nonstop time limits estimate a probability rate of 1.25% for the limits. In actual usage, the incidence rates are below 0.001%, because users do not dive to the limits generally.

Table 2. Nonstop Time Limits For 1% And 5% DCI Probability

| depth $d\ (fsw)$ | nonstop limit $t_n\ (min)$ $p = .05$ | nonstop limit $t_n\ (min)$ $p = .01$ | nonstop limit $t_n\ (min)$ US Navy |
|---|---|---|---|
| 30 | 240 | 170 | |
| 40 | 170 | 100 | 200 |
| 50 | 120 | 70 | 100 |
| 60 | 80 | 40 | 60 |
| 70 | 80 | 25 | 50 |
| 80 | 60 | 15 | 40 |
| 90 | 50 | 10 | 30 |
| 100 | 50 | 8 | 25 |
| 110 | 40 | 5 | 20 |
| 120 | 40 | 5 | 15 |
| 130 | 30 | 5 | 10 |

For the past 10-15 years, a probabilistic approach to assessing risk in diving has been in vogue. Sometimes this can be confusing, or misleading, since definitions or terms, as presented, are often mixed. Also confusing are risk estimates varying by factors of 10 to 1,000, and distributions serving as basis for analysis, also varying in size by the same factors. So, before continuing with a risk analysis of recreational profiles, a few comments are germane.

Any set of statistical data can be analyzed directly, or sampled in smaller chunks. The smaller sets (samples) may or may not reflect the parent distribution, but if the analyst does his work correctly, samples reflecting the parent distribution can be extracted for study. In the case of dive profiles, risk probabilities extracted from sample profiles try to reflect the incidence rate, $p$, of the parent distribution ($N$ profiles, and $p$ underlying DCI rate). The incidence rate, $p$, is the most important metric, followed by the shape of the distribution in total as measured by

the variance, $s$. For smaller sample of profile size, $K < N$, we have mean incidences, $Q$, for sample incidence rate, $r$,

$$Q = rK \tag{47}$$

and variance, $v$,

$$v = r(1-r)K \tag{48}$$

By the central limit theorem, the distribution of sample means, $Q$, is normally distributed about parent (actual) mean, $M$, with variance, $v = s/K$. Actually, the distribution of sample means, $Q$, is normally distributed no matter what the distribution of samples. This important fact is the basis for error estimation with establishment of confidence intervals, $\chi$, for $r$, with estimates denoted, $r_{\pm}$,

$$r_{\pm} = r \pm \chi \left[\frac{s}{K}\right]^{1/2} \tag{49}$$

$$0 < \chi < 1 \tag{50}$$

The sample binomial probability, $B(k)$, is analogously,

$$B(k) = \frac{K!}{k!\ j!} r^k (1-r)^j \tag{51}$$

with $k + j = K$, for $k$ the number of DCI hits, normalized,

$$\sum_{k=1}^{K} B(k) = 1 \tag{52}$$

and with property, if $K \to \infty$, then $B(k) \to 0$, when, $r << 1$.

For example, if 12 cases of DCI are reported in a parent set of 7,896 profiles, then,

$$N = 7896 \tag{53}$$

$$p = \frac{12}{7896} = .0015 \tag{54}$$

Smaller samples might be used to estimate risk, via sample incidence, $r$, with samples possibly chosen to reduce computer processing time, overestimate $p$ for conservancy sake, focus on a smaller subregion of profiles, or any other reason. Thus, one might nest all 12 DCI incidence profiles in a smaller sample, $K = 1,000$, so that the sample risk, $r = 12/1,000 = 0.012$, is larger than $p$. Usually though the analyst wishes to mirror the parent distribution in the sample. If the parent is a set of benign, recreational, no decompression, no multiday dive profiles, and the sample mirrors the parent, then both risks, $p$ and $r$, are are reasonably true measures of actual risk associated with recreational diving. If sample distributions chosen are not representative of the class of diving performed, risk estimates are not trustworthy. For instance, if a high risk set of mixed gas decompression profiles were the background against which recreational dive profiles were compared, all estimates would be skewed and faulty (actually underestimated in relative risk, and overestimated in absolute risk). For this parent set, $N$ is large, $p$ is small, with mean, $M = pN = 0.0015 \times 7896 = 12$, and the applicable binomial statistics smoothly transition to Poisson representation, convenient for logarithmic and covariant numerical analysis (on a computer). Additionally, any parent set may be a large sample of a megaset, so that $p$ is itself an estimate of risk in the megaset.

Turns out that our parent distribution above is just that, a subset of larger megaset, namely, the millions and millions of recreational dives performed and logged over the past 30 years, or so. The above set of profiles was collected in training and vacation diving scenarios. The set is recreational (no decompression, no multiday,

light, benign) and representative, with all the distribution metrics as listed. For reference and perspective, sets of recreational profiles collected by others (Gilliam, NAUI, PADI, YMCA, DAN) are similar in context, but larger in size, $N$, and smaller in incidence rate, $p$. Data and studies reported by many sources quote, $N > 1,000,000$, with, $p < 0.00001 = 0.001\%$. Obviously our set has higher rate, $p$, though still nominally small, but the same shape. So our estimates will be liberal (overestimate risk).

To perform risk analysis, a risk estimator need be employed. For diving, dissolved gas and phase estimators are useful. Two, detailed earlier, are used here. First is the dissolved gas supersaturation ratio, historically coupled to Haldane models, $\phi$,

$$\phi = \kappa \, \frac{p - \lambda p_a}{p_a} \tag{55}$$

and second, $\psi$, is the separated phase, invoked by phase models,

$$\psi = \gamma \left[ 1 - \frac{r}{\xi} \right] G \tag{56}$$

For simplicity, the asymptotic exposure limit is used in the likelihood integrals for both risk functions,

$$1 - r(\kappa, \lambda) = exp \, \left[ - \int_0^\infty \phi(\kappa, \lambda, t) dt \right] \tag{57}$$

$$1 - r(\gamma, \xi) = exp \, \left[ - \int_0^\infty \psi(\gamma, \xi, t) dt \right] \tag{58}$$

with $hit - no \; hit$, likelihood function, $\Omega$, of form,

$$\Omega = \prod_{k=1}^{K} \Omega_k \tag{59}$$

$$\Omega_k = r_k^{\delta_k} (1 - r_k)^{1 - \delta_k} \tag{60}$$

where, $\delta_k = 0$ if DCI does not occur in profile, $k$, or, $\delta_k = 1$ if DCI does occur in profile, $k$. To estimate $\kappa$, $\lambda$, $\gamma$, and $\xi$ in maximum likelihood, a modified Levermore-Marquardt algorithm is employed (*SNLSE*, Common Los Alamos Applied Mathematical Software Library), just a nonlinear least squares data fit to an arbitrary function (minimization of variance over $K$ datapoints here), with $L1$ error norm. Additionally, using a random number generator for profiles across 1,000 parallel SMP (Origin 2000) processors at LANL, we construct 1,000 subsets, with $K = 2,000$ and $r = 0.006$, for separate likelihood regression analysis, averaging $\kappa$, $\lambda$, $\gamma$, and $\xi$ by weighting the inverse variance.

For recreational diving, both estimators are roughly equivalent, because little dissolved gas has separated into free phases (bubbles). Analysis shows this true for all cases examined, in that estimated risks for both overlap at the 95% confidence level. The only case where dissolved gas and phase estimators differ (slightly here) is within repetitive diving profiles. The dissolved gas estimator cues on gas buildup in the slow tissue compartments (staircasing for repets within an hour or two), while the phase estimator cues on bubble gas diffusion in the fast compartments (dropping rapidly over hour time spans). This holding true within all recreational diving distributions, we proceed to the risk analysis.

Nonstop limits (NDLs), denoted $t_n$ as before, from the US Navy, PADI, and NAUI Tables, and those employed by the Oceanic decometer provide a set for comparison of relative DCI risk. Listed below in Table 3 are the NDLs and corresponding risks, $r_n$, for the profile, assuming ascent and descent rates of 60 $fsw/min$ (no safety stops). Haldane and RGBM estimates vary little for these cases, and only the phase estimates are included.

Table 3. Risk Estimates For Various NDLs.

| $d$ $(fsw)$ | USN $t_n$ $(min)$ | PADI $t_n$ $(min)$ | NAUI $t_n$ $(min)$ | Oceanic $t_n$ $(min)$ |
|---|---|---|---|---|
| 35 | 310 (4.3%) | 205 (2.0%) | | 181 (1.3%) |
| 40 | 200 (3.1%) | 140 (1.5%) | 130 (1.4%) | 137 (1.5%) |
| 50 | 100 (2.1%) | 80 (1.1%) | 80 (1.1%) | 80 (1.1%) |
| 60 | 60 (1.7%) | 55 (1.4%) | 55 (1.4%) | 57 (1.5%) |
| 70 | 50 (2.0%) | 40 (1.2%) | 45 (1.3%) | 40 (1.2%) |
| 80 | 40 (2.1%) | 30 (1.3%) | 35 (1.5%) | 30 (1.3%) |
| 90 | 30 (2.1%) | 25 (1.5%) | 25 (1.5%) | 24 (1.4%) |
| 100 | 25 (2.1%) | 20 (1.3%) | 22 (1.4%) | 19 (1.2%) |
| 110 | 20 (2.2%) | 13 (1.1%) | 15 (1.2%) | 16 (1.3%) |
| 120 | 15 (2.0%) | 13 (1.3%) | 12 (1.2%) | 13 (1.3%) |
| 130 | 10 (1.7%) | 10 (1.7%) | 8 (1.3%) | 10 (1.7%) |

Risks are internally consistent across NDLs at each depth, and agree with the US Navy assessments in Table 2. Greatest underlying and binomial risks occur in the USN shallow exposures. The PADI, NAUI, and Oceanic risks are all less than 2% for this set, thus binomial risks for single DCI incidence are less than 0.02%. PADI and NAUI have reported that field risks ($p$) across all exposures are less than 0.001%, so considering their enviable track record of diving safety, our estimates are liberal. Oceanic risk estimates track as the PADI and NAUI risks, again, very safely.

Next, the analysis is extended to profiles with varying ascent and descent rates, safety stops, and repetitive sequence. Table 4 lists nominal profiles (recreational) for various depths, exposure and travel times, and safety stops at 5 *msw*. DCI estimates, $r$, are tabulated for both dissolved gas supersaturation ratio and bubble numebr excess risk functions.

Table 4. Dissolved And Separated Phase Risk Estimates For Nominal Profiles.

| profile $(depth/min)$ | descent rate $(msw/min)$ | ascent rate $(msw/min)$ | safety stop $(depth/min)$ | $r$ RGBM | $r$ ZHL |
|---|---|---|---|---|---|
| 14 *msw*/38 *min* | 18 | 9 | 5 *msw*/3 *min* | .0034 | .0062 |
| 19 *msw*/38 *min* | 18 | 9 | 5 *msw*/3 *min* | .0095 | .0110 |
| 37 *msw*/17 *min* | 18 | 9 | 5 *msw*/3 *min* | .0165 | .0151 |
| 18 *msw*/31 *min* | 18 | 9 | 5 *msw*/3 *min* | .0063 | .0072 |
| | 18 | 9 | | .0088 | .0084 |
| | 18 | 18 | | .0101 | .0135 |
| | 18 | 18 | 5 *msw*/3 *min* | .0069 | .0084 |
| 17 *msw*/32 *min* SI 176 *min* | 18 | 9 | 5 *msw*/3 *min* | | |
| 13 *msw*/37 *min* SI 174 *min* | 18 | 9 | 5 *msw*/3 *min* | | |
| 23 *msw*/17 *min* | 18 | 18 | 5 *msw*/3 *min* | .0127 | .0232 |

The ZHL (Buhlmann) NDLs and staging regimens are widespread across decompression meters presently, and are good representation for Haldane risk analysis. The RGBM is newer and more modern (and more physically correct), and is coming online in decometers and associated software. For recreational exposures, the RGBM collapses to a Haldane dissolved gas algorithm. This is reflected in the risk estimates above, where estimates for both models differ little.

Simple comments hold for the analyzed profile risks. The maximum relative risk is 0.0232 for the 3 dive repetitive sequence according to the Haldane dissolved risk estimator. This translates to 0.2% binomial risk, which is comparable to the maximum NDL risk for the PADI, NAUI, and Oceanic NDLs. Again, this type of dive profile is common, practiced daily on liveaboards, and benign. According to Gilliam, the absolute incidence rate for this type of diving is less than 0.02%. Again, our figures overestimate risk.

Effects of slower ascent rates and safety stops are noticeable at the 0.25% to 0.5% level in relative surfacing risk. Safety stops at 5 *m* for 3 *min* lower relative risk an average of 0.3%, while reducing the ascent rate from 18 $msw/min$ to 9 $msw/min$ reduces relative risk an average of 0.35%.

Staging, NDLs, and other contraints imposed by decometer algorithms are entirely consistent with acceptable and safe recreational diving protocols. The estimated absolute risk associated across all ZHL NDLs and diver staging regimens analyzed herein is less than 0.232%, and is probably much less in actual practice. That is, we use $p = 0.006$, and much evidence suggests $p < 0.0001$, some ten times safer.

Implicit in such formulations of risk tables are the assumptions that a decompression stress is more likely to produce symptoms if it is sustained in time, and that large numbers of separate events may culminate in the same probability after time integration. Though individual schedule segments may not be replicated enough to offer total statistical validation, categories of predicted safety can always be grouped within subsets of corroborating data. Since the method is general, any model parameter or meaningful index, properly normalized, can be applied to decompression data, and the full power of statistical methods employed to quantify overall risk. While powerful, such statistical methods are neither deterministic nor mechanistic, and cannot predict on first principles. But as a means to table fabrication with quoted risk, such approaches offer attractive pathways for analysis.

**Model Validation**

Validation procedures for schedules and tables can be quantified by a set of procedures based on statistical decompression analysis:

1. select or construct a measure of decompression risk, or a probabilistic model;
2. evaluate as many dives as possible, especially dives similar in exposure time, depth, and environmental factors;
3. conduct limited testing if no data is available;
4. apply the model to the data using maximum likelihood;
5. construct appropriate schedules or tables using whatever incidence of decompression sickness is acceptable;
6. release and then collect profile statistics for final validation and tuning.

Questions of what risk is acceptable to the diver vary. Sport and research divers would probably opt for very small risk (0.01% or less), while military and commercial divers might live with higher risk (1%), considering the nearness of medical attention in general. Many factors influence these two populations, but fitness and acclimatization levels would probably differ considerably across them. While such factors are difficult to fold into any table exercise or analysis, the simple fact that human subjects in dive experiments exhibit higher incidences during testing phases certainly helps to lower the actual incidence rate in the field, noted by Bennett and Lanphier.

**Dissolved And Free Phase Models**

Certainly there is considerable latitude in model assumptions, and many plausible variants on a theme. Many models are correlated with diving exposure data, using maximum likelihood to fit parameters or other valid statistical approaches, but not all. Most have been applied to profiles outside of tested ranges, when testing has been performed, in an obvious extrapolation mode. Sometimes the extrapolations are valid, other times not. RPs represent just that sort of extrapolation process for the bulk of these models, since RP testing has not been extensive. So, now consider the 6 models:

1. Perfusion Limited Model (PLM)

   Exchange of inert gas, controlled by blood flow rates across regions of varying concentration, is driven by the gas gradient, that is, the difference between the arterial blood tension, $p_a$, and the instantaneous tissue tension, $p$. This behavior is modeled in time, $t$, by classes of exponential response functions, bounded by $p_a$ and the initial value of $p$, denoted $p_i$. These multitissue functions satisfy a differential perfusion rate equation,

   $$\frac{\partial p}{\partial t} = -\lambda(p - p_a) \tag{61}$$

   and take the form, tracking both dissolved gas buildup and elimination symmetrically,

   $$p - p_a = (p_i - p_a)\ exp\ (-\lambda\, t) \tag{62}$$

   $$\lambda = \frac{.693}{\tau} \tag{63}$$

   with perfusion constant, $\lambda$, linked to tissue halftime, $\tau$. Compartments with 1, 2.5, 5, 10, 20, 40, 80, 120, 180, 240, 360, 480, and 720 minute halftimes, $\tau$, are employed, and are independent of pressure.

   In a series of dives or multiple stages, $p_i$ and $p_a$ represent extremes for each stage, or more precisely, the initial tension and the arterial tension at the beginning of the next stage. Stages are treated sequentially, with finishing tensions at one step representing initial tensions for the next step, and so on. To maximize the rate of uptake or elimination of dissolved gases the *gradient*, simply the difference between $p_i$ and $p_a$, is maximized by pulling the diver as close to the surface as possible. Exposures are limited by requiring that the tissue tensions never exceed $M$, written,

   $$M = M_0 + \Delta M\ d \tag{64}$$

   at depth, $d$, for $\Delta M$ the change per unit depth. A set of $M_0$ and $\Delta M$ are listed in Table 5. In absolute units, the corresponding critical gradient, $G$, and critical ratio, $R$, are given by,

   $$G = \frac{M}{.79} - P \tag{65}$$

   $$R = \frac{M}{P} \tag{66}$$

   with $P$ ambient pressure.

Table 5. Classical US Navy Surfacing Ratios And Critical Tensions

| halftime $\tau\ (min)$ | critical ratio $R_0$ | critical tension $M_0\ (fsw)$ | tension change $\Delta M$ |
|---|---|---|---|
| 5 | 3.15 | 104 | 2.27 |
| 10 | 2.67 | 88 | 2.01 |
| 20 | 2.18 | 72 | 1.67 |
| 40 | 1.76 | 58 | 1.34 |
| 80 | 1.58 | 52 | 1.26 |
| 120 | 1.55 | 51 | 1.19 |

At altitude, some critical tensions have been correlated with actual testing, in which case, the depth, $d$, is defined in terms of the absolute pressure,

$$d = P - 33 \tag{67}$$

with absolute pressure, $P$, at altitude, $z$, given by ($fsw$),

$$P = 33\ exp\ (-0.0381z) = 33\ \alpha^{-1} \tag{68}$$

$$\alpha = exp\ (0.0381z) \tag{69}$$

and $z$ in multiples of 1000 $feet$. However, in those cases where the critical tensions have not been tested nor extended to altitude, an exponentially decreasing extrapolation scheme, called *similarity*, has been employed. Extrapolations of critical tensions, below $P = 33\ fsw$, then fall off more rapidly then in the linear case. The similarity extrapolation holds the ratio, $R = M/P$, constant at altitude. Denoting an equivalent sea level depth, $\delta$, at altitude, $z$, one has for an excursion to depth $d$,

$$\frac{M(d)}{d + 33\alpha^{-1}} = \frac{M(\delta)}{\delta + 33} \tag{70}$$

so that the equality is satisfied when,

$$\delta = \alpha d \tag{71}$$

$$M(\delta) = \alpha M(d). \tag{72}$$

Considering the minimum surface tension pressure of bubbles, $G^{min}$ (near $10\,fsw$), as a limit point, the similarity extrapolation should be limited to 10,000 $feet$ in elevation, and neither for decompression, nor heavy repetitive diving.

As described previously, depth-time exposures are often limited by a law of the form,

$$dt_n^{1/2} = H \tag{73}$$

with $t_n$ the nonstop time limit, and $400 \leq H \leq 500\ fsw\ min^{1/2}$. One can obtain the corresponding tissue constant, $\lambda$, controlling the exposure at depth $d$, for nonstop time $t_n$, by differentiating the tissue equation with respect to depth, $d$, and setting the result to zero. With $p_a = .79(d + 33)$ at sea level, there results,

$$1 - exp\ (-\lambda t_n)(1 + 2\,\lambda t_n) = 0. \tag{74}$$

Corresponding critical tensions, $M$, are then easily obtained using $d$, $\lambda$, and $t_n$. In the above case, the transcendental equation is satisfied when,

$$\lambda t_n = 1.25 \tag{75}$$

Time remaining before a stop, time at a stop, or surface interval before flying can all be obtained by inverting the tissue equation. Denoting the appropriate critical tension at some desired stage, $M$, and the instantaneous tension at that time, $p$, at stage, $p_a$, the time remaining, $t_r$, follows from,

$$t_r = \frac{1}{\lambda} \ln \left[ \frac{p - p_a}{M - p_a} \right] \tag{76}$$

for each compartment, $\lambda$. Obviously, the smallest $t_r$ controls the ascent.

The PLM forms the basis for most table and meter algorithms, and has been extensively tested for different profiles and gas loadings. First tests on the modern approaches were performed by the US Navy.

2. Diffusion Limited Model (DLM)

Exchange of inert gas, controlled by diffusion across regions of varying concentration, is also driven by the local gradient. As before, denoting the arterial blood tension, $p_a$, and instantaneous tissue tension, $p$, the gas diffusion equation takes the form in one dimensional planar geometry,

$$D \frac{\partial^2 p}{\partial x^2} = \frac{\partial p}{\partial t} \tag{77}$$

with $D$ a single diffusion coefficient appropriate to the media. Using standard techniques of separation of variables, with $\omega^2$ the separation constant (eigenvalue), the solution is written,

$$p - p_a = (p_i - p_a) \sum_{n=1}^{\infty} W_n \, sin \, (\omega_n x) \, exp \, (-\omega_n^2 D t) \tag{78}$$

assuming at the left tissue boundary, $x = 0$, we have $p = p_a$, and with $W_n$ a set of constants obtained from the initial condition. First, requiring $p = p_a$ at the right tissue boundary, $x = l$, yields,

$$\omega_n = \frac{n\pi}{l} \tag{79}$$

for all $n$. Then, taking $p = p_i$ at $t = 0$, multiplying both sides of the diffusion solution by $sin \, (\omega_m x)$, integrating over the tissue zone, $l$, and collecting terms gives,

$$W_{2n} = 0 \tag{80}$$

$$W_{2n-1} = \frac{4}{(2n-1)\pi} \tag{81}$$

Averaging the solution over the tissue domain eliminates spatial dependence, that is $sin \, (\omega_n x)$, from the solution, giving a bulk response,

$$p - p_a = (p_i - p_a) \sum_{n=1}^{\infty} \frac{8}{(2n-1)^2 \, \pi^2} exp \, (-\omega_{2n-1}^2 D t). \tag{82}$$

The expansion resembles a weighted sum over $effective$ tissue compartments with time constants, $\omega_{2n-1}^2 D$, determined by diffusivity and boundary conditions.

Diffusion models fit the time constant, $K$,

$$\kappa = \pi^2 D l^2 \tag{83}$$

to exposure data, with a typical value employed by the Royal Navy given by,

$$\kappa = 0.007928 \, min^{-1}. \tag{84}$$

The approach is aptly single tissue, with equivalent tissue halftime, $\tau_D$,

$$\tau_D = \frac{.693}{\kappa} = 87.5\ min \tag{85}$$

close to the US Navy 120 *minute* compartment used to control saturation, decompression, and repetitive diving. Corresponding critical tensions in the bulk model, take the form,

$$M = \frac{709\ P}{P + 404} \tag{86}$$

falling somewhere between fixed gradient and multitissue values. At the surface, $M = 53\ fsw$, while at $200\,fsw$, $M = 259\ fsw$. A critical gradient, $G$, satisfies,

$$G = \frac{M}{.79} - P = \frac{P\ (493 - P)}{(P + 404)}. \tag{87}$$

The limiting features of bulk diffusion can be gleaned from an extension of the above slab model in the limit of thick tissue region, that is, $l \to \infty$. Replacing the summation over $n$ with an integral as $l \to \infty$, we find

$$p - p_a = (p_i - p_a)\ \bar{erf}\ [l/(4Dt)^{1/2}] \tag{88}$$

with $\bar{erf}$ the average value of the $error - function$ over $l$, having the limiting form (Abramowitz and Stegun),

$$\bar{erf}\ [l/(4Dt)^{1/2}] = 1 - (4Dt)^{1/2} l \pi^{1/2} \tag{89}$$

for short times, and

$$\bar{erf}\ [l/(4Dt)^{1/2}] = \frac{l}{(4\pi Dt)^{1/2}} \tag{90}$$

for long times.

Unlike the perfusion case, the diffusion solution, consisting of a sum of exponentials in time, cannot be formally inverted to yield time remaining, time at a stop, nor time before flying. Such information can only be obtained by solving the equation numerically, that is, with computer or hand calculator for given $M$, $p$, and $p_a$.

If we wrap the above planar geometry around into a hollow cylinder of inner radius, $a$, and outer radius, $b$, we generate Krogh geometry. The hollow cylindrical model retains all the features of the planar model, and additionally includes curvature for small $a$ and $b$, with $l = b - a$ from before. Assigning the same boundary conditions at $a$ and $b$, namely, the tissue tension, $p$, equals the arterial tension, $p_a$, writing the diffusion equation in radial cylindrical coordinates,

$$D\frac{\partial^2 p}{\partial r^2} + \frac{D}{r}\frac{\partial p}{\partial r} = \frac{\partial p}{\partial t} \tag{91}$$

and solving yields,

$$p - p_a = (p_i - p_a) \sum_{n=1}^{\infty} X_n\ U_0(\varepsilon_n r)\ exp\ (-\varepsilon_n^2 Dt) \tag{92}$$

with $X_n$ a constant satisfying initial conditions, $U_0$ the cylinder functions (Abramowitz and Stegun), and $\varepsilon_n$ the eigenvalues satisfying,

$$U_0(\varepsilon_n a) = \frac{\partial U_0(\varepsilon_n b/2)}{\partial r} = 0 \tag{93}$$

Averaging over the tissue region, $a \le r \le b$, finally gives,

$$p - p_a = (p_i - p_a)\ \frac{4}{(b/2)^2 - a^2} \sum_{n=1}^{\infty} \frac{1}{\varepsilon_n^2}\ \frac{J_1^2(\varepsilon_n b/2)}{J_0^2(\varepsilon_n a) - J_1^2(\varepsilon_n b/2)}\ exp\ (-\varepsilon_n^2 Dt) \tag{94}$$

with $J_1$ and $J_0$ Bessel functions, order 1 and 0. Typical vascular parameters are bounded by,

$$0 < a \leq 4 \; microns \tag{95}$$

$$10 \leq b \leq 32 \; microns. \tag{96}$$

The DLM was introduced and tested extensively by the Royal Navy, roughly on the same time scales as the US Navy tested the PLM.

3. Thermodynamic Model (TM)

The thermodynamic model couples both the tissue diffusion and blood perfusion equations. Cylindrical symmetry is assumed in the model. From a boundary vascular zone of thickness, $a$, gas diffuses into the extended extravascular region, bounded by $b$. The radial diffusion equation is given by,

$$D\frac{\partial^2 p}{\partial r^2} + \frac{D}{r}\frac{\partial p}{\partial r} = \frac{\partial p}{\partial t} \tag{97}$$

with the tissue tensions, $p$, equal to the venous tensions, $p_v$, at the vascular interfaces, $a$ and $b$. The solution to the tissue diffusion equation is given previously,

$$p - p_v = (p_i - p_v)\ \frac{4}{(b/2)^2 - a^2}\ \sum_{n=1}^{\infty}\ \frac{1}{\varepsilon_n^2}\ \frac{J_1^2(\varepsilon_n b/2)}{J_0^2(\varepsilon_n a) - J_1^2(\varepsilon_n b/2)}\ exp\ (-\varepsilon_n^2 D t) \tag{98}$$

with $\varepsilon_n$ eigenvalue roots of the boundary conditions,

$$J_0(\varepsilon_n a)\ Y_1(\varepsilon_n b/2) - Y_0(\varepsilon_n a)\ J_1(\varepsilon_n b/2) = 0 \tag{99}$$

for $J$ and $Y$ Bessel and Neumann functions, order 1 and 0. Perfusion limiting is applied as a boundary condition through the venous tension, $p_v$, by enforcing a mass balance across both the vascular and cellular regions at $a$,

$$\frac{\partial p_v}{\partial t} = -\kappa(p_v - p_a) - \frac{3}{a}S_p D\left[\frac{\partial p}{\partial r}\right]_{r=a} \tag{100}$$

with $S_p$ the ratio of cellular to blood gas solubilities, $\kappa$ the perfusion constant, and $p_a$ the arterial tension. The coupled set relate tension, gas flow, diffusion and perfusion, and solubility in a complex feedback loop.

The thermodynamic trigger point for decompression sickness is the volume fraction, $\chi$, of separated gas, coupled to mass balance. Denoting the separated gas partial pressure, $P_{N_2}$, under worse case conditions of zero gas elimination upon decompression, the separated gas fraction is estimated,

$$\chi\ P_{N_2} = S_c\ (p - P_{N_2}) \tag{101}$$

with $S_c$ the cellular gas solubility. The separated nitrogen partial pressure, $P_{N_2}$ is taken up by the inherent unsaturation, and given by ($fsw$),

$$P_{N_2} = P + 3.21 \tag{102}$$

in the original Hills formulation, but other estimates have been employed. Mechanical fluid injection pain, depending on the injection pressure, $\delta$, can be related to the separated gas fraction, $\chi$, through the tissue modulus, $K$,

$$K\chi = \delta \tag{103}$$

so that a decompression criteria requires,

$$K\chi \leq \delta \tag{104}$$

with $\delta$ in the range, for $K = 3.7 \times 10^4$ *dyne* $cm^{-2}$,

$$0.34 \leq \delta \leq 1.13 \ fsw. \tag{105}$$

Identification of the separated phase volume as a critical indicator is a significant development in decompression theory.

The TM has been applied to computational studies of air and helium deep saturation data, and was the first model to suggest deep stops to control bubble growth.

4. Varying Permeability Model (VPM)

The critical radius, $r_i$, at fixed pressure, $P_0$, represents the cutoff for growth upon decompression to lesser pressure. Nuclei larger than $r_i$ will all grow upon decompression. Additionally, following an initial compression, $\Delta P = P - P_0$, a smaller class of micronuclei of critical radius, $r$, can be excited into growth with decompression. If $r_i$ is the critical radius at $P_0$, then, the smaller family, $r$, excited by decompression from $P$, obeys,

$$\frac{1}{r} = \frac{1}{r_i} + \frac{\Delta P}{158} \tag{106}$$

with $\Delta P$ measured in $fsw$, and $r$ in *microns*. Table 6 lists critical radii, $r$, excited by sea level compressions ($P_0 = 33 \ fsw$), assuming $r_i = .8$ *microns*. Entries also represent the equilibrium critical radius at pressure, $P$.

Table 6. Micronuclei Excitation Radii

| pressure $P$ $(fsw)$ | excitation radius $r_s$ $(\mu m)$ | pressure $P$ $(fsw)$ | excitation radius $r_s$ $(\mu m)$ |
|---|---|---|---|
| 13 | .89 | 153 | .49 |
| 33 | .80 | 173 | .46 |
| 53 | .72 | 193 | .44 |
| 73 | .66 | 213 | .41 |
| 93 | .61 | 233 | .39 |
| 113 | .57 | 253 | .37 |
| 133 | .53 | 273 | .36 |

The permissible gradient, $G$, is written for each compartment, $\tau$, using the standard formalism,

$$G = G_0 + \Delta G d \tag{107}$$

at depth $d = P - 33 \ fsw$. A nonstop bounce exposure, followed by direct return to the surface, thus allows $G_0$ for that compartment. One set $G_0$ and $\Delta G$ are tabulated in Table 7, with $\Delta G$ suggested by Buhlmann. The minimum excitation, $G^{min}$, initially probing $r$, and taking into account regeneration of nuclei over time scales $\tau_r$, is $(fsw)$,

$$G^{min} = \frac{2\,\gamma\,(\gamma_c - \gamma)}{\gamma_c\, r(t)} = \frac{11.01}{r(t)} \tag{108}$$

with,

$$r(t) = r + (r_i - r)\,[1 - exp\,(-\lambda_r t)] \tag{109}$$

$\gamma$, $\gamma_c$ film, surfactant surface tensions, that is, $\gamma = 17.9$ *dyne*/*cm*, $\gamma_c = 257$ *dyne*/*cm*, and $\lambda_r$ the inverse of the regeneration time for stabilized gas micronuclei (many days). Prolonged exposure leads to saturation, and the largest permissible gradient, $G^{sat}$, takes the form $(fsw)$, in all compartments,

$$G^{sat} = \frac{58.6}{r} - 49.9 = .372\,P + 11.01. \tag{110}$$

On the other hand, $G^{min}$ is the excitation threshold, the amount by which the surrounding tension must exceeed internal bubble pressure to just support growth.

Although the actual size distribution of gas nuclei in humans is unknown, experiments *in vitro* suggest that a decaying exponential is reasonable,

$$n = N\ exp\ (-\beta r) \tag{111}$$

with $\beta$ a constant, and $N$ a convenient normalization factor across the distribution. For small values of the argument, $\beta r$,

$$exp\ (-\beta r) = 1 - \beta r \tag{112}$$

as a nice simplification. For a stabilized distribution, $n_0$, accommodated by the body at fixed pressure, $P_0$, the excess number of nuclei, $\Lambda$, excited by compression-decompression from new pressure, $P$, is,

$$\Lambda = n_0 - n = N\beta r_i \left[1 - \frac{r}{r_i}\right]. \tag{113}$$

For large compressions-decompressions, $\Lambda$ is large, while for small compressions-decompressions, $\Lambda$ is small. When $\Lambda$ is folded over the gradient, $G$, in time, the product serves as a critical volume indicator and can be used as a limit point in the following way.

The rate at which gas inflates in tissue depends upon both the excess bubble number, $\Lambda$, and the gradient, $G$. The critical volume hypothesis requires that the integral of the product of the two must always remain less than some limit point, $\alpha\, V$, with $\alpha$ a proportionality constant,

$$\int_0^\infty \Lambda G dt = \alpha V \tag{114}$$

for $V$ the limiting gas volume. Assuming that gradients are constant during decompression, $t_d$, while decaying exponentially to zero afterwards, and taking the limiting condition of the equal sign, yields simply for a bounce dive, with $\lambda$ the tissue constant,

$$\Lambda G\ (t_d + \lambda^{-1}) = \alpha V. \tag{115}$$

In terms of earlier parameters, one more constant, $\delta$, closes the set, defined by,

$$\delta = \frac{\gamma_c\ \alpha\ V}{\gamma\ \beta\ r_i\ N} = 7180\ fsw\ min \tag{116}$$

so that,

$$\left[1 - \frac{r}{r_i}\right]\ G\ (t_d + \lambda^{-1}) = \delta\ \frac{\gamma}{\gamma_c} = 500.8\ fsw\ min. \tag{117}$$

The five parameters, $\gamma$, $\gamma_c$, $\delta$, $\lambda_r$, $r_i$, are five of the six fundamental constants in the varying permeability model. The remaining parameter, $\lambda_m$, interpolating bounce and saturation exposures, represents the inverse time contant modulating multidiving. Bubble growth experiments suggest that $\lambda_m^{-1}$ is in the neighborhood of an hour.

The depth at which a compartment controls an exposure, and the excitation radius as a function of halftime, $\tau$, in the range, $12 \leq d \leq 220\ fsw$, satisfy,

$$\frac{r}{r_i} = .9 - .43\ exp\ (-\zeta\tau) \tag{118}$$

with $\zeta = 0.0559\ min^{-1}$. The regeneration constant, $\lambda_r$, is on the order of inverse days, that is, $\lambda_r = 0.0495\ days^{-1}$. Characteristic halftimes, $\tau_r$ and $\tau_h$, take the values $\tau_r = 14\ days$ and $\tau_h = 12.4\ min$. For large $\tau$, $r$ is close to $r_i$, while for small $\tau$, $r$ is on the order of .5 $r_i$. At sea level, $r_i = .8\ microns$ as discussed.

The VPM has been applied extensively to nitrox and heliox staging in ranges down to 250 $fsw$, with parameters fitted to exposures in this range.

5. Reduced Gradient Bubble Model (RGBM)

The phase (limit) integral for multiexposures is written,

$$\sum_{j=1}^{J} \left[ \Lambda G\, t_{d_j} + \int_0^{t_j} \Lambda G dt \right] \leq \alpha\, V \tag{119}$$

with the index $j$ denoting each dive segment, up to a total of $J$, and $t_j$ the surface interval after the $j^{th}$ segment. For the inequality to hold, that is, for the sum of all growth rate terms to total less than $\alpha V$, obviously each term must be less the $\alpha\, V$. Assuming that $t_J \to \infty$, gives,

$$\sum_{j=1}^{J-1} \left[ \Lambda G \left[ t_{d_j} + \lambda^{-1} - \lambda^{-1} exp\, (-\lambda t_j) \right] \right] + \Lambda G\, (t_{d_J} + \lambda^{-1}) \leq \alpha\, V. \tag{120}$$

Defining $G_j$,

$$\Lambda G_j\, (t_{d_j} + \lambda^{-1}) = \Lambda G\, (t_{d_j} + \lambda^{-1}) - \Lambda G\, \lambda^{-1} exp\, (-\lambda t_{j-1}) \tag{121}$$

for $j = 2$ to $J$, and,

$$\Lambda G_1 = \Lambda G \tag{122}$$

for $j = 1$, it follows that

$$\sum_{j=1}^{J} \Lambda\, G_j\, (t_{d_j} + \lambda^{-1}) \leq \alpha\, V \tag{123}$$

with the important property,

$$G_j \leq G. \tag{124}$$

This implies we employ reduced gradients extracted from bounce gradients by writing,

$$G_j = \xi_j\, G \tag{125}$$

with $\xi_j$ a *multidiving* fraction requisitely satisfying,

$$0 \leq \xi_j \leq 1 \tag{126}$$

so that, as needed,

$$\Lambda G_j \leq \Lambda G. \tag{127}$$

The fractions, $\xi$, applied to $G$ always reduce them. As time and repetitive frequency increase, the body's ability to eliminate excess bubbles and nuclei decreases, so that we restrict the permissible bubble excess in time,

$$\Lambda(t_{j-1}^{cum}) = N\beta r_i \left[ 1 - \frac{r(t_{j-1}^{cum})}{r_i} \right] = \Lambda\, exp\, (-\lambda_r t_{j-1}^{cum}) \tag{128}$$

$$t_{j-1}^{cum} = \sum_{i=1}^{j-1} t_i \tag{129}$$

with $t_{j-1}^{cum}$ cumulative dive time. A reduction factor, $\eta_j^{reg}$, accounting for creation of new micronuclei is taken to be the ratio of present excess over initial excess, written,

$$\eta_j^{reg} = \frac{\Lambda(t_{j-1}^{cum})}{\Lambda} = exp\, (-\lambda_r t_{j-1}^{cum}) \tag{130}$$

For reverse profile diving, the gradient is restricted by the ratio (minimum value) of the bubble excess on the present segment to the bubble excess at the deepest point over segments. The gradient reduction, $\eta_j^{exc}$, is then,

$$\eta_j^{exc} = \frac{(\Lambda)_{max}}{(\Lambda)_j} = \frac{(rd)_{max}}{(rd)_j} \tag{131}$$

with *rd* the product of the appropriate excitation radius and depth. Because bubble elimination periods are shortened over repetitive dives, compared to intervals for bounce dives, the gradient reduction, $\eta_j^{rep}$, is proportional to the difference between maximum and actual surface bubble inflation rate, that is,

$$\eta_j^{rep} = 1 - \left[1 - \frac{G^{min}}{G}\right] exp\ (-\lambda_m t_{j-1}) \tag{132}$$

with $t_{j-1}$ consecutive surface interval time, $\lambda_m^{-1}$ on the order of an hour, and $G^{min}$ the smallest $G_0$ in Table 7.

Finally, for multidiving, the gradient reduction factor, $\xi$, is defined bt the product of the three $\eta$,

$$\xi_j = \eta_j^{exc}\ \eta_j^{rep}\ \eta_j^{reg} = \frac{(\Lambda)_{max}}{(\Lambda)_j}\ \left[1 - \left(1 - \frac{G^{min}}{G}\right) exp\ (-\lambda_m t_{j-1})\right] exp\ (-\lambda_r t_{j-1}^{cum}) \tag{133}$$

with $t_{j-1}$ consecutive dive time, and $t_{j-1}^{cum}$ cumulative dive time, as noted. Since bubble numbers increase with depth, reduction in permissible gradient is commensurate. Multiday diving is mostly impacted by $\lambda_r$, while repetitive diving mostly by $\lambda_m$. Obviously, the critical tension, $M$, takes the form,

$$M = \xi(G_0 + \Delta G d) + P. \tag{134}$$

Table 7 tabulates a set of critical gradients, $G_0$ and $\Delta G$.

Table 7. Critical Phase Volume Gradients

| halftime $\tau\ (min)$ | threshold depth $\delta\ (fsw)$ | surface gradient $G_0\ (fsw)$ | gradient change $\Delta G$ |
|---|---|---|---|
| 2 | 190 | 151.0 | .518 |
| 5 | 135 | 95.0 | .515 |
| 10 | 95 | 67.0 | .511 |
| 20 | 65 | 49.0 | .506 |
| 40 | 40 | 36.0 | .468 |
| 80 | 30 | 27.0 | .417 |
| 120 | 28 | 24.0 | .379 |
| 240 | 16 | 23.0 | .329 |
| 480 | 12 | 22.0 | .312 |

The RGBM extends the VPM to multidiving and depths to roughly 500 $fsw$ for nitrox, heliox, and trimix. Parameters are correlated with diving exposure date using maximum likelihood regression.

6. Tissue Bubble Diffusion Model (TBDM)

Bubbles shrink or grow according to a simple radial diffusion equation linking total gas tension, $\Pi$, ambient pressure, $P$, and surface tension, $\gamma$, to bubble radius, $r$,

$$\frac{\partial r}{\partial t} = \frac{DS}{r}\left[\Pi - P - \frac{2\gamma}{r}\right] \tag{135}$$

with $D$ the gas diffusion coefficient, and $S$ the gas solubility. Bubbles grow when the surrounding gas tension exceeds the sum of ambient plus surface tension pressure, and vice versa. Higher gas solubilities and diffusivities enhance the rate. Related bubble area, $A$, and volume, $V$, changes satisfy,

$$\frac{\partial A}{\partial t} = 8\pi r \frac{\partial r}{\partial t} \tag{136}$$

$$\frac{\partial V}{\partial t} = 4\pi r^2 \frac{\partial r}{\partial t} \tag{137}$$

Using Fick's law, a corresponding molar current, $J$, of gas into, or out of, the bubble is easily computed assuming an ideal gas,

$$J = -\frac{DS}{RTh}\left[\Pi - P - \frac{2\gamma}{r}\right] \tag{138}$$

for $R$ the ideal gas constant, $T$ the temperature, and $h$ an effective diffusion barrier thickness. And the molal flow rate is just the molal current times the interface area, that is,

$$\frac{\partial n}{\partial t} = JA \tag{139}$$

for $n$ the number of moles of gas. The change in pressure and volume of the bubble, due to gas diffusion, follows simply from the ideal gas law,

$$\frac{\partial (PV + 2\gamma r^{-1} V)}{\partial t} = R\frac{\partial (nT)}{\partial t} \tag{140}$$

for $V$ the bubble volume.

Obviously, the above constitute a coupled set of differential equations, solvable for a wide range of boundary and thermodynamic conditions connecting the state variables, namely, $P$, $V$, $\Pi$, $r$, $n$, and $T$.

In the TBDM, a bubble dose, based on the hypothetical volume of an expanding test bubble, is linked to decompression data for the exposure. Maximum likelihood regression is used to correlate bubble dose with DCI risk.

## COMPARATIVE REVERSE DIVE PROFILES

### Nominal Reverse Profiles

Employing the above described algorithms, we consider model predictions for RPs, extract underlying features and tendencies, and draw comparisons. The code, *DECOMP*, containing a number of model kernels, is employed for calculations.

The RPs (100/60 and 60/100) are normalized to roughly the same NDLs so that the nonstop time limits at 100 $fsw$ and 60 $fsw$ are 15 *min* and 50 *min*, respectively. This normalization leans slightly toward the conservative side as far as NDLs are concerned. Table 8 encapsulates the results for the PLM, DLM, TM, VPM, RGBM, and TBDM. Typically, tracking bubble growth and dissolved gas buildup and elimination, phase models require slightly more decompression times for the RPs. The PLM and DLM are comparable, the TM, VPM, and TBDM also track closely, and the RGBM is most conservative. These profiles are relatively shallow, and the RP increment is small ($\Delta d = 40$ $fsw$). Generally,

Table 8. Comparative RPs And Algorithmms

| Algorithm | Dive 1 | Deco 1 | Surface Interval | Dive 2 | Deco 2 |
|---|---|---|---|---|---|
| PLM | 100/15 | none | 30 | 60/30 | 10/2 |
| DLM | | none | | | 10/2 |
| TM | | none | | | 10/1 |
| VPM | | none | | | 10/2 |
| RGBM | | none | | | 10/4 |
| TBDM | | none | | | 10/3 |
| PLM | 60/30 | none | | 100/15 | 10/2 |
| DLM | | none | | | 10/2 |
| TM | | none | | | 10/2 |
| VPM | | none | | | 10/3 |
| RGBM | | none | | | 10/5 |
| TBDM | | none | | | 10/3 |
| PLM | 100/15 | none | 60 | 60/30 | 10/1 |
| DLM | | none | | | 10/1 |
| TM | | none | | | 10/1 |
| VPM | | none | | | 10/2 |
| RGBM | | none | | | 10/4 |
| TBDM | | none | | | 10/2 |
| PLM | 60/30 | none | | 100/15 | 10/1 |
| DLM | | none | | | 10/1 |
| TM | | none | | | 10/1 |
| VPM | | none | | | 10/3 |
| RGBM | | none | | | 10/6 |
| TBDM | | none | | | 10/2 |
| PLM | 100/15 | none | 120 | 60/30 | none |
| DLM | | none | | | none |
| TM | | none | | | 10/1 |
| VPM | | none | | | 10/1 |
| RGBM | | none | | | 10/3 |
| TBDM | | none | | | 10/1 |
| PLM | 60/30 | none | | 100/15 | 10/1 |
| DLM | | none | | | 10/1 |
| TM | | none | | | 10/1 |
| VPM | | none | | | 10/2 |
| RGBM | | none | | | 10/4 |
| TBDM | | none | | | 10/2 |
| PLM | 100/15 | none | 240 | 60/30 | none |
| DLM | | none | | | none |
| TM | | none | | | none |
| VPM | | none | | | none |
| RGBM | | none | | | 10/1 |
| TBDM | | none | | | 10/1 |
| PLM | 60/30 | none | | 100/15 | none |
| DLM | | none | | | none |
| TM | | none | | | none |
| VPM | | none | | | 10/1 |
| RGBM | | none | | | 10/2 |
| TBDM | | none | | | 10/1 |

bubble models affect deep and prolonged exposures the most, requiring deeper stops, but usually shorter overall decompression times. The effect is not seen here trendwise, but will reappear as the RP increments increase. Bubble and

Haldane models overlap for short and shallow exposures, such as these RPs, and entries in Table 8 are no exception. The observation has often been made that not much free gas phase has been excited during short and shallow exposures, and then, bubble models should collapse to dissolved gas phase models in the limit.

**Extreme Reverse Profiles**

When exposures are deeper and RP increments are greater than 40 $fsw$, model differentiations between dissolved gas and dual phase models appear in the staging regimens, as seen in Table 9, contrasting the PLM and RGBM only for 160/40 and 40/160 RPs. Clearly phase models (RGBM) require deeper staging but shorter times, as seen in Table 9 for the same surface intervals in Table 8. The bottom times are 7 *min* and 100 *min* at 160 $fsw$ and 40 $fsw$ respectively in Table 9.

Table 9. Comparative PLM And RGBM (Deep) RPs

| Algorithm | Dive 1 | Deco 1 | Surface Interval | Dive 2 | Deco 2 |
|---|---|---|---|---|---|
| PLM | 160/7 | 10/3 | 30 | 40/100 | none |
| RGBM | | 10/1 | | | 10/4 |
| PLM | 40/100 | none | | 160/7 | 10/11 |
| RGBM | | none | | | 30/1,20/1,10/2 |
| PLM | 160/7 | 10/3 | 60 | 40/100 | none |
| RGBM | | 10/1 | | | 10/3 |
| PLM | 40/100 | none | | 160/7 | 10/3 |
| RGBM | | none | | | 20/1,10/2 |
| PLM | 160/7 | 10/3 | 120 | 40/100 | none |
| RGBM | | 10/1 | | | 10/2 |
| PLM | 40/100 | none | | 160/7 | 10/3 |
| RGBM | | none | | | 20/1,10/1 |
| PLM | 160/7 | 10/3 | 240 | 40/100 | none |
| RGBM | | 10/1 | | | 10/1 |
| PLM | 40/100 | none | | 160/7 | 10/3 |
| RGBM | | none | | | 20/1,10/1 |

**NEST Reverse Profiles Data**

The Nuclear Emergency Strategy Team (NEST) is involved in counterterrorism and countermeasures related to nuclear and biological threats. Exercises and tests have yielded scattered data about RPs across a spectrum of breathing gas mixtures (nitrox, heliox, trimix). Recent activities use trimix as bottom and ascent gas, with pure oxygen breathed at 20 $fsw$. Mixtures range 13-17% helium, 53-61% nitrogen, and 16-36% oxygen. RP increments, $\Delta d$, vary from 40 - 120 $fsw$, and surface intervals are nominally greater than 60 *min*. The RGBM is the staging algorithm.

Table 10 tabulates results of NEST field activities, with nominal surface intervals of an hour or more. Maximum bottom depth is 250 $fsw$, and exposures are near trimix NDLs. Dives are grouped in RP categories of 40 $fsw$. The NDLs computed from the RGBM for trimix in the range down to 250 $fsw$ are roughly:

| | |
|---|---|
| 100 $fsw$ | 8 - 10 *min* |
| 150 $fsw$ | 5 - 7 *min* |
| 200 $fsw$ | 4 - 6 *min* |
| 250 $fsw$ | 2 - 3 *min* |

similar in duration to Haldane trimix NDLs. The ascent profile is different under the RGBM, as compared to standard Haldane staging. And this is well known. The incidence rate, $p$, in Table 10 is 6.7%, with highest count in the 40 - 120 $fsw$ increment range. There are many variables here, such as staging depth, gas mixture, exposure time, and surface interval not tabulated separately.

Table 10. NEST RP Risk Table

| Dives | RP Increment ($fsw$) | Probable Hits |
|---|---|---|
| 36 | 0 - 40 | 0 |
| 18 | 40 -80 | 2 |
| 6 | 80 - 120 | 2 |

Practices for the deeper increments may border the yo-yo category, though no prior history of repetitive diving existed. Exercises continue, and data will grow. Trends are apparent in the above Table 10, but further analysis is required.

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