Int J Biomed Comput, 26 (1990) 237—256
Elsevier Scientific Publishers Ireland Ltd.

REDUCED GRADIENT BUBBLE MODEL

B.R. WIENKE

Applied Theoretical Physics Division, Los Alamos National Laboratory, Los Alamos, NM 87545 (U.S.A.)

(Received June 6th, 1990)
(Revision received July 13th, 1990)
(Accepted July 13th, 1990)

An approach to decompression modeling, the reduced gradient bubble model (RGBM), is developed from the critical phase hypothesis. The phase limit is introduced, extended, and applied within bubble-nucleation theory proposed by Yount. Much is different in the RGBM algorithm, on both theoretical and applied sides, with a focus on permissible bubble excesses rather than just dissolved gas buildup, something of a departure from traditional models. Overall, the approach is conservative, with changes in parameter settings affording flexibility. Marginal profiles permitted by tables and meters are restricted by the bubble algorithm. Highlighted features of the conservative algorithm include: (1) reduced no-stop time limits from the varying-permeability model (VPM); (2) short safety stops (or shallow swimming ascents) in the 10—20 feet of sea water (fsw) zone; (3) ascent and descent rates of 60 fsw/min, or slower; (4) restricted repetitive exposures, particularly beyond 100 fsw, based on reduced permissible bubble excess; (5) restricted spike (shallow-to-deep) exposures based on excitation of additional micronuclei; (6) restricted multi-day activity based on regeneration of micronuclei; (7) consistent treatment of altitude diving within model framework; (8) algorithm linked to bubble-nucleation theory and experiment. Coupled to medical reports about the long term effects of breathing pressurized gases and shortcomings in dissolved gas models, conservative modeling seems prudent.

Keywords: Decompression; Phase models; Bubbles and nucleation; Critical phase hypothesis; Meter algorithms; Varying permeability model; Reduced critical tensions; Gradients

1. Introduction

From an operational viewpoint, the decompression algorithm proposed years ago by Haldane [1] forms the basis for most dive tables and meters [2—6]. Based on empirical limits for dissolved gases (critical tensions) across phenomenological tissue compartments, which can be backfitted to almost any set of exposure data, this algorithm is not entirely consistent with contemporary notions of dissolved and free gas mechanics under decompression. Parameter latitude in fitting data nicely imparts flexibility to the model, but skirts some vital issues [7—16], trouble points when tables and meters are pushed outside operational limits. Such considerations root the RGBM and meter implementation.

Correspondence to: B.R. Wienke, LANL MS-F664, Group X-4, Los Alamos, N.M. 87545, U.S.A.

0020-7101/90/$03.50

2. Decompression Review and History

Discussion of the Haldane model and issues first deserve attention, with model underpinnings and limitations a good starting point. Much of the following is now the lore and a closer look at the basis is illuminating. In diving literature, it has become customary to quote depth and pressure units in feet of sea water (fsw) with the implication that 33 fsw = 1 atm. We consistently employ the same units for depth and pressure.

2.1. Haldane model

Tables and schedules for diving are traced to the English physiologist, John Scott Haldane [1]. He observed that goats, saturated to depths of 165 fsw did not develop decompression sickness (DCS) if subsequent decompression was limited to half the ambient pressure. Extending to humans, researchers suggested that tissues tolerate a factor of 2 over-pressure before the onset of symptoms. Haldane then constructed schedules limiting the saturation ratio to 2 in hypothetical tissues. Tissue compartments were characterized by their half-life, τ, that is, the time required for the compartment to halve (loose) or double (gain) existent nitrogen. Five compartments (5, 10, 20, 40, 75 min) were employed in calculations and staged procedures for 50 years. Later, in performing deep dives and expanding table ranges, workers advocated the use of six tissues (5, 10, 20, 40, 80, 120 min) in constructing schedules, with each compartment having its own critical pressure (M-value). Uptake and elimination of inert gas was based on models addressing only macroscopic aspects of gas exchange between blood and tissue.

Exact bubble production mechanisms, interplay of free and dissolved gas phases and related transport phenomena were not quantified since they were neither known nor understood. As such, the Haldane approach is a perfusion model, essentially fitted to exposure data for bounce and decompression dives. It has been a workhorse, maligned by careless application outside tested limits.

Supersaturation models focus on dissolved gas buildup (tensions), but neither bubbles nor gas micronuclei (free phases), suspected to be the immediate causes of DCS. By limiting the degree of supersaturation, these models regulate diving activity. From fundamental perspectives, gas phase concerns encompass all activity, with recent investigations concluding that tissues store *persistent* gas micronuclei, normally stabilized at fixed pressure but growth-excitable under pressure changes. Problems associated with neglecting phase dynamics and nucleation tend to surface in multi-day, multi-level and repetitive cases [7,8,17,18]. When models neglect free phases, they are not optimal nor global. As part fixup, one can incorporate free phase limiters, such as maximum bubble number, separated gas volume, or bubble growth rate, to extrapolate them, or introduce full-blown nucleation-bubble models. Safety stops and slower ascent rates are always prudent, along with reduced no-stop time limits. The RGBM algorithm accordingly employs a modern nucleation-bubble model, with shorter no-stop time limits and safety stops in the 10—20 fsw zone. Although designed for no-decompression diving, the algorithm will also function in the decompression mode if necessary. The full spectrum of diving activity (bounce,

multi-level, repetitive, multi-day, near-decompression) is more handled within the bubble algorithm. Altitude diving is treated no differently than sea-level diving within the model, excepting that ambient pressure is reduced.

Testing is central to diving and much testing of bounce (single), no-stop diving has transpired. Repetitive, multi-level and multi-day exposures can neither claim, nor reap, the same benefits. Application of the Haldane algorithm in the latter cases has witnessed slightly higher bends statistics than in the former one, as reported by Vann [7] in DAN newsletters and discussed at diving workshops [17,18]. Reasons appear tractable. The Haldane approach is a dissolved gas model and so long as the bulk of tissue gas remains in the dissolved state, the more correct and useful will prove such an approach. But as increasing proportions of free phase grow, by direct excitation of critical micronuclei or more gradual bubble coalescing transitions, the classical algorithm loses predictive capability. Invariably, such conditions might attend diving *extrapolated* outside model and test ranges, sometimes as a surprise. The fact that some divers are using meters in ways that they could never use, nor test, tables underscores the need to devise more globally correct algorithms, such as bubble models. In lock step, procedures such as shorter no-stop time limits, safety stops and slow ascent rates are consistent with bubble mechanics, reducing bubble growth rates and free gas buildup because of greater effective pressure at the end of the dive [13,18]. Dissolved gas buildup in the faster tissues is also reduced by such procedures.

2.2. Validation and testing

Certainly any algorithm can be piecewise safe over tested ranges, but not always globally. Some implementations, as pointed out by Weathersby [5], may not be statistically rigorous, relying on too small a set of exposure data to confidently predict outcome. Models not strongly correlated with tests can promulgate wide variation in predictive capability. Similarly, models can often *interpolate* within data, while failing to *extrapolate* outside the data. And then we must modify procedures to accommodate the extrapolation. A good point in question is the repetitive use of the USN tables. It is clear that single, no-decompression dives, followed possibly by one more repetitive dive, form the test basis of the no-stop parts of the schedules. Yet, multiple, repetitive dives, permitted by the tables, incur higher bends statistics, particularly in the deeper categories. This results from both model shortcomings and less reliable statistics. Adequate testing of any algorithm is requisite, that is, descent rate, exposure profile, ascent rate, surface interval and repetitive loading. Because of model and testing limitations, a menu, used in conjunction with present table and meter protocols, is suggested, having been the subject of discussion at recent workshops and technical forums [7,17,18]: (1) limit repetitive dives to a maximum of 3/day, not exceeding the 100 fsw level; (2) avoid multi-day, multi-level, or repetitive dives to increasing depths; (3) wait 12 h before flying after nominal diving, 24 h after heavy diving (taxing, near decompression, or prolonged repetitive) activity; (4) avoid multiple surface ascents and short repetitive dives (*spikes*) within surface intervals of 1 h; (5) surface intervals of more than an hour are recommended for repetitive diving; (6) safety stops for 2—4 min in the 10—20 fsw zone are advisable for all

diving, but particularly for deep (near 100 fsw), repetitive and multi-day exposures; (7) do not dive at altitudes above 304 m (10 000 feet), employing linear dive table extrapolations; (8) dive conservatively, remembering that tables and meters are not bends-proof.

Procedures such as those above are helpful and effective in dealing with shortcomings in the classical model. In the broad sense, they are fixes for an incomplete theory when extrapolated outside test range. The RGBM addresses these concerns more directly within bubble mechanics. The bubble model is fitted to customary diving data (bounce, repetitive and saturation), but relies on a reduction in permissible bubble parameters to restrict multiday, multi-level, deeper spike and repetitive diving. Such types of activity have only seen limited testing at this time. Ongoing accumulation of data will help in refining future versions of the meter, also benefitting the diving community with multi-diving data, now suffering from a paucity of information.

3. Bubble Model Rationale and Issues

Tables and present meters are based on dissolved gas limits points, as discussed. However, we suspect that the primary cause of DCS is bubbles and therefore we might consider free phases and their complicated interactions with dissolved gases. Trigger points addressing free phase accumulations are desirable. Such concerns are addressed in the RGBM implementation according to the following rationale.

3.1. Supersaturation and dissolved gas dynamics

Haldane models are based on limited (permissible) gas supersaturation in tissues, with gas exchange controlled by blood flow rates (perfusion) in assumed homogeneous media. Exchange of inert gas across regions of varying concentration is driven by the gradient, the difference between the arterial blood tension and the instantaneous tissue tension. Behavior can be modeled in time by mathematical classes of exponential tissue functions, bounded by ambient pressure and the initial tissue tension (see Section 3). Eleven compartments with 2, 5, 10, 20, 40, 80, 120, 180, 240, 360 and 480 min half-lives, τ, are routinely employed in application and half-lives are assumed to be independent of pressure. A one-to-one correspondence between compartments and specific anatomical entities is neither established, nor implied. Inert gas washout experiments by physiologists suggest that the eleven compartment spectrum of half-lives (2—480 min) represents a realistic range. To transport arterial gases to tissue, the exponential function, Eqn. (2), is employed with the above set of half-lives, τ.

For large values of τ, tissue uptake and elimination of inert gas is relatively slow according to the response function. For small values of τ, gas uptake and elimination proceed much more rapidly. To maximize the rate, the gradient is maximized. Maximization is, however, constrained. Fits to the exposure data, mainly for nonstop diving, empirically limit degrees of compartment supersaturation by critical values, M, having a modern range, $223 \leqslant M \leqslant 41$ fsw in absolute units. Gradient criteria, however, require differences between the tissue tension and ambient pressure to remain less than another bends trigger point, G. Gradient criteria link more natu-

rally to bubble tests, while critical tensions are ill-defined in heterogeneous media, such as tissues. Though τ and M are not fundamental, they do enjoy widespread popularity in diving. Sets of half-lives and critical tensions evolved from routine application of tissue response functions to exposure data, that is, trial and error bootstrapping of model equations to observed exposure time limits. Newer compilations ultimately extend older ones in like manner. The widely employed sets of critical tensions detailed by Buhlmann [2] and Workman [3], for arbitrary compartments at depth, are popular realizations of the algorithm. These critical tensions increase linearly with pressure and were tested at sea-level. The Workman (USN) set is parameterized by the fit [12,16], in absolute pressure units (fsw),

$$M = 193.3\tau^{-1/4} + 4.11\tau^{-1/4}d, \tag{1}$$

with d the depth and τ the half-life. While such trigger points can be modified in workable fashion to accommodate all types of diving, the RGBM trigger points are linked to bubble mechanics.

Buhlmann [2] and Bell and Borgwardt [6] also tested M-values at altitude. The extension is a study in itself and also a reflection of limitations of sea-level compilations. Wienke [12] proposed exponential extrapolations of critical tensions back through zero absolute pressure, an intuitively conservative scheme. But, proposed extrapolations of critical parameters require testing, and altitude is no exception. Such is also the case for deep exposures. Based on reductions in VGE counts in select exposures, Spencer [4] pioneered a modern trend to reduce no-stop time limits and subsequent critical tensions extracted from them. Pilmanis [9] also noted sizable reductions in VGE counts following safety stops in diving activities off Catalina, though the test population was rather small ($n = 1$).

3.2. Bubbles and free gas dynamics

Internal pressures in bubbles exceed ambient pressures by amounts equal to the effective surface tensions of the bubbles. To eliminate bubbles, or reduce growth, increasing ambient pressure is requisite not only to restrict size, but also to drive the gas by diffusion out of the bubble and across the tissue-bubble interface. The shorter the desired time of elimination, the greater must be the ambient pressure. Experiments conducted in decompressed gels, notably by Yount [19—22] and Strauss [14,23], Kunkle and Beckman [24], bear testimony to the fact. The smaller the bubble, the shorter the dissolution time. Here implication for diving is simple. In the presence of even threshold amounts of free phases, increased pressure is prudent. With any pressure, the length of time required to dissolve bubbles of 250 μm diameter is significantly shorter than that required to dissolve larger bubbles. Immediate recompression, within < 5 min, is adequate treatment for bubbles < 100 μm in diameter, and forms the basis for Hawaiian [25] and Australian [26] emergency in-water recompression procedures. Such facts prop arguments for safety stops when conventional tables are pushed to limits, timewise or repetitively.

Bubbles, which are unstable, might grow from stable, micron size, gas nuclei which resist collapse due to elastic skins of surface-activated molecules (sur-

factants), or possibly reduction in surface tension at tissue interfaces. Families of these micronuclei persist, varying in size and surfactant content. Large pressures (somewhere near 10 atm) are necessary to crush them. Micronuclei are small enough to pass through the pulmonary filters, yet dense enough not to float to the surfaces of their environments, with which they are in both hydrostatic (pressure) and diffusion (gas flow) equilibrium. When nuclei are stabilized and not activated to growth or contraction by external pressure changes, the skin (surfactant) tension offsets both the Laplacian (film) tension and any mechanical help from surrounding tissue. Then all pressures and gas tensions are equal. However, on decompression, the seed pockets are surrounded by dissolved gases at high tension and can subsequently grow (bubbles) as surrounding gas diffuses into them. The rate at which bubbles grow, or contract, depends directly on the difference between tissue tension and local ambient pressure, effectively the gradient, G, discussed a moment ago. At some point in time, a critical volume of bubbles, or separated gas, is established and symptoms of decompression sickness become more probable. On compression, the micronuclei are crunched down to smaller sizes across families, apparently stabilizing at new reduced size. Bubbles are also crunched by increasing pressure because of Boyle's law and then additionally shrink, if gas diffuses out of them. As bubbles get smaller and smaller, they possibly restabilize as micronuclei. Such dynamics are central to bubble models.

Nucleation theory is consistent with a number of diving observations. Divers can significantly increase tolerance against bubble formation and therefore DCS, by following three simple practices, originally observed by Walder [8] and later reiterated by Beckman and coworkers [24,25], simply make the first dive a deep, short (crush) dive, make succeeding dives progressively more shallow and make frequent dives. If nucleation sites are extinguished, reduced in number, or ill-disposed to excitation, bubble formation and risk are lessened. Regeneration times for classes of micronuclei are estimated to be near a week, under-scoring physiological adaptation to recurring pressure environments. The mechanics of nucleation, stabilization and bubble growth are fairly complex, with stabilization mechanisms having been recently elucidated. Source and generation mechanisms before stabilization are not well understood. Some candidates include cosmic radiation and charged particles, dissolved gases in fluids we drink, lymph draining tissues into veins, collisional coalescence, blood turbulence and vorticity, exercise, the stomach and the thin air-blood endothelium in the lungs. Once formed, micronuclei apparently stabilize rapidly with surfactant material. Passing through the pulmonary filters of the lungs, only sub-micron sizes might survive.

Very slow tissue compartments with small critical tensions can be treatment compartments, tracking both free and dissolved gas exchange in poorly perfused regions. But attempts to track free phases within dissolved phase models are not optimal, often requiring divergent procedures. One consistent approach is to slow ascent rates and/or introduce safety stops strategically. As far as net gas exchange is concerned, most combinations of stops and rates can be equivalenced to almost any other set at given pressure, so there is always some leeway. Growth minimization and free phase elimination favor slow ascents, but very slow ascent rates are difficult

maneuvers at best and most divers pay lip service to 60 fsw/min. Additionally, ascent rates of 60 fsw/min are part of tested schedules. Therefore, a rate of 60 fsw/min is employed in the algorithm, with safety stops. Slower rates are still encouraged, with no detriment implied in the bubble algorithm.

Based on reported calculations by Wienke [13] and Hamilton [17] safety stops for 2—4 min in the 10—20 fsw zone help to restrict bubble growth while having relatively small impact on dissolved gas buildup in the slow tissues. The reduction in growth parameters far outstrips any dissolved gas buildup. Stops, slow ascent rates and reduced non-stop limits, appear beneficial in multi-day, multi-level and repetitive diving. Slow ascent rates afford additional advantages, but safety stops are easier and more efficient. The combination of the two affords conservatism.

4. Algorithm Equations and Parameters

The foregoing can be placed into a compact framework for calculations and digital meter implementation. Both dissolved and free phases are necessarily factors in any realistic decompression algorithm, table or meter based. The RGBM factors both into calculations in the following way.

4.1. Dissolved gas transfer

Exchange of inert gas, by random molecular motion across regions of varying concentration, is driven by the local gradient, that is, the difference between the arterial blood tension, p_a, and the instantaneous tissue tension, p. Such behavior is modeled in time, t, by mathematical classes of exponential response functions, bounded by p_a and the initial value of p, denoted p_i. These multi-tissue functions take a very simple form, tracking both dissolved gas build-up and elimination symmetrically,

$$\dot{p} = p_a + (p_i - p_a) \exp(-\lambda t),$$
$$\lambda = \frac{0.6931}{\tau}, \tag{2}$$

with the perfusion constant, λ, related to the tissue half-time, τ. Eleven compartments with 2, 5, 10, 20, 40, 80, 120, 180, 240, 360 and 480 min half-lives, τ, are employed, and half-lives are independent of pressure.

In a series of dives or multiple stages, p_i and p_a represent extremes for each stage, or more precisely, the initial tension and the arterial tension at the beginning of the next stage. Stages are treated sequentially, with finishing tensions at one step representing initial tensions for the next step and so on. To maximize the rate of uptake or elimination of dissolved gases the gradient, simply the quantity $g = p_i - p_a$, is maximized. In the bubble model, this maximization is tempered by considerations of free phase elimination and not just rotely maximized by pulling the diver as close to the surface as possible in all cases. Most contemporary models, however, limit exposures by requiring that the tissue tensions never exceed the M-values, written,

$$M = M_0 + \Delta M\, d, \tag{3}$$

as a function of depth, d, for ΔM the change per unit depth. In absolute units, the corresponding critical gradient, G, is given by,

$$G = M - P, \tag{4}$$

with P ambient pressure and M given in absolute pressure. The RGBM employs gradients extracted from a bubble model with reduced non-stop limits, among the more conservative today.

4.2. Nucleation and bubbles

Previous tables for humans were based upon unsupported assumptions because many of the underlying processes by which dissolved gas is liberated from blood and tissues were poorly understood. Some of those assumptions, as enumerated by Yount and Strauss [14], Hills [11] and Wienke [13], are now known to be wrong. Recent development of a bubble nucleation model has made it possible to calculate diving tables from established principles. To evaluate the approach, Yount and Hoffman [19] developed a comprehensive set of air tables and compared them to the USN and RN conventional tables. Decompressions, bounce diving, altitude bends and saturation diving are all successfully described by one setting of four global parameters, replacing the usual set of M-values. This set is remarkably self-consistent, permitting accurate interpolation and extrapolation. We draw upon this methodology in extending the model to multi-diving.

The main outcome of studies [19—22] has been the development of a varying-permeability bubble model (VPM), in which cavitation nuclei of stabilized spherical gas phases are small enough to remain in solution and strong enough to resist collapse, their stability provided by elastic skins or membranes consisting of surface-active molecules. Ordinarily, VPM skins are permeable to gas, but can become impermeable when subjected to large compressions (near 10 atm). By tracking the changes in nuclear radius that are caused by increases or decreases in ambient pressure, the VPM provides precise quantitative descriptions of several bubble-counting experiments carried out in supersaturated gel. The model has also been used to trace levels of incidence of DCS in animal species such as shrimp, salmon, rats, and humans. Microscopic evidence has also been obtained which indicates the spherical gas nuclei, those persistent microbubbles, actually do exist and possess physical properties consistent with earlier assignments. For example, nuclear radii are on the order of 1 μm (10^{-6} m), or less and their number density in bio-media decreases exponentially with increasing radius, characteristic of a system VPM nuclei in equilibrium with their surroundings at the same temperature.

The critical radius, r_0, at fixed pressure, P_0, represents the cutoff for growth upon decompression to lesser pressure. Nuclei larger than R_0 will grow upon decompression. Additionally, following an initial compression, $\Delta P = P - P_0$, a smaller class of micronuclei of critical radius, r, can be excited into growth with decompression. If r_0 is the critical radius at P_0, then, the smaller family, r, excited by decompression from P, obeys,

$$\frac{1}{r} = \frac{1}{r_0} + \frac{\Delta P}{158}, \tag{5}$$

TABLE I

EXCITATION RADII

Pressure P (fsw)	Excitation radius r (μm)	Pressure P (fsw)	Excitation radius r (μm)
13	0.890	153	0.498
33	0.800	173	0.468
53	0.726	193	0.442
73	0.665	213	0.419
93	0.614	233	0.397
113	0.569	253	0.378
133	0.531	273	0.361

with ΔP measured in fsw and r in μm. Table I lists critical radii, r, excited by sea-level compressions ($P_0 = 33$ fsw), assuming $r_0 = 0.8\ \mu$m. Entries also represent the equilibrium critical radius at pressure, P. Deeper decompressions excite smaller, more stable, nuclei.

The primary bubbles formed directly from nuclei may lead to secondary bubbles via fission in the blood, or by the creation of rosaries in the interstitial spaces of firmer tissues. Since tissue deformation and impairment of circulation should depend upon both the size and number of bubbles, it seems plausible that the total volume of evolved gas would serve as an effective criteria in any bubble model, such as the VPM. Today, constant bubble number hypotheses have been replaced by critical volume hypotheses, with bubble numbers fluctuating accordingly. For shorter decompression times, bubble nuclei have little time to inflate. The permissible critical radius is then smaller and the allowed supersaturation larger, resulting in many small bubbles. Conversely, during long decompressions, bubbles may grow very large, so that only a few are permitted. Because size distribution and number density of nuclei in vivo are unknown, bubble models use an iterative procedure in algorithms.

4.3. Trigger points

Critical tensions fitted to bounce data must decrease with time when applied to saturation exposures, while corresponding critical tensions fitted to saturation data must increase when applied to bounce exposures. One way to explain this is through permissible bubble excess, greater on short deep dives and lesser on long shallow dives, reasoning outside dissolved gas approaches. The RGBM requires that repetitive gradients are systematically reduced to scale tissue loading, bubble growth, and phase separation over appropriate intervals. For bounce dives, permissible gradients can be extracted from the non-stop limits, t_{nd}, summarized in Table II. Different sets of non-stop time limits obviously yield different sets of permissible gradients, ostensibly small in difference when differences in the time limits are also small.

The bounce gradient, G, is written for each compartment, τ, using the standard formalism,

$$G = M - P = G_0 + \Delta G\, d, \tag{6}$$

TABLE II

BOUNCE TIME LIMITS

Depth d (fsw)	Time limit t_{nd} (min)	Depth d (fsw)	Time limit t_{nd} (min)
30	230.0	120	11.0
40	108.0	130	9.0
50	65.0	140	8.0
60	40.0	150	7.0
70	30.0	160	6.0
80	24.0	170	5.5
90	18.0	180	5.0
100	15.0	190	4.5
110	13.0	200	4.0

at depth $d = P - 33$ fsw. A non-stop bounce exposure, followed by direct return to the surface, thus allows G_0 for that compartment. Both G_0 and ΔG are tabulated in Table III, with ΔG suggested by Buhlmann [2]. Maximum tensions occur at threshold depths, d_{th}, for time limits, t_{nd}. The minimum excitation, G^{min}, initially probing r and taking into account regeneration of nuclei over time scales τ_r, is (fsw),

$$G^{min} = \frac{2\gamma(\gamma_c - \gamma)}{\gamma_c r(t)} = \frac{11.01}{r(t)}, \tag{7}$$

with,

$$r(t) = r + (r_0 - r)[1 - \exp(-\lambda_r t)], \tag{8}$$

TABLE III

BOUNCE GRADIENTS.

Half-life τ (min)	Threshold depth d_{th} (fsw)	Surface gradient G_0 (fsw)	Gradient change ΔG
2	190	150.0	0.519
5	135	93.9	0.518
10	95	64.9	0.516
20	65	45.6	0.512
40	40	33.8	0.468
80	30	25.9	0.417
120	28	22.2	0.379
180	25	18.1	0.354
240	23	15.5	0.329
360	18	12.1	0.313
480	15	10.5	0.303

γ, γ_c film, surfactant surface tensions, that is, $\gamma = 0.0179$ N/m, $\gamma_c = 0.257$ N/m and λ_r the inverse of the regeneration time for stabilized gas micronuclei (many days). Nuclei probed depend on depth according to Eqn. (5). Prolonged exposure leads to saturation and the largest possible gradient, G^{sat}, then takes the form (fsw),

$$G^{sat} = \frac{58.6}{r} - 49.9 = 0.372P + 11.01. \tag{9}$$

Near saturation, G^{sat} is the largest permissible gradient in any compartment. On the other hand, G^{min} is the excitation threshold, the amount by which the surrounding tension must exceed internal bubble pressure to just support growth. The parameterization of G^{sat}, deduced from diving exposures and given by Eqn. (9), is also consistent with the bubble model gradient for the 120-min compartment, extracted from the non-stop time limits. This behavior is also common to *M*-value algorithms, that is, the *M*-values extracted from the no-stop limits for one slow compartment are very similar in content to Eqn. (9). Also recall that the USN tables (and others) employ the 120-min compartment to control multi-exposures [3], while the Swiss tables use the 635-min compartment [2].

Across the fuller exposure spectrum, bounce to saturation diving, G will approach G^{sat}. This reflects the body's ability to support greater numbers of bubbles for short times and lesser numbers as time progresses. Part of this adaptation might occur in the pulmonary circulation, which seems to accomodate overloads for a short period of time. This has not been established as the mechanism, but the fact remains that permissible bounce gradients exceed saturation gradients by a large amount. At the surface, $G^{min} = 13.8$ fsw and $G^{sat} = 23.3$ fsw, while at 240 fsw, $G^{min} = 30.4$ fsw and $G^{sat} = 100.3$ fsw. What is also seen here is a reflection of the body's ability to maintain higher degrees of supersaturation with increased pressure. Bubble and micronuclei tend to both shrink and stabilize under pressure, permitting increased levels of supersaturation because of greater surface tension. Under decompression smaller bubbles and nuclei also grow more slowly for the same reason. Surface tension pressure, varying inversely as the spherical radius, r, helps to expel gas in the pocket by squeezing and building up a diffusion gradient across the film boundary. Unless nuclei are stabilized so that the net surface tension pressure is zero, all nuclei would eventually collapse upon themselves because of this squeeze. When nuclei are squeezed by increasing pressure, experiments established that they stabilize at new smaller radius, not growing back to earlier size unless ambient pressure is reduced.

Although the actual size distribution of gas nuclei in humans is unknown, experiments in vitro suggest that a decaying exponential is reasonable,

$$n = N \exp(-\beta r), \tag{10}$$

with β a VPM constant and N a convenient normalization factor across the distribution. For small values of the argument, βr,

$$\exp(-\beta r) \approx 1 - \beta r, \tag{11}$$

as a nice simplification. For a stabilized distribution, n_0, accommodated by the body at fixed pressure, P_0, the excess number of nuclei, Δn, excited by compression-decompression from new pressure, P, is, with Eqn. (5) tracking change in radius, r,

$$\Delta n = n_0 - n \approx N\beta r_0\left(1 - \frac{r}{r_0}\right). \tag{12}$$

For large compressions-decompressions, Δn is large, while for small compressions-decompressions, Δn is small. When Δn is folded over the gradient, G, in time, the product serves as a critical volume indicator and can be used as a limit point in the following way.

The rate at which gas inflates in tissue depends upon both the excess bubble number, Δn and the gradient, G. The critical volume hypothesis requires that the integral of the product of the two must always remain less than some limit point, αV^{crit}, with α a proportionality constant. According to Yount and Hoffman [19], this requires,

$$\int_0^\infty \Delta n\, G\, dt = \alpha V^{\text{crit}}, \tag{13}$$

for V^{crit} the limiting gas volume. Using Eqn. (2), assuming that gradients are constant during decompression, t_d, while decaying exponentially to zero afterwards and taking the limiting condition of the equal sign, yields simply for a bounce dive,

$$\Delta n\, G\,(t_d + \lambda^{-1}) = \alpha V^{\text{crit}}. \tag{14}$$

For bounce exposures with linear ascent rate, v, we have $t_d = d/v$. For saturation diving, Eqn. (13) has to be evaluated iteratively over component decompression stages, specifying optimal G at each stage. In evaluating the integral over time, it was assumed that the bubble number, Δn, depends only on the initial, final and intermediate pressures, that is surface P_0, compression to P and decompression back to intermediate pressures, typical of bounce or saturation (slow bleed) exposures. For multi-diving, the permissible bubble number will also vary with exposure profile, but the profiles will certainly not be monotonic in general.

In terms of earlier parameters, one more constant, δ, closes the set, defined by,

$$\delta = \frac{\gamma_c \alpha V^{\text{crit}}}{\gamma \beta r_0 N} = 7180\, fsw \text{ min}, \tag{15}$$

so that Eqn. (14) is recast, using Eqn. (12),

$$\left(1 - \frac{r}{r_0}\right) G(t_d + \lambda^{-1}) = \delta \frac{\gamma}{\gamma_c} = 500.8\, fsw \text{ min}. \tag{16}$$

The five parameters, γ, γ_c, δ, λ_r, r_0, are five of the six fundamental constants in the

bubble model (VPM). The remaining parameter, λ_m, interpolating bounce and saturation exposures, represents the inverse time contant modulating multi-diving. Bubble growth experiments suggest that λ_m^{-1} is in the neighborhood of an hour.

In terms of Eqn. (5) and the depth at which a compartment controls the exposure according to Table III, the radii of nuclei excited as a function of controlling half-life, τ, in the range, $12 \leqslant d \leqslant 220$ fsw, satisfy,

$$\frac{r}{r_0} = 0.9 - 0.43 \exp(-\lambda_h \tau) \tag{17}$$

with $\lambda_h = 0.0559$ min^{-1}. The regeneration constant, λ_r, is on the order of inverse days, that is, $\lambda_r = 0.0495$ days^{-1}. Characteristic half-times, τ_r and τ_h, take the values $\tau_r = 14$ days and $\tau_h = 12.4$ min. For large τ, r is close to r_0, while for small τ, r is on the order of $0.5\, r_0$. As mentioned above, τ_m is on the order of hours.

One way to address the interpolation of critical parameters between bounce and saturation limits, areas into which repetitive, spike, multi-level and multi-day diving fall, is to reduce the bounce gradients, G, with something like exponential smoothing, employing bubble growth/elimination parameters. Any phenomenology can be backfitted to the data, contemporary multi-diving exposure data such as collected by NEDU, RNPL, and DSAT and exponentials are natural to diffusive phenomena, such as gaseous transfer across thin film boundaries. As seen, exponentials parameterize the distribution of nuclear sizes, r, in tissue and blood.

To effect this reduction, we first extend the critical volume criterion to multi-diving, that is, the integral of Eqn. (13) to multi-exposures,

$$\sum_{j=1}^{J} \left[\Delta n\, G\, t_d + \int_0^{t_j} \Delta n\, G\, dt \right] \leqslant \alpha V^{\text{crit}}, \tag{18}$$

with the index j denoting each dive segment, up to a total of J, and t_j the surface interval after the jth segment. Particular G are general and not necessarily the set derived for bounce and saturation diving. However, it is useful to extract G from the standard set for meter convenience and computational simplicity, effectively generating constraints on multi-exposures as a function of time, depth and controlling half-life. If we knew the multi-dive profiles in advance, we could optimize Eqn. (18) so that the equality sign held, effectively dictating the optimized G. But diving habits and meter operation preclude this in general, so a safe estimate of the permissible gradients must be made on each dive segment. The bounce gradients in Table III then nicely serve as an upper bound, seen in the following manner.

For the inequality to hold, that is, for the sum of all growth rate terms in Eqn. (18) to total less than αV^{crit}, obviously each term must be less the αV^{crit}. Performing the indicated operations, assuming $t_j \to \infty$, gives,

$$\sum_{j=1}^{J-1} [\Delta nG[t_{d_j} + \lambda^{-1} - \lambda^{-1} \exp(-\lambda t_j)]] + \Delta nG(t_{d_j} + \lambda^{-1}) \leqslant \alpha V^{\text{crit}}. \tag{19}$$

Defining G_j.

$$\Delta n\, G_j(t_{d_j} + \lambda^{-1}) = \Delta n\, G(t_{d_j} + \lambda^{-1}) - \Delta n\, G\lambda^{-1} \exp(-\lambda t_{j-1}), \tag{20}$$

for $j = 2$ to J, and,

$$\Delta n\, G_1 = \Delta n\, G, \tag{21}$$

for $j = 1$, Eqn. (19) can be rewritten

$$\sum_{j=1}^{J} \Delta n\, G_j(t_{d_j} + \lambda^{-1}) \leq \alpha V^{\text{crit}}, \tag{22}$$

with the important property

$$G_j \leq G. \tag{23}$$

The gradients, G_j and G, differ only by the effects of the surface interval, t_{j-1}. As seen, G requires long surface intervals to eliminate excess bubbles, so that G_j must be reduced to compensate for the fact that long surface intervals are not available for bubble elimination on repetitive exposures.

The criterion, Eqn. (22), looks like a constraint on multiple bounce dives, with repetitive growth rate, $\Delta n\, G_j$, less than bounce growth rate, $\Delta n\, G$, tabulated in Table III for a set of non-stop exposures. This implies we might use the gradients in Table III for multi-diving provided they are reduced at successive exposures, by writing,

$$G_j = \xi_j G, \tag{24}$$

with ξ_j a *multi-diving* fraction requisitely satisfying Eqn. (23), that is,

$$0 \leq \xi_j \leq 1, \tag{25}$$

so that, as needed,

$$\Delta n\, G_j \leq \Delta n\, G. \tag{26}$$

As surface time intervals decrease, appropriate ξ_j should get smaller and staging approach saturation limits as $J \to \infty$. As surface time intervals increase, ξ_j should get larger and staging approach bounce limits as $t_j \to \infty$. In between, behavior depends on total elapsed time, total surface interval, tissue compartment and profile. Considering interpolating behavior, a checklist of properties of ξ_j, correlating with diving practice, is desirable: (1) $\xi_j = 1$ for a bounce dive, remaining < 1 for repetitive dives within some interval; (2) ξ_j decrease monotonically with increasing exposure time; (3) ξ_j increase monotonically with increasing surface interval time; (4) ξ_j scale

faster tissue compartments the most; (5) ξ_j decrease with depth of repetitive dive segment; (6) ξ_j scale deeper-than-previous dives the most; (7) ξ_j change with every dive segment, but only within any dive segment when a greater depth is reached; (8) ξ_j decrease as micronuclei are regenerated; (9) the time constant controlling ξ_j is linked to λ, λ_m and λ_r.

Allowing the bubble numbers, Δn, to vary with regeneration and permitting excitation of additional bubble nuclei at increasing depth, the situation acquires more degrees of freedom. As time increases repetitively, we might surmise that the body's ability to eliminate excess bubbles and nuclei decreases. At each successive segment, we restrict the permissible bubble excess according to Eqn. (8),

$$\Delta n(t_{j-1}^{\text{cum}}) = N\beta r_0\left(1 - \frac{r(t_{j-1}^{\text{cum}})}{r_0}\right) = \Delta n \exp(-\lambda_r t_{j-1}^{\text{cum}}),$$

$$t_{j-1}^{\text{cum}} = \sum_{i=1}^{j-1} t_i, \tag{27}$$

so that the reduction factor, η_j^{exc}, is written,

$$\eta_j^{\text{reg}} = \frac{\Delta n(t_{j-1}^{\text{cum}})}{\Delta n}. \tag{28}$$

For deeper-than-previous diving, the gradient is restricted by the ratio (minimum value) of the bubble excess on the present segment to the bubble excess at the deepest point over segments. The gradient reduction, η_j^{exc}, is then written,

$$\eta_j^{\text{exc}} = \frac{(\Delta n)_{\max}}{(\Delta n)_j} = \frac{(rd)_{\max}}{(rd)_j}, \tag{29}$$

with rd the product of the appropriate excitation radius and depth. Because bubble elimination periods are shortened over repetitive dives, compared to intervals for bounce dives, the gradient reduction, η_j^{sur}, is proportional to the difference between maximum and actual surface bubble inflation rate, that is,

$$\eta_j^{\text{sur}} = 1 - \left(1 - \frac{G^{\min}}{G}\right)\exp(-\lambda_m t_{j-1}), \tag{30}$$

with λ_m^{-1} on the order of hours and $G^{\min}$ the smallest G_0 in Table III.

For multi-diving, we therefore define for the above,

$$\xi_j = \eta_j^{\text{exc}}\eta_j^{\text{sur}}\eta_j^{\text{exc}} = \frac{(\Delta n)_{\max}}{(\Delta n)_j}\left[1 - \left(1 - \frac{G^{\min}}{G}\right)\exp(-\lambda_m t_{j-1})\right]\exp(-\lambda_r t_{j-1}^{\text{cum}}), \tag{31}$$

with t_{j-1} consecutive interval time and t_{j-1}^{cum} cumulative interval time, as noted. Since bubble numbers increase with depth, reduction in permissible gradient is commensurate. Deeper repetitive dives are more constrained by the bubble model, but all repetitive dives suffer reductions in permissible supersaturation gradient, increasing with repetitivity and short surface time. All terms in Eqn. (28) are bounded by zero and one, with $(\Delta n)_{max}$ evaluated at the deepest point over consecutive segments and $(\Delta n)_j$ equal to $(\Delta n)_{max}$ until such time as a greater depth is reached. Multi-day diving is mostly impacted by λ_r while repetitive diving mostly by λ_m.

The gradients are reduced by amounts that reflect shorter times for bubble elimination, compared to very long bounce surface intervals and regeneration of micronuclei over those same surface intervals. Consistent with recent workshop recommendations and flying-after-diving studies, factors in Eqn. (31) relax to one after 48 h of continuous surface interval, that is, $\xi_j = 1$ after any 48-h period of non-diving. Bubble and dissolved gas elimination should equilibrate with ambient pressure in such time intervals.

In the RGBM, the reduced gradients, G_j, replace the bounce set in Table III for repetitive, multi-level, deeper spike and multi-day activities. Unlike the bounce (fixed) set in Table III, the multi-set, G_j, change with depth, time and dive profile. They are computed on the fly in the meter. The Table set, G, are stored in memory, with the six constants, that is, parameters γ, γ_c, δ, r_0, λ_r and λ_m the basic set. The first four are fitted in the VPM to the bounce and saturation data, while the last two are related to bubble inflation and regeneration rates over surface intervals in the RGBM. All can be tweaked to fit data, but the multi-dive and multi-day parameters, λ_m and λ_r, are the most flexible, affording variable time scales over which to reduce gradients to compensate for reduced bubble elimination in repetitive activities.

5. Implementation and Application

The implementation of the foregoing set of model equations is rather straightforward. Free and dissolved gas buildup, multi-diving fractions, bubble excesses and critical gradients in the RGBM are the crux of the implementation.

5.1. Algorithm

Defining the RGBM critical tensions, M, from Eqn. (6),

$$M = \xi(G_0 + \Delta G(P - 33)) + P, \tag{32}$$

with P absolute pressure, G_0 and ΔG listed in Table III and an effective depth, d, obviously defined to be,

$$d = P - 33, \tag{33}$$

the instantaneous tissue tensions in all compartments must be maintained below the M-values computed at that level, or anticipated level for an upward or surfacing excursion. For excursions directly to the surface, the tensions, p must not exceed M

in Eqn. (32) at the surfacing value of P. If a decompression stop is required, tensions must not exceed the M-value at the stop, with greater P than at the surface. When p are greater than M at a particular level, a decompression stop, prior (deeper) to that level, is required, with the tissue compartment exhibiting the largest tension in excess of M controlling the procedure. As the ascent progresses, other compartments may take over control.

Diving at altitude, $P < 33$ fsw, is no different than diving at sea-level, as far as calculations are concerned, provided the effective depth, d, is computed from Eqn. (33) consistently. At the surface at altitude, effective depth is negative, rapidly becoming positive as the diver descends. However, there is one constraint on RGBM critical gradients and tensions that comes into play at altitude, namely, for model consistency, compartment M-values have a lower limit of 43.5 fsw. Altitude values computed from Eqn. (32) can drop below 43.5 fsw, but then must be restored to 43.5 fsw. This is a bubble mechanical effect, serving as an absolute lower limit. At altitude, z, ambient pressure, P, is given by [12,13].

$$P = 33 \exp(-0.038z), \tag{34}$$

with z measured in increments of 300 m and P again fsw. At sea-level, $z = 0$ and $P = P_0 = 33$ fsw.

On first dives, or any other dives separated by at least 48 h, the fraction, ξ, is one. In other cases, $\xi \leqslant 1$, computed according to Eqns. (28—31) for $\lambda_r^{-1} = 14$ days and $\lambda_m^{-1} = 40$ min. If the fraction is one on all dives, the bare bones gradients in Table III and M-values, compare favorably with the Buhlmann critical tensions, conservative in themselves and tested at altitude. Surface intervals between dives and cumulative surface interval over repetitive dives are required to estimate ξ from the same equations. Similarly, the bubble excesses at depth, according to Eqns. (5) and (12), are necessary to scale deeper-than-previous dives, through ξ. In our case, G_{min} is taken to be 10.5 fsw from Table III, more conservative than G^{min} at sea-level from Eqn. (7).

5.2. Multi-day repetitive exposure

As a simple demonstration of the RGBM, consider two repetitive dives per day, 120 fsw for 10 min separated by 2-h surface intervals, over three consecutive days. This profile, extended to three repetitive dives a day, has produced bends in 3 out of 4 cases on the first day, according to Leitch and Barnard [27] so it is not an academic exercise at this point. The model reduces the permissible gradients in each tissue compartment, on each segment of the six dives, according to Table IV, listing ξ at the start of each repetitive and multi-day segment. Systematic reduction in ξ is clearly seen.

Reductions in critical gradients approach 20% in the fast compartments and 15% in the slower ones, on the last dive. On the first day, reductions in the fast compartments approach 5% on the second dive and near 10% on the second dive of the second day. Smaller reductions, by a few percent, are seen in the slow compartments. Exposures in the 120 fsw range are controlled by the 10-min compartment (Table

TABLE IV

MULTI-DIVING FRACTIONS (120/10, 0/120, 120/10 for 3 days)

τ (min)	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6
2	1.00	0.95	0.93	0.88	0.86	0.81
5	1.00	0.95	0.93	0.88	0.86	0.81
10	1.00	0.95	0.93	0.89	0.86	0.82
20	1.00	0.95	0.93	0.89	0.86	0.82
40	1.00	0.96	0.93	0.89	0.86	0.82
80	1.00	0.96	0.93	0.89	0.86	0.82
120	1.00	0.96	0.93	0.90	0.86	0.82
180	1.00	0.97	0.93	0.90	0.86	0.83
240	1.00	0.97	0.93	0.90	0.86	0.84
360	1.00	0.98	0.93	0.91	0.86	0.84
480	1.00	0.98	0.93	0.91	0.86	0.85

III), with 11 min the non-stop time limit on the first dive ($\xi = 1$), from Table II. On the sixth dive ($\xi = 0.82$), the non-stop limit drops to 6 min. On dives 2—5, non-stop time limits decrease monotonically between 11 and 6 min.

6. Conclusions

Repetitive, deeper-than-previous, multi-day and multi-level diving present problems for the Haldane model which might be lessened in impact by a systematic reduction in critical gradients, or tensions, consistent with bubble mechanics and the phase volume limit. Reductions are based on possible excitation and regeneration of micronuclei and bubble inflation rates, and not dissolved gas build-up per se. A model, called the reduced gradient bubble model, RGBM for short, has been described and applied to a marginal multi-day profile, illustrating systematic reductions in critical gradients and tensions and hence non-stop time limits, across individual dive segments. Six adjustable parameters codify the model, with five of them the original VPM parameters and the sixth appropriate to the RGBM, under study and meter development. Certainly the fractions, ξ, can be freed from any model connection, indeed, fitted to repetitive diving data. That avenue is also being pursued, mainly the correlations between data and Eqns. (28—31). What is nice about a model for ξ is the predictive capability, as seen for the multi-day profiles. That is necessary for calculations on the fly underwater.

Acknowledgments

We thank colleagues and friends for their help and advice, including Tom Kunkle (LANL), David Yount (University of Hawaii), Brian Hills (University of Adelaide), Val Hempleman (RNPL), Charles Lehner and Ed Lanphier (University of Wisconsin), as well as the National Association of Underwater Instructors (NAUI), the

Professional Association of Diving Instructors (PADI), the YMCA, the Biophysics and Reactor Safety Groups (LANL) and the Los Alamos National Laboratory for personal support. Very special thanks to Steven Shankle (INSIGHT) and Doug Toth (SCUBAPRO).

References

1 Boycott AE, Damant GCC and Haldane JS: The prevention of compressed-air illness, *J Hyg,* 8 (1908) 342—443.
2 Buhlmann AA: *Decompression/Decompression Sickness,* Springer-Verlag, Berlin, 1984.
3 Workman RD: *Calculation Of Decompression Schedules For Nitrogen-Oxygen And Helium-Oxygn Dives,* USN Experimental Diving Unit Research Report, NEDU 6-65, Washington, D.C., 1965.
4 Spencer MP: Decompression limits for compressed air determined by ultrasonically detected blood bubbles, *J Appl Physiol,* 40 (1976) 229—235.
5 Weathersby PK, Homer LD and Flynn ET: On the likelihood of decompression sickness, *J Appl Physiol,* 57 (1984) 815—825.
6 Bell RL and Borgwardt RE: The theory of high altitude correction to the US Navy Standard Decompression Tables: I. The cross corrections, *Undersea Biomed Res,* 3 (1976) 1—23.
7 Vann RD, Dovenbarger J, Wachholz C and Bennett PB: Decompression sickness in dive computer and table use, *DAN Newsletter,* (1989) 3—6.
8 Walder DN: Adaptation to decompression sickness in caisson work, *Biometeor,* 11 (1968) 350—359.
9 Pilmanis AA: *Intravenous Gas Emboli In Man After Compressed Air Ocean Diving, Office of Naval Research Contract Report,* N00014-67-A-0269-0026, Washington, D.C., 1976.
10 Halsey MJ: *MRC Decompression Sickness Panel: Long Term Health Effects Working Group,* Medical Research Council Report, University of Hawaii, Honolulu, 1988.
11 Hills BA: *Decompression Sickness,* John Wiley & Sons Inc., New York, 1977.
12 Wienke BR: N_2 transfer and critical pressures in compartments, *Math Comp Mod,* 12 (1989) 1—15.
13 Wienke BR: Modeling dissolved and free phase gas dynamics under decompression, *Int J Biomed Comput,* 25 (1990) 193—205.
14 Yount DE and Strauss RH: Bubble formation in gelatin: a model for decompression sickness, *J Appl Phys,* 47 (1976)5081—5089.
15 Hennessy TR and Hempleman HV: An examination of the critical released gas concept in decompression sickness, *Proc Roy Soc London B,* 197 (1977) 299—313.
16 Wienke BR: Tissue gas exchange models and decompression computations: a review, *Undersea Biomed Res,* 16 (1989) 53—89.
17 Lang MA and Hamilton RW: *Proceedings Of The American Academy Of Underwater Sciences Dive Computer Workshop,* University Of Southern California Sea Grant Publication, USCSG-TR-01-89, Los Angeles, 1989.
18 Lang MA and Egstrom GH: *Proceedings of The American Academy of Underwater Sciences Biomechanics of Safe Ascents Workshop,* Diving Safety Publication AAUSDSP-BSA-01-90, Costa Mesa, 1990.
19 Yount DE and Hoffman DC: On the use of a bubble formation model to calculate diving tables, *Aviat Space Environ Med,* 57 (1986) 149—156.
20 Yount DE: On the evolution, generation, and regeneration of gas cavitation nuclei, *J Acoust Soc Am,* 71 (1982) 1473—1481.
21 Yount DE: Skins of varying permeability: a stabilization mechanism for gas cavitation nuclei, *J Acoust Soc Am,* 65 (1979) 1431—1439.
22 Yount DE, Yeung CM and Ingle FW: Determination of the radii of gas cavitation nuclei by filtering gelatin, *J Acoust Soc Am,* 65 (1979) 1440—1450.
23 Yount DE and Strauss RH: Bubble formation in gelatin: a model for decompression sickness, *J Appl Phys,* 47 (1976) 5081—5089.
24 Kunkle TD and Beckman EL: Bubble dissolution physics and the treatment of decompression sickness, *Med Phys,* 10 (1983) 184—190.

25 Farm FP, Hayashi EM and Beckman EL: *Diving And Decompression Sickness Treatment Practices Among Hawaii's Diving Fisherman,* University of Hawaii Sea Grant Report, UNIHI-SEA-GRANT-TP-86-01, Honolulu, 1986.
26 LeMessurier DH and Hills BA: Decompression sickness: a study of diving techniques in the Torres Strait, *Hvaldradets Skrifter,* 48 (1965) 54—84.
27 Leitch DR and Barnard EEP; Observations on no-stop and repetitive air and oxynitrogen diving, *Undersea Biomed Res,* 9 (1982) 113—129.
28 Wienke BR and Hills BA: *Bubbles and Bends and Bio-Transport,* Proc. Conf. Transport Theory Invariant Imbedding, And Integral Equations, New York, Marcel Dekker Inc. 1989.
29 Wienke BR: Computational decompression models, *Int J Bio-Med Comp,* 21 (1987) 205—221.