Marlow Anderson

The Physics of Scuba Diving

Nottingham
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This book provides a fun introduction to the mathematics and physics needed to really understand scuba diving; it was written by an enthusiastic scuba diver, who is also a mathematician.

It is written for the lay person, and assumes only a willingness to express basic scientific relationships using mathematical equations. It describes all of the basics regarding pressure, depth and density, covered more completely than in the typical open water diving course. But in addition, it looks at the mathematical background for dive tables and dive computers, tools divers use to avoid getting decompression sickness. The basic Haldane theory of nitrogen on-gassing and off-gassing is explained, in more depth than is found in popular literature.

The Physics of Scuba Diving will appeal to divers interested in a bit more background than is available in the popular literature, and to people interested in how basic science and mathematics has a large impact on the sport of scuba diving. The book has been used as a textbook for a popular introduction to mathematical thinking.

Contents

A professor of maths at The Colorado College, in Colorado Springs, Marlow received his undergraduate degree from Whitman College, studied algebra at the University of Kansas and received his PhD in 1978. He has co-authored a technical monograph and a textbook in maths, and has co-edited two books on its history. Scuba diving since 1996, he became a PADI assistant instructor in diving in 1998, and has been able to share his passion for both maths and scuba diving with many students.

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Preface

When taking my Open Water training course in scuba diving, I was immediately captivated by the columns and rows of numbers that make up the dive tables we were learning how to use. As a mathematician and educator, I naturally wondered: where do these numbers come from? They were obviously based on physics and mathematics somehow. My personal quest to understand those dive tables has resulted in this book.

After training, my wife and I immediately became enthusiastic divers, and now have 15 years of great scuba diving adventures to look back on. Along the way, we continued our diving education, eventually becoming PADI Assistant Instructors. Getting involved in diving education was natural for me, since I love to share my passions with other people. In this book I am able to share two passions at once: science and scuba diving! I have had the great opportunity to teach this material a number times in a course at The Colorado College, the liberal arts college where I am a mathematics professor. The course I teach does not assume much mathematical or scientific background, and is intended to entice students into appreciating the power of scientific thinking in understanding the real world, in this particular case the wonderful underwater world of scuba diving.

The students in my course, and the readers of this book, need only have a willingness to represent physical phenomena by using equations, and to manipulate these equations with a little algebra. With these tools, the reader will be able to better appreciate the rules and protocols she might have learned in her dive training. On the other hand, I have also had students of science or mathematics who had no dive training, who have enjoyed seeing how scientific thinking illuminates the practical endeavour of diving underwater. I always arrange a chance for such students to try out scuba diving in a swimming pool.

The fundamental science behind scuba diving encompasses a wide range of topics from physics including: pressure, Boyle's Law, buoyancy, the behaviour of ideal gases, the structure of water, and the movement of sound and light in water. By presenting these important principles of physics in the context of scuba diving I have found that students generally were motivated to understand and appreciate interesting and practical applications of physics.

To understand the numbers on the dive tables, we need to use the idea of exponential decay, a powerful mathematical model which describes many important phenomena in biology, physics and economics. To equip readers with this tool, there is one chapter which is mostly about mathematics and physics (Chapter 6). However, the pay-off in terms of understanding is rich indeed. In this chapter I talk about the rate of change of exponential functions; more mathematically sophisticated readers might recognize that a little calculus is sneaking into our discussion. However, the book is written for readers who have only some algebra in their mathematical backgrounds. Once the exponential function is introduced, it would be helpful to have a scientific calculator handy. For some of the computations which are repeated often, I recommend the use of a spreadsheet like MicroSoft Excel.

The material in this book can be covered easily in a single semester or quarter, even for students with quite modest scientific and mathematical backgrounds. Alternatively, it can be effectively used as a unit or project in a physics, calculus or mathematical modeling course for students with more background. Each chapter ends with a list of exercises, which will help test the reader's understanding of the text. There is no new material presented in the exercises, and so they are quite dispensable for the general reader. However, the best way to completely understand physics or mathematics is to actually do some computations.

When I first started thinking about teaching the course and writing the book, I was frustrated by the popular literature about diving, which in discussing the dive tables stops just when things get interesting scientifically. On the other hand, the technical literature is mostly written by scientists for scientists, and is consequently difficult to read for the layman. This book is intended to fill that gap. And so whether you are an avid diver with some scientific curiosity, or a reader interested in science who likes to see new applications of scientific ideas, I hope you will find this book interesting.

This is not a scholarly work, and is consequently not encumbered with scientific citations. However, I have included a guide to further reading, which includes my commentary on the books in the scuba literature I found helpful and valuable.

Many people have helped me in the creation of this book, but I owe a special thanks to Rona Culp, the diving instructor who trained me in most of my PADI diving courses. I learned a lot from Rona, both above and below the surface. My wife and I have for a long time been associated with a dive shop called Underwater Connection, in Colorado Springs, Colorado. We have helped with dive training, and also done a lot of dive travel with the staff at this store. The store is owned by Troy Juth, who has taught me a lot about diving, whether while relaxing between dives on a liveaboard dive boat, or while nursing a beer in a bar with water nearby. Troy has on several occasions given a practical and personal lecture on the history of diving equipment for my class. Other dive buddies from Underwater Connection from whom I have learned much include Luther Huffman, Bill Bailey, Pam McInturff, Peg Lewandowski and Maria Martelotto. I owe a big thank you to one instructor at Underwater Connection: Sam Wheatman. Sam gave a draft of this book an extremely close reading, and provided me with many pages of careful and valuable commentary. His efforts improved the book a great deal. Incidentally, Sam also attended some of my class sessions in one version of the course taught out of this book, and also gave a guest lecture. I was also honoured to have Ken Kurtis of Reef Seekers in Los Angeles have a look at my manuscript. I appreciate the advice of this nonpareil diver.

It has been my privilege to teach in the Department of Mathematics at Colorado College since 1982. The administration and my departmental colleagues have offered me their full support for teaching a course as crazy as "The Science of Scuba Diving", expecting me to make the course both fun and academically rigorous. My students over the years have had an important impact on the emphasis, style and content of the book.

I am grateful for the help and encouragement I have encountered from the people of Nottingham University Press, including Clifford Adams, Ros Webb and Sarah Keeling; I've enjoyed working with them. To Sarah I owe special thanks for turning my manuscript and rough diagrams into the beautiful book you hold in your hands. I also owe a debt of gratitude to Trevor Lipscombe, an editor at another press who suggested Nottingham to me, and contacted them on my behalf. I'm happy to see my book go alongside Trevor's Nottingham book The Physics of Rugby. I'm grateful to my friend Tom Culp, who helped me with the photograph of scuba equipment which appears in the book.

Last of all, I have to thank my usual dive buddy, best friend and wife Audrey Anderson. She has given the text a careful reading, and has given me lots of valuable advice and encouragement too. She's the best diver I know! I look forward to many more exciting dive adventures with her.

Marlow Anderson
Manitou Springs, Colorado
April, 2011

CHAPTER 1

PRESSURE

Anyone who has flown in an airplane or swum to the bottom of a swimming pool has encountered the effect of changing pressure. Your ears sense the lesser pressure in the cabin of the airplane, and the greater pressure at the bottom of the swimming pool. When diving in the ocean with scuba equipment, the change in pressure can be even greater, and this change has important physical and physiological effects. To understand these effects, we must first inquire carefully into what this pressure is, and how we measure it.

AIR & WATER PRESSURE

It's easy when sitting in an easy chair in a quiet room to forget that the air surrounding us is a physical substance that exerts force upon us. But we need only to step outside on a windy day to be reminded of this. *Force* is a quantity that physicists measure using units like *pounds*. An object weighs 10 pounds precisely because we need to exert a force of 10 pounds to move it. We've all seen winds powerful enough to move objects weighing 10 pounds.

Now obviously, some objects weighing 10 pounds are easily moved by wind, like a kite for example. On the other hand, a lump of lead weighing 10 pounds is unlikely to be moved by the wind. In other words, measuring force alone is not enough. Instead, we should really consider the force exerted *per unit area*; this gives us the notion of *pressure*. Because the area of the kite is larger, it takes much less *wind pressure* to move it than it does to move the lump of lead.

For scuba divers in the United States, a standard measure of pressure is *pounds per square inch*, or *psi* for short. Although we will use the abbreviation "psi", we should point out here that a more mathematical way to write this would be lb/in^2, where "lb" is the standard (rather peculiar) abbreviation for pounds. This more mathematical formulation makes it clear that to compute pressure, we need to measure the total force exerted in pounds, and divide that by the total area on which the force is exerted.

As an example of a pressure calculation, consider the force you exert on the floor when standing erect. Let's suppose that you weigh 150 pounds, and that the area of the bottom of your feet is about 75 square inches. You then exert a pressure of 150/75 = 2 pounds per square inch on the floor. Architects design floors precisely to withstand a certain maximum pressure exerted by a uniformly distributed "live load".[1] For residential purposes, a common limit is 40 pounds per square foot. We have here encountered a first case when we must change the units we're measuring things in. Each square foot consists of 12 × 12 = 144 square inches. The architectural limit can thus be translated into psi as follows:

$$40\text{lb/ft}^2 \times 1\text{ft}^2/144\ \text{in}^2 = .28\ \text{lb/in}^2$$

It appears that you have exceeded the architectural limit just by standing! However, architects are really more interested in a *distributed* load over the entire floor; one cannot pack enough people into an apartment to put a load of 2 pounds per square inch on every square inch of the floor!

But let's return to the question of pressure exerted by air. It's quite surprising to realize that the air surrounding us exerts a force of 14.7 pounds per square inch.[2] When you consider how many square inches of area there are on a human body, this sounds like an impossibly heavy force bearing down upon us. But we were after all born and raised under this pressure, and have evolved to withstand it. We consequently do not even notice it, unless there's a heavy wind changing the pressure, or we change altitude.

The air in which we live is a *fluid.* Speaking anthropomorphically, a fluid always seeks to equalize pressure everywhere; mathematicians and physicists say that a fluid seeks *equilibrium.* This means that the air pressure of 14.7 psi is exerted in all possible directions. When you hold your hand in a horizontal position, there is just as much pressure exerted by the air around us on the back of your hand as there is on your palm. This is why you don't notice the pressure the air exerts on your hand. But if you hold your hand up in a windstorm, the air does exert more force on one side of your hand than the other – we can feel this force then, precisely because the fluid consisting of the atmosphere is *not* in equilibrium.

So suppose we have a fluid with only the downward vertical force of gravity acting upon it. A given horizontal area will then have only the *weight of the fluid directly above it acting upon it.* So if we imagine a horizontal surface with an area of 1 square inch, then the total weight of the air above it (extending directly upward until there's no atmosphere left), is 14.7 pounds.

With precise measurements, a physicist at sea level can determine that a cubic foot of air weighs only .081 pounds, and a cubic inch then weighs only $.081/12^3 = .000047$ pounds! But there are miles and miles of air above our tiny one square inch of horizontal area. That makes it at least plausible that this air might weigh as much as 14.7 pounds. Later we'll do a calculation to verify this.

So now let's dive into another fluid – the ocean! Seawater is also a fluid, and so it exerts its pressure in all possible directions. This means that we can compute the additional pressure exerted by the water upon us by calculating the weight of the water above us.

It's an empirical fact that 1 cubic foot of seawater weighs 64 pounds.[3] Since there are $12^3 = 1728$ cubic inches in a cubic foot, this means that a cubic inch of water weighs 64/1728 = .037 pounds. So let's suppose that we are x feet deep in the ocean. Imagine now a one square inch patch on your body. What is the weight of the skinny box of water directly above you? The volume of a box is just its cross-sectional area (1 square inch), times its height in inches ($12x$ inches). This means that our skinny box of water above our 1 square inch patch must weigh

$$.037 \times 12x = .4444x$$

pounds. If we happen to dive to 33 feet, then this skinny box of water will weigh .4444 × 33 = 14.7 pounds.

But wait! The pressure on us will include not only the skinny box of 33 feet of seawater, but also the miles-long box of air above that. Consequently, at 33 feet of seawater we will be experiencing exactly *twice* the pressure as we would at the surface. So each 33 feet of depth in seawater will add an additional 14.7 psi to the pressure we experience. So this means that at 99 feet under the water, we will be experiencing 4 times as much pressure as we would at the surface!

As another example of this, consider a scuba diver at 45 feet of seawater. How much pressure is she experiencing at this depth? Let's do this calculation twice. As we saw above, the weight of water above one square inch at 45 feet will weight .4444 × 45 = 20.0 pounds. The scuba diver in question will then be experiencing 20 + 14.7 = 34.7 psi of pressure. But here is a second approach, which takes advantage of the fact that 33 feet of seawater *doubles* the total pressure. We then have that 45 feet contributes 45/33 = 1.36 as much pressure as does the 33 feet. The *total* pressure the diver experiences is then 1.36+1 = 2.36 as much pressure as the person on the surface. This is 2.36 × 14.7 = 34.7 psi.

The second approach to this problem suggests why scuba divers often use a *different unit of pressure than psi*. We define 1 *atmosphere* of pressure as 14.7 psi. So when we are at 99 feet in the ocean, we will be experiencing 4 atmospheres of pressure (1 because of the air above us, and 3 for the 3 × 33 = 99 feet of water above us). So we say that at this depth we experience 4 atmospheres of pressure, or 4 ata for short.[4]

How much pressure is exerted upon us, if we are diving to a depth of 74 feet in the ocean? This becomes a simple problem in proportions. We have 74/33= 2.24 atmospheres of pressure due to the water. When we add on the air pressure, we obtain a total pressure of 3.24 = 1+2.24 atmospheres, or 3.24 × 14.7 = 47.6 psi.

This kind of calculation shows why we feel the change of pressure even when swimming down to the bottom of a ten foot pool. In this case we have 10/33 = .30 atmospheres of additional pressure. That is, even at 10 feet we are experiencing 30% more pressure than we are at the surface. This additional pressure on the delicate membrane called the eardrum is enough to cause discomfort or even pain.

There is actually a technicality we avoided in the previous paragraph. We were assuming there that the pool we were swimming in was filled with seawater at sea level. Actually, most pools are filled with fresh water, and fresh water weighs only 62.4 pounds per cubic foot. This really means that in fresh water we will only experience an additional atmosphere of pressure if we dive to

$$(64/62.4) \times 33 = 34$$

feet of fresh water. This is a small but important difference, as we will explore more later.

HOW IS AIR PRESSURE MEASURED?

How could we experimentally determine that air pressure at sea level is 14.7 psi? To do this is we use a *barometer*, which is precisely a scientific instrument designed to measure air pressure. There are actually many types of barometers, but the first one was designed by the seventeenth-century Italian mathematician and physicist Evangelista Torricelli. Torricelli's mentor and teacher was Galileo, and he brought to Torricelli's attention the curious fact that dock workers could only raise seawater 33 feet while using an air pump; Galileo asked Torricelli to explain why.

Torricelli took a tube closed at one end and filled it with the heavy liquid metal mercury. When he inverted this tube into an open basin of mercury, some of the mercury in the tube flowed into the basin. But not all of it did; instead, a column of mercury remained in the tube, with a space above it.[5] This column was held up exactly by the force of the air exerted on the mercury in the open basin. When Torricelli measured the height of the column of mercury so supported, he discovered that it was 30 inches tall.

How does this experiment lead to our figure of 14.7 psi for air pressure? One cubic inch of mercury weighs .49 pounds. Consequently, a column of mercury 30 inches tall will weigh 30 × .49 × A, where A is the cross-sectional area of the tube. We may as well assume that we are using a tube with a cross-sectional area of exactly 1 square inch. In that case, our mercury weighs 30 × .49 = 14.7 pounds, and so we arrive at our value of the pressure on the bowl of mercury as 14.7 psi.

Shortly after Torricelli's experiments, the great French mathematician and philosopher Blaise Pascal conjectured correctly that air pressure should decrease as we move to higher altitudes. To verify this, he sent his brother-in-law to a mountain top with a mercury-filled tube and sure enough, the mercury rose to a level less that 30 inches, as Pascal had predicted it would. We'll later inquire more carefully into the mathematical relationship between altitude and atmospheric pressure.

This leads us to another standard unit for pressure: "inches of mercury" or "inHg".[6] Indeed, weathercasters in the United States usually report air pressure using this unit, although they often report these numbers without explicitly referring to the unit they are using, saying instead something like "the barometer stands at 31 inches tonight." Assuming that our weathercaster is at sea level, he is thus reporting that the air pressure is a bit higher than he would expect when just considering the weight of air above him. Anyone who has watched many weather forecasts knows that these local variations in the pressure found in the fluid of the atmosphere can have important practical effects on our weather!

For scuba divers interested in diving in seawater, the more natural unit to use is the height of a column of seawater supported by air pressure, rather than the height of a column of mercury. As we've seen already, that height is 33 feet, and so we will often use the unit "feet of seawater" or "fsw".

Of course, this provides the explanation of the restriction imposed on the dock workers observed by Galileo. They raised seawater by displacing it with air moved in place by a piston pump. The hand-operated pump did not condense or pressurize the air, which instead stayed at the pressure of 1 ata, or 33 fsw.

To summarize, we can refer to air pressure at sea level with several different numbers, depending on the unit of measurement we use:

$$14.7 \text{ psi} = 1 \text{ ata} = 30 \text{ inHg} = 33 \text{ fsw}.$$

In the exercises you will be asked to compute equivalent measures of air pressure in all these units, and indeed more units that we will encounter below.

The Scuba Cylinder

An important way in which a scuba diver encounters units of pressure is when discussing his *scuba cylinder* (or *scuba tank*). This is a rigid metal cylinder containing ordinary air, but kept at a pressure greater than atmospheric pressure. We'll discuss the scuba tank considerably later on, but for now we will only remark that the most commonly used cylinder in the United States contains air that can be pressurized to as much as 3000 psi. This pressure is

$$3000/14.7 = 204.08$$

atmospheres – roughly 200 times as much pressure as the surrounding air. Modern scuba divers always have a gauge attached to this tank which reads the pressure inside the scuba cylinder. As we shall see in Chapter 3, this pressure reading amounts to telling the diver how much air he has available for the remainder of his dive. That's why a typical scuba diver might report that she has "1000 pounds of air left". This of course does not mean that the air in her tank weighs 1000 pounds; instead, it actually means that the *pressure* of the air in her tank is

$$1000 \text{ psi} = 1000/14.7 = 68 \text{ ata}.$$

Metric Units

Among scuba divers in the United States, pressure is almost always measured in either psi or atmospheres or fsw, and consequently we will mostly adhere to these units of measurement. We will call this system of measurement the *imperial system* (reflecting its origins in Britain).

In Europe (and in the scientific literature), pressure is measured in metric units instead. The *International System of Units* (or SI for short) is cleverly designed based on powers of ten. Sensible definitions relating units for length, area, volume, mass, force and pressure are made, with the object of minimizing the number of arbitrary constants needed to relate one unit to another. Indeed, water plays an important role in these definitions; this actually makes many scuba-related computations much easier in the SI system than in the imperial system.

In the SI system the basic unit of length is the *meter* instead of the inch; we abbreviate meters by m. One meter is approximately equal to 39.37 inches (or 39.37/12 = 3.28 feet). For shorter distances, we use the *centimeter* instead. The prefix "centi" means one-hundredth, and so one centimeter is one-hundredth the length of a meter, and is consequently about .3937 inches. The basic unit of force is the *newton* instead of the pound; we abbreviate newtons by N. One newton is approximately .2248 pounds. The basic unit of pressure in the SI system is then 1 newton of force per square meter. Appropriately enough this unit is called the *pascal*, and is abbreviated by Pa. We consequently have the following:

$$1 \text{ pascal} = 1 \text{ Pa} = 1\frac{\text{N}}{\text{m}^2}$$

Let's now express one atmosphere of pressure using pascals. We have that

$$1\,\text{Pa} = 1\frac{\text{N}}{\text{m}^2} = \frac{.2248\,\text{lb}}{(39.37)^2\text{in}^2} = .000145\frac{\text{lb}}{\text{in}^2} = .000145\,\text{psi}.$$

That is, a pascal is a much smaller unit of pressure than is a psi. To translate 1 atmosphere (or 14.7 psi) into pascals, we need only divide: 14.7 psi is equivalent to

$$14.7/.000145 = 101325 \text{ Pa}.$$

The answer we got in this example is large enough that we customarily take advantage of the way SI units make use of powers of ten. The prefix "kilo" means one thousand, and so a *kilopascal* (kPa for short) is precisely

$$1 \text{ kPa} = 1000 \text{ Pa} = 10^3 \text{ Pa}.$$

Thus, 1 atmosphere of pressure is 101.325 kPa, or more approximately, 100 kPa.

Scuba divers in Europe have an even better unit for pressure. A *bar* of pressure is defined to be 10^5 Pa. Then 1 atmosphere of pressure is equal to 1.01325 bars. In practice, divers tend to consider 1 bar of pressure to equal 1 atmosphere, although actually 1 bar is equal to only 14.7/1.01325 = 14.5 psi. Indeed, while gauges on scuba cylinders in the United States (and much of the Caribbean) measure pressure in psi, gauges on European equipment are calibrated in bars.

For example, suppose that a French diver reports that her pressure gauge gives a reading of 160 bar. What would this translate to in terms of psi? Here is the required arithmetic:

$$160 \text{ bar} = 1.6 \times 10^7 \text{ Pa} = 1.6 \times 10^7 \times .000145 \text{ psi} = 2320 \text{ psi}.$$

Although translating from one of these unit systems to the other is just a matter of multiplying (or dividing) by the appropriate constant, it can in practice be confusing; you'll have ample opportunity to practice these conversions in the exercises. In Appendix A you will find a table describing unit conversions in detail.

BREATHING UNDERWATER

As we all know, in order to survive human beings must breathe almost continuously. This means that our stays underwater can only be of short duration if we breath-hold — for most of us a minute or less. If we wish to stay underwater longer, we must obtain a continuing supply of fresh air for us to breathe.

One method that works well is a *snorkel*, which is basically just a tube with one end above the surface of the water, and the other placed in the swimmer's mouth. A swimmer equipped with a snorkel can float face down in the water indefinitely, because the snorkel tube allows the swimmer to inhale fresh air through the snorkel tube and into the mouth and from there into the lungs. This allows the swimmer to observe what's happening underwater, without having to lift her head out of the water to take a breath.

It has occurred to many people over the years that it might be possible to lengthen the snorkel tube, and thus allow the swimmer to remain fully immersed in the water. However, the additional pressure experienced by the swimmer at depth makes this impossible. The pressure of the water (in addition to the atmosphere above) presses down upon the swimmer's chest; experiments have shown that a human being can only effectively inhale when the additional pressure is no more than 2 psi greater than the internal lung pressure (coming from the surface air being inhaled). But

$$2 \text{ psi} = (2/14.7) \times 33 \text{ fsw} = 4.5 \text{ fsw},$$

and so a snorkel could certainly not work at any great depth.

But even a snorkel as short as 4.5 feet will not work, because there are a couple of other practical reasons that limit the length of a snorkel even more. Air drawn through a slender tube encounters considerable air resistance created by the sides of the tube; as the tube gets longer, it takes more and more work on the part of the breather to draw air through the tube. Indeed, modern snorkel designers take pains to minimize the air resistance of their best models, even though they are no more than 17 inches long.

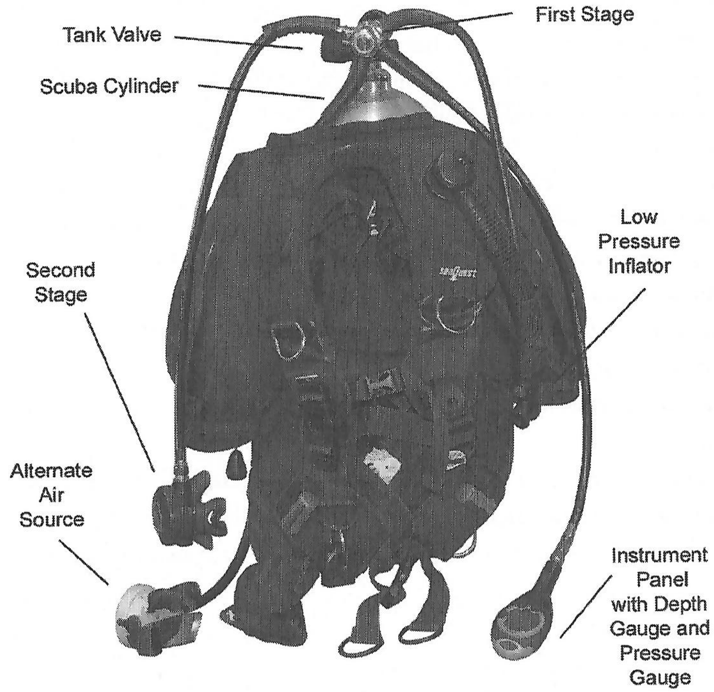
Another practical limitation on snorkels is related to the exhalation part of the breathing cycle. An exhalation (which needs to be forceful because of air resistance) will fill a long snorkel tube with oxygen-poor, stale air, and this will result in the swimmer breathing in air that is more and more stale. Scuba divers call the interior of the snorkel a *dead air space*, which must be cleared with each breath, in order to maintain a steady supply of fresh air from the surface. As the tube gets longer and longer, it because more and more difficult (and eventually impossible) for the snorkeler to clear the dead air space.

This is not to say that it is impossible to breathe via a tube or hose supplied by fresh air from the surface. The traditional military or industrial deep sea diver with the large brass helmet and lead boots breathes in exactly this way. But the air supplied from the surface is pumped into the hose, *at a pressure equal to the pressure the diver is experiencing at depth*. The diver then encounters no difficulty breathing, because the air drawn into the lungs has pressure in equilibrium with the ambient pressure he is experiencing in the water. This is not a system designed for casual recreational use, however, because the diver requires a tender at the surface to insure that a steady supply of fresh air at the appropriate pressure is supplied to the diver via his hose.[7] In the old days, such pumps were operated by manpower; today electrical or diesel-powered compressors are used.[8]

So, to design a SCUBA system – that is, a *Self Contained Underwater Breathing Apparatus* – it is necessary to overcome this problem of breathing while under pressure greater than that of the topside air. There are really only two technological possibilities. The first is to encase the diver in a rigid enough suit that the larger ambient pressure does not directly impose itself upon the diver; this is the model of the submarine. There are incredibly cumbersome and elaborate diving suits based on this model, typically used today for extremely deep diving (and hence extremely large surrounding pressure). But the alternative is technologically much less daunting: merely design a system that supplies air to the diver at exactly the surrounding pressure!

From this observation was born the *demand regulator*, a valve that on inhalation ("on demand") supplies air to the diver at exactly the right pressure. It was French naval officer Jacques Cousteau and an engineer named Emile Gagnan who designed the first scuba system (or "aqualung") on this basis, making use of Gagnan's design of a valve that delivered natural gas to the carburetor of military vehicles.

In the Cousteau-Gagnan device, the diver descends with a scuba cylinder, a rigid metal tank filled with air compressed to considerable pressure (as we discussed above, in modern systems this pressure can be 200 ata or even more). A valve (the *regulator*) then releases air at the appropriate ambient pressure, only when the diver breathes in. So when a scuba diver is at 50 fsw, the demand regulator will supply air to her at 83 = 50+33 fsw, which she can then inhale effortlessly.



Scuba technology has evolved a lot since the original Cousteau-Gagnan aqualung. The modern scuba regulator reduces the pressure in the tank in two stages. The *first stage* valve (mounted at the tank stem) supplies air at a more moderate pressure to the *second stage*, which the diver holds in his mouth and breathes through. The second stage supplies air at the ambient pressure on demand. A modern system also includes a redundant second stage (the *alternate air source*), which can be used to supply air to another diver who might be out of air. Such a system also includes a *pressure gauge* to measure pressure in the tank, and a *depth gauge* to measure the depth in the water. The tank is attached to the diver's *buoyancy vest*,[9] which can be inflated with air either orally, or by means of a valve and low-pressure hose that is connected to the first stage. But these refinements are only elaborations of the original aqualung, which supplies air of appropriate pressure to the diver.

Now our bodies have evolved to breathe air at about 1 atmosphere of pressure. Are there physiological effects that result from breathing air at higher pressures? As we shall see in future chapters, the answer is yes. Some of these effects have potential to harm the diver. But fortunately, understanding the mathematics and physics of these effects will enable us to dive safely and avoid any adverse consequences.

EXERCISES

1. Translate the following measurements of length, volume and force, into the suggested units (some of these problems involve units you are probably familiar with, which are not mentioned in the text):

   (a) 176 inches, into meters.
   (b) 15 centimeters, into inches.
   (c) 14 meters, into yards.
   (d) 145 newtons, into pounds.
   (e) 1 ton, into newtons.
   (f) 450 cubic centimeters (cc), into cubic inches.

2. In the text we considered a person weighing 150 pounds, standing erect with total shoe area of 75 square inches. What happens to the pressure on the floor, if the person stands on one leg? What if he stands on one leg, and on tiptoe? Now suppose that our 150 pound person is wearing high heels. At an appropriate moment in the stride, all weight may well rest on one heel. Suppose that the heel is 1/4 inch in diameter. How much pressure is exerted on the floor in psi? in ata?

3. Suppose that a scuba diver is on a dive in the ocean at 79 feet. How much pressure is this diver experiencing? Give your answer in the following units:

   (a) psi.
   (b) fsw.
   (c) kilopascals.
   (d) bars.

4. Repeat the previous exercise, under the assumption that the diver is in 79 feet of *fresh* water.

5. A scientist conducts an experiment and reports a pressure of 1157 Pa. What is that pressure, expressed in psi?

6. A mechanic inflates a tire to 30 psi. What is this pressure[10], measured in the following units?

   (a) ata.
   (b) fsw.
   (c) kilopascals.
   (d) bars.

7. One atmosphere of pressure is equal to 33 fsw. How many meters is this?[11]

8. One atmosphere of pressure is equal to 30 inHg. In the SI system, this is measured in millimeters of mercury instead (mmHg). How many mmHg is 1 atmosphere?[12]

9. Glycerin is a liquid that weighs 1.26 times as much as pure water, per unit volume (as we shall see in the next chapter, we say that glycerin has *specific gravity* 1.26). How many inches of glycerin would be supported by 1 atmosphere of pressure?

10. As we shall see later, breathing pure oxygen can be hazardous at pressures equal to 1.6 ata. In how many feet of seawater could breathing pure oxygen from a scuba apparatus be hazardous?

11. A room in a house at sea level is 8 feet high, 12 feet wide, and 15 feet long. How much does the air filling this room weigh?

12. Suppose that the room in the previous exercise is filled with seawater. How much would that seawater weigh? How much would that amount of fresh water weigh?

13. A scuba diver on an ocean dive notes that she is experiencing 37.8 psi of pressure. In how many feet of seawater is she?

14. Suppose a snorkel is 17 inches long, and one inch in diameter. What is the volume of this snorkel? (Assume the snorkel is a circular cylinder – this isn't quite true in practice, but is a good approximation).

    Now translate this into cubic centimeters. Compare this dead air space to the typical human respiration, which is about 500 cc.

15. The top of a submarine is in 50 fsw. The interior of the submarine is pressurized at 1 ata. A circular hatch on the top of the submarine is 4 feet in diameter. Suppose that the hatch itself weighs 50 pounds. How many pounds of force will a scuba diver require to open the door from the outside? What if the the top of the submarine is in 20 fsw? [13]

ENDNOTES

1 This excludes the "dead load" consisting of the weight of the floor system itself.

2 This is the figure for those us who live at sea level; it's less for people at higher elevations, and we will explore this more later.

3 This is an average and approximate figure, because the salt content (or *salinity*) of the ocean varies considerably from place to place. This varying composition of seawater in turns affects its weight.

4 The notation "ata" stands for "atmospheres absolute", where the word "absolute" indicates that we are computing all the pressure, and not just the *relative* pressure added by the water.

5 An interesting side effect of this experiment is that the space at the top of the tube vacated by the mercury is now filled with neither mercury nor air, and indeed with nothing! We call this a *vacuum*.

6 "Hg" is the chemist's notation for the element mercury.

7 In such a system, the stale exhaled air is vented out of the breathing supply system, and into the water.

8 Actually, there are some recreational systems based on these principles, typically designed for introductory diving for the young or inexperienced. But such divers have considerable limitation in movement, because they are literally on a short leash, no more than 15 or 20 feet deep. These are called *hookah* systems.

9 This is often called the *buoyancy control device*, or BCD for short.

10 Incidentally, the air gauge used by the mechanic will read 0 psi if the tire is completely deflated, but actually complete deflation means only that the pressure inside the tire is equal to the pressure outside the tire, namely 14.7 psi. The mechanic's gauge is really measuring the *relative change* in air pressure, as compared to the ambient air. Divers speak of this as a measurement of *gauge pressure*, as opposed to *absolute pressure*.

11 The answer to this exercise shows another reason why scuba calculations are easier in SI units; note that all these calculations are approximate, and scuba divers use the nearest whole number figure.

12 Once again, these conversions are approximate, and the usual figure people use is to the nearest 10 millimeters.

13 Of course, in practice submarines have a double door system with an intervening chamber that can be flooded with water, in communication with the water outside. This equalization of pressure reduces the force necessary to move the door to the weight of the door.

CHAPTER 2
BUOYANCY

We have already seen how a fluid seeks to equalize pressures and reach equilibrium. One extremely important consequence of this is that if we submerge an object in water, the water exerts an upward force on the object exactly equal to the weight of the water the object displaces. This is known as *Archimedes' Principle*, named after the famous ancient Greek mathematician who first enunciated it. This is important enough that we will re-state it with emphasis:

> **Archimedes' Principle** *In an environment subject to gravity, an object submersed in a fluid is buoyed upward by a force exactly equal to the weight of the fluid the object displaces.*

As we shall see, this principle has significant consequences for scuba divers.

SPECIFIC GRAVITY

Archimedes' Principle makes it important to decide how much an object weighs, *relative to water*. This idea is conveniently measured by the concept of specific gravity. A substance's *specific gravity* is the ratio of weight of a given volume of the substance to the weight of the same volume of pure water.[1]

It is then clear that water itself has a specific gravity of exactly 1. But consider seawater: we know that a cubic foot of pure water weighs 62.4 pounds, while a cubic foot of seawater weighs 64 pounds. Consequently, the specific gravity of seawater is 64/62.4=1.026. In the previous chapter we reported that one cubic inch of mercury weighs .49 pounds. Consequently, a cubic foot of mercury will weigh $.49 \times 12^3 = 846.7$ pounds. (We are here changing from cubic inches to cubic feet: there are $12^3 = 1728$ cubic inches in a cubic foot.) Consequently, the specific gravity of mercury is 846.7/62.4

= 13.6. Note also that in one of the exercises in the previous chapter, we specified the weight of glycerin by giving its specific gravity (which happens to be 1.26). Consult Appendix B for a table listing the specific gravity of some common substances.

Now consider an object with specific gravity of 1, submersed in pure water. The force of gravity downward is then exactly counterbalanced by the buoyant upward force of the water. We call such an object *neutrally buoyant.* Absent any other force acting on a neutrally buoyant object, it will float motionless in mid-water.

On the other hand, if an object has specific gravity less than 1, the force of gravity downward is now *less* than the buoyant upper force. We call such an object *positively buoyant.* The buoyant force of the water will push the object to the surface of the water. It floats![2]

Finally, if an object has specific gravity greater than 1, the force of gravity downward will now be *greater* than the buoyant upper force. We call such an object *negatively buoyant.* Gravity will push the object downward, until it reaches the bottom of the water. It sinks!

For scuba divers, the concept of buoyancy is crucial: a scuba diver seeks to be neutrally buoyant. If she is negatively buoyant, she must swim upward to maintain her depth in the water. On the other hand, if she is positively buoyant, she must swim downward to stay at the same depth. But if she is neutrally buoyant, she experiences a "weightless" state, with no force pushing her up or down. This makes hovering in mid-water effortless.[3] Scuba divers adjust their buoyancy to achieve this state by adding or removing lead weights from their weight belts,[4] and adding or releasing air from their inflatable buoyancy vests.

Note that the buoyancy of an object depends only on its volume and its weight, and in general these factors do not depend on depth. Suppose that a piece of wood weighs one pound less than a corresponding volume of water. We then say that the wood has a net positive buoyancy of 1 pound of force, and this will apply according to Archimedes' Principle, whether it is at 50 feet or at 100 feet. We are assuming here that the *volume* of the wood does not change as we change its depth; this is a reasonable assumption, at least at pressures which might be encountered by a scuba diver. However, there are

compressible objects (like a balloon filled with air) whose volumes are affected by depth; this in turn affects their buoyancy. We will inquire into this in the next chapter.

There is an alternative way to think about specific gravity, and that is by using the idea of *density.* Speaking intuitively, we think of a material with a larger specific gravity than some other material as being more dense. Density itself can be measured as *weight per unit volume*[5]. We have already encountered this idea several times: we reported that air at sea level has a *density* of .081 pounds per cubic foot, and that pure water has a *density* of 62.4 pounds per cubic foot. The advantage of specific gravity is that the number we arrive at does not depend on the units we choose to use. The SI system of units takes full advantage of this, because 1 cubic centimeter of water has mass of 1 gram; in this system of units water has density 1 (grams per cubic centimeter), and so the specific gravity of a given substance is the same numerically as the mass of 1 cc of the substance[6].

A famous story about Archimedes illustrates these ideas. He was asked whether an ornate crown was in fact made of pure gold, as the artisan claimed. Archimedes knew what a given volume of pure gold would weigh – using modern language, he understood its density (or specific gravity). But to determine the volume of the crown seemed impossible, except by melting it down (and destroying it). But he then happened on the idea that he could determine the volume of the crown by submersing it in water, and measuring the water displaced. He could then determine whether the crown was made of dense pure gold, or less dense gold adulterated with silver. By legend, he was in the bath when he came to this conclusion; he leapt out, and ran through the streets of Syracuse naked, crying "Eureka!" ("I have found it" in Greek).

It is important to note that buoyancy depends on the *overall* weight and volume of the object. This means that we can construct a floating ship out of steel, a material with specific gravity much larger than 1. We need only build an enclosed hull (filled with air). The ship then displaces water weighing much more than the total weight of the ship.

As an example of this, we will now consider a boat made of sheet metal, and decide whether it will float in fresh water. Let us build an open box out of quarter inch iron sheet metal.[7] The box is 8 feet long, 4 feet wide and 3 feet high. Will this box float in fresh water? If so, where will the water line be on our box?

To solve this problem, we must first compute the weight of the box. The bottom has surface area 4 × 8 = 32 ft^2. The two ends have surface area 2 × 4 × 3 = 24 ft^2. Finally, the sides have a total surface area of 48 ft^2. Thus the box has a total surface area of 32+24+48 = 104. Since the sheet metal is 1/4 inch thick, this means that the metal of the box has volume[8]

$$104\ \text{ft}^2 \times \frac{1}{4}\text{in} \times \frac{1\ \text{ft}}{12\text{in}} = 2.167\ \text{ft}^3$$

Now it turns out that iron has specific gravity 7.87. This means that our box weighs

$$7.87 \times 62.4 \frac{\text{lbs}}{\text{ft}^3} \times 2.167\ \text{ft}^3 = 1064\ \text{lbs}$$

where we have used the fact that one cubic foot of fresh water weighs 62.4 pounds.

But actually, we do not really need to explicitly compute this weight. Instead, we only need determine the volume of water which weighs the same amount! Since iron is 7.87 times as heavy as water, this volume will be 7.87 × 2.167 = 17.05 cubic feet.

Now consider the box placed in the water. We hope that our box will float. Suppose that it does, and the water line on the box is h feet from the bottom. Then the portion of the box that is submerged has volume $h \times 4 \times 8 = 32h\ \text{ft}^3$. This is of course the volume of the water that the box displaces. By Archimedes' Principle, the volume of water should be equal to 17.05 cubic feet. But then $32h = 17.05$, and so $h = .53$ feet. Our box floats, and indeed, rides quite high in the water! You will explore this example a little further in the Exercises.

One of the interesting side-effects of Archimedes' Principle is that scuba divers are able to move objects underwater that they could not budge on the surface. This is simply because they are assisted by the buoyant force of the water in which they are immersed.

Even if an object is quite heavy, scuba divers can still bring it to the surface, by attaching bags to it, which they then inflate with air. This strategy vastly increases the volume of the object, while increasing its weight very little. Eventually, this makes the once-heavy object weightless; then it can be moved with little effort. Divers call these *lift bags*.

Suppose that an automobile is submersed in a lake, in 80 feet of fresh water. The automobile weighs 4000 pounds, and its volume is 30 cubic feet.[9] Now 30 cubic feet of fresh water weighs 30 × 62.4 = 1872 pounds. Consequently, the car is 4000−1872 = 2128 pounds negatively buoyant. In order to make the car neutrally buoyant, we need to counteract this 2128 pounds.

Now 2128 pounds of fresh water has volume 2128/62.4 = 34.1 cubic feet. Consequently, we must attach a lift bag (or in practice, lift bags), whose total volume is 34.1 cubic feet.[10] Notice that in performing this calculation we did not need to know the depth of the lake, because the weight and the volume of the car are not affected by its depth. However, in the next chapter we will see how much air from our scuba tanks needs to be added to the lift bag in order to achieve the volume necessary to make the automobile neutrally buoyant; this calculation *will* depend on the depth.

There are many additional practical problems facing divers actually trying to lift an automobile off the bottom of the lake. One is that such an object may be embedded in mud, which exerts a negatively buoyant force on the automobile in addition to its weight. Once this force is counteracted by the lift bag, the object may abruptly become positively buoyant and rocket without control to the surface. There is also the problem of balance: in practice, several lift bags in different locations will need to be attached to the object being lifted.

However, the most important practical problem for the divers attempting to lift this automobile from the bottom of the lake is that their lift bags will *expand* as the automobile moves towards the surface. This could lead to a dangerous uncontrolled ascent. This is a consequence of *Boyle's Law*, which we will discuss in the next chapter.

Buoyancy of the human body

The human body has a specific gravity of approximately one; this is because we are mostly composed of water. However, each of us has a slightly different buoyancy, depending on our body type and age. Fat cells are positively buoyant, bones are negatively buoyant, and muscle cells are (slightly) negative. Consequently, some human beings float easily, while others "sink like a stone".

Furthermore, seawater weighs a little more than fresh water; as we indicated above, the specific gravity of seawater is about 1.026. Consequently, the buoyant force in salt water, being the weight of the given volume of the object, is about 2.6% more than in fresh water. Those of us who have swum in a fresh-water pool and also in the ocean have experienced this — we float more easily in the ocean. The practical effect of this for scuba divers is that when moving from fresh to salt water, they must add a little weight to their equipment in order to counteract the additional buoyancy of salt water.

Suppose that a geared-up diver weighs 150 pounds, and is neutrally buoyant in fresh water. How much additional weight should she add, in order to be neutrally buoyant when she dives in salt water? The neutrally buoyant diver in fresh water must have volume

$$\frac{150\text{lb}}{62.4\frac{\text{lb}}{\text{ft}^3}} = 2.4\ \text{ft}^3$$

But 2.4 ft^3 of seawater weighs 2.4 × 64 = 153.6 lb. Consequently, the diver needs to add 3.6 pounds in order to be neutrally buoyant.

You might prefer to rephrase the previous calculation in terms of proportions. The weight of the diver in fresh water (150 lbs) is to the weight (62.4) of one cubic foot of fresh water, as is the weight of the diver in salt water (w) is to the weight (64) of one cubic foot of salt water. That is, 150/62.4= w/64, or w = 153.6, as before. By all means use the reasoning which you find most persuasive!

Actually, the previous calculation assumes that we can add 3.6 pounds of weight without adding any volume at all. In practice, scuba divers increase

their weight by adding lead to their equipment. Lead is so dense (has such a large specific gravity) that for a small increase the additional volume can be considered negligible. In one of the exercises you will explore this tiny difference.

Exercises

1. A gallon contains 231 cubic inches. How much does a gallon of fresh water weigh? How about a gallon of salt water?

2. A liter of fresh water (1000 cubic centimeters) has mass of 1 kilogram.[11] How much mass does a liter of mercury have?

3. An object has a volume of 1 cubic inch. This object weighs .6 ounces. Does it float in fresh water or not? (There are 16 ounces in a pound.)

4. An object has a volume of 400 cc, and has mass 500 grams. Will it float or sink if it is suspended in a vat filled with glycerine? (Recall that glycerine has a specific gravity of 1.26.)

5. Two cubic feet of a given substance weighs 187 pounds. What is the density of this substance (in pounds per cubic foot)? What is its specific gravity? What is its density (in grams per cubic centimeter)? Why would it be silly to ask for its specific gravity, in SI units?

6. An object weighing 750 pounds and displacing 10 cubic feet lies in 50 feet of fresh water. A 55 gallon drum is to be used to lift the object to the surface. How much air (in gallons) must be added to the drum? (You may disregard the weight of the drum, which is negligible compared to the weight of the sunken object.)

7. Where would the water line be, if we place our rectangular sheet metal box into saltwater?

8. How many 200 pound passengers will our rectangular sheet metal box floating in fresh water accommodate, before it sinks?

9. A geared-up diver weighs 180 pounds; he is neutrally buoyant in fresh water. He is wearing a thin, neutrally buoyant dive suit. The

suit weighs 5 pounds. He trades the suit in for a thicker suit which is positively buoyant – its specific gravity is .75. The new suit weighs 12 pounds. How much weight must he add to remain neutrally buoyant? (We will assume that the additional volume of the weight added is negligible.)

10. A 120 pound diver embarks on a fitness program; when she begins, she is neutrally buoyant in seawater. She turns 5 pounds of fat into 3 pounds of muscle. How does this affect her buoyancy? Assume the specific gravity of fat is .8, and the specific gravity of muscle is 1.08. *Hint:* Compute her total volume, and then compare the volume of 5 pounds of fat to the volume of 3 pounds of muscle.

11. A geared-up diver weighs 200 pounds, and is neutrally buoyant in salt water. He knows that he would be negatively buoyant in fresh water. How much weight should he remove from his weight belt, in order to be neutral in fresh water? (You may disregard the volume of lead removed.)

12. Suppose as in the example in the text that a 150 pound diver is neutrally buoyant in fresh water, and wishes to add weight to ensure that she is neutrally buoyant in salt water. How much lead should she add to ensure this? This time, include the volume of the lead in your calculation. You will need to know that the specific gravity of lead is 11.35. Your answer should be slightly more than the number computed in the text!

Endnotes

1 Physicists will complain that we should really speak of mass, rather than weight — but this important distinction is not so important for us here.

2 Note furthermore that once the object is floating, the portion of it *below* the water line will displace a quantity of water which weighs exactly what the entire object weighs.

3 Indeed, part of astronaut training involves scuba diving, to experience in some measure the weightlessness an astronaut experiences when outside the influence of gravity. The experiences are similar, but not identical. A scuba diver still has a sense of up and down, which comes from the subtly different pressure experienced by the upper and lower parts of the body.

4 Actually most recreational divers do not wear their lead on a belt anymore, but instead stow it in removable pockets in their vests. Most divers find this more comfortable and convenient than the traditional weight belt.

5 More correctly, it should be mass per unit volume.

6 Technically, we say that specific gravity is a unitless quantity, or pure number.

7 Of course, this would be a very unsatisfactory design from the point of view of propulsion through the water; this design will make our math much easier, however.

8 We are actually double counting the welds at the corners, but these have negligible volume. Indeed, an engineer would undoubtedly reinforce these joints, adding some additional weight that we will disregard.

9 The volume of the automobile is a tricky concept. We are not including the volume of the passenger cabin, which after all is filled with water at depth, but only the volume of the engine, frame and other parts. It's easy to decree this volume in a problem, but we might have to take Archimedes' approach to the crown in order to determine this quantity in practice!

10 We are here assuming that the weight of the lift bag, and for that matter the weight of the air in the lift bag, is negligible relative to the weight of the object being lifted. That is certainly the case in this example.

11 As we've seen, this is no accident, but part of the design of the SI system.

Chapter 3
Boyle's Law

Chemists tell us that a well-mixed gas consists of an unimaginably large number of tiny particles called molecules, which are in continuous random motion. Suppose that the gas is confined in a container. More specifically, you might think of the gas as confined inside a scuba tank. As the molecules move at random, they will strike the interior surface of the tank. Each such molecule will thus exert a tiny force on that surface. These millions of tiny molecule drum-beats will in aggregate assert a measurable force on the tank, and if the gas is well-mixed, this force will be fixed, for any given surface area on the interior of the tank. As we've already discussed, force per unit area is pressure. And so our well-mixed gas in our container exerts a constant pressure on the interior of the tank.

Gas in a rigid container

Now an aluminum (or steel) scuba tank is a *rigid* container. This means that its shape and size, and hence its volume, will not change, even if the pressure of the gas within it is considerable.[1] A scuba tank typically has a valve at the top, which can be closed to confine the gas within it. If we open this valve, gas will rush out until we reach equilibrium— that is to say, until the pressure inside the tank is the same as that outside the tank. This is 1 ata (at sea level).

If we now use a *compressor* to introduce air into the tank, we can easily increase the pressure of the air inside, which is then confined if we keep the valve closed. A typical scuba tank is engineered to maintain its shape and volume, even if the pressure of the air within reaches as much as 3000 psi. Recall that 3000 psi = 3000/14.7 ata = 204 ata. The pressure inside the tank is over 200 times as much as outside!

What does the compressor actually do to the gas to obtain this high pressure? To increase the number of molecules striking the interior of the tank, we need only increase the number of molecules.[2] In other words, there are more molecules per unit volume inside the tank than there are outside the tank. This means the air inside the tank is *compressed*, and is *denser*. We could actually measure the density of the air using units like the number of molecules per unit volume, and chemists actually perform such counts! But as we've seen, it is easier for us to measure density in units like pounds per cubic foot, or newtons per cubic meter.[3]

How does a compressor carry out this increase in pressure? Actually it is just a piston pump, like that used by the dock workers observed by Galileo. The big difference is that a compressor requires a metal piston milled to fine tolerances (to prevent leakage), and also a mechanical source of power such as a steam engine or electrical motor. This was the technology lacking in seventeenth century Italy.

So, as the compressor works, it introduces more and more molecules into the tank. This means that the density increases, and the pressure increases. Indeed, our argument about the drum-beat of the molecules implies that density is directly proportional to pressure. As we already discussed, the density of air at sea level is .081 lb/ft^3. But in a scuba tank pressurized at 3000 psi, that density is then .081 × 204 = 16.52 lb/ft^3.

Let's now consider a particular scuba cylinder often encountered in recreational diving, a tank called an "aluminum 80". What this jargon means is that the tank is made of aluminum, and that it contains 80 cubic feet[4], *at its operating pressure* of 3000 psi. The actual volume of the tank (at 1 ata) is then only 80/204 = .39 cubic feet. Consequently the air in this full scuba tank will weigh .39 ft^3 × 16.52 lb/ft^3 =6.4 lb.[5]

So when a diver is asked about how much air remains in her tank, and she responds "I have 1000 pounds left", she is reporting on the pressure in her tank. But pressure is proportional to density, which in turn is proportional to the number of molecules of air in her tank (since the *volume* of the tank does not change). This means that a pressure gauge measurement of 1000 psi really does reflect the fact that the diver has consumed two-thirds of her available air. When the tank is "empty" and the gauge reads 0 psi, the pressure in the tank will really be 1 ata or 14.7 psi – at equilibrium with

the air at the surface. The *gauge pressure* of the air in the tank is 0, while the *absolute pressure* is 14.7 psi.

Now an empty aluminum 80 tank weighs about 32 pounds. The difference between 32 and 38.4 pounds is actually enough so that experienced divers can detect whether a tank is full or empty just by heft, especially if there is a full tank available for comparison.

During a typical recreational dive, a scuba diver might use up as much as 80% of the air in his tank. Consequently, the geared diver will weigh about 5 pounds less at the end of the dive than at the beginning. But notice that there will be no corresponding change in volume (the tank is a rigid container): this means a change in buoyancy! A change of 5 pounds is enough to have a significant effect. If the diver is perfectly neutral at the beginning of the dive, he will then be quite positively buoyant at its end. This might make it difficult for the diver to control his depth, especially at shallower depths, where pressure changes significantly with small changes in depth. This is the reason why training agencies recommend that divers adjust their weighting so that they will be neutrally buoyant *at the end of the dive.*

GAS IN A FLEXIBLE CONTAINER AND BOYLE'S LAW

Suppose now that we introduce our gas into a flexible container, like a rubber balloon. Such a container cannot withstand the pressure exerted by the additional gas inside, and so the container increases in volume to accommodate. It will continue to expand, until the pressure inside the container is exactly equal to the pressure outside. For a balloon inflated by air, when the balloon reaches its equilibrium size, the pressure inside and outside will be 1 ata. Equivalently, the density of the air will be same inside as outside.[6]

Let's now take our inflated balloon on a dive into the water. As we descend into the water, the ambient pressure surrounding the balloon will increase; indeed, at 33 fsw, the surrounding pressure will be twice what it was at the surface. In this case, the balloon again changes size, only this time by shrinking. To double the pressure inside the balloon, the air molecules there need to be closer together in order to strike the interior of the balloon often enough to exert twice the pressure. The volume of the balloon is half what it once was; the air inside is twice as dense as it once was.

We can formulate these observations more formally as follows:

> **Boyle's Law** *Suppose a given quantity of a well-mixed gas held at a fixed temperature is confined in a volume V, and the gas exerts a pressure P. Then P and V are inversely proportional; that is, $PV = k$, where k is a constant.*

Boyle's Law is named after the seventeenth-century Irish physicist Robert Boyle, who first enunciated it. This law is actually only a part of a more general gas law, which explains the value of the constant *k*; we shall explore this law in the next chapter.[7]

Note that Boyle's Law applies to (most) gases, but most definitely does not apply to liquids. Indeed, the liquid with which we are mostly concerned (water), is essentially *incompressible* under the pressures a scuba diver is likely to experience. This means that a diver (who is mostly water) need not worry about being compressed by the additional pressure brought on by diving underwater; it is only the air spaces inside the body that might be compressed.

Suppose that a balloon filled with air has volume of 1 gallon at 50 fsw. What is its volume if it is taken to the surface? What would its volume be at 100 fsw?

At 50 fsw, the pressure P_1 of the gas in the balloon is equal to the ambient pressure at that depth, which is 50+33 = 83 fsw. Clearly the volume of the balloon is just V_1 =1 gallon, and so P_1V_1 = 83. If we now consider the situation at the surface, we have that the new pressure is P_2 = 33; we then have $P_2V_2 = P_1V_1$ = 83, and so the new volume is V_2 = 83/33 = 2.52 gallons.[8] On the other hand, the volume at 100 fsw (where the total pressure is 133 fsw) is then 83/133 = .62 gallons.

Notice that we chose rather peculiar units for volume (gallons) and pressure (fsw), just because they happened to be the units readily at hand for this problem. As long as we are consistent in the unit choice, we will have no

problems. Changing our system of units will only change *the numerical value* of the constant *k* — it will not change the important fact that the product PV is constant.[9]

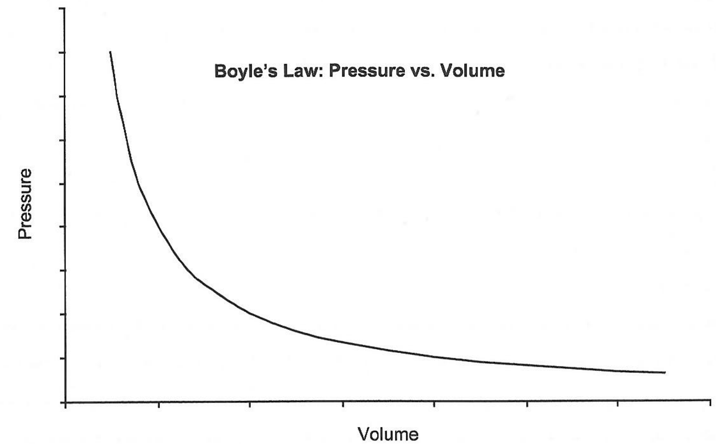
It is useful to think about Boyle's law in a slightly different way, where we suppress mention of the constant k altogether. For if

$$P_1V_1 = k = P_2V_2,$$

then we have the proportion

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

This language of proportionality is really what we used in our introductory example, and for many it is the most natural way to think about Boyle's Law.



For example, suppose that a scientist has a flexible container filled with gas, at a pressure of 10 bar, and a volume of 12 liters. If she increases the pressure to 18 bar, what will the volume be? Since the pressure has increased in the proportion 18/10, the volume will decrease in that proportion, giving us 12 x 10/18 = 6.66 liters.

Consequences of Boyle's Law for divers

Lift Bags

Let's now return to an example in the previous chapter:

Recall that an automobile is submersed in a lake, in 80 feet of fresh water. The automobile weighs 4000 pounds, and its volume is 30 cubic feet. We computed in the previous chapter that in order to make this automobile neutrally buoyant, we needed to add 2128 pounds of lift. Furthermore, we discovered that we needed to displace 34.1 cubic feet of fresh water with lift bags in order to accomplish this. Now let's compute the ambient pressure at 80 feet in the lake. This time let's choose to use atmospheres as our unit of pressure. Since 34 feet of fresh water makes 1 atmosphere, and 80/34 = 2.35, this means the total pressure at 80 ffw is 3.35 ata. Consequently, the needed volume of 34.1 cubic feet of air would actually expand to 34.1 × 3.35 = 114.24 cubic feet at the surface.

This latter calculation is of considerable practical consequence for the dive team attempting to lift the automobile, because a typical scuba tank only contains 80 cubic feet of air (at 1 ata). It is thus likely that the divers involved in this project will need to bring down additional scuba tanks, rather than use their own air supply, or else they might run out of air to breathe!

There is another important practical consideration that Boyle's Law forces upon us. Once the lift bags are expanded to displace 34.1 cubic feet, the automobile will be neutrally buoyant, and hence easy to move — but this is only true at 80 feet of depth! As soon as the divers start to move the automobile upward, the lift bags will expand according to Boyle's Law. Once this expansion takes place, the upper buoyant force will *exceed* the downward force due to gravity. This could lead to an uncontrolled ascent for the automobile, with potentially disastrous results for the dive team. If the lift bags reach the surface outside human control, they will probably abruptly deflate, which would lead to the automobile plummeting back to the bottom.

For this reason, commercially prepared lift bags are equipped with a *dump valve* that a diver can use to regularly deflate the bag, to keep the automobile approximately neutrally buoyant on its entire ascent to the surface.

Lungs

But even recreational scuba divers who never attempt the lift of heavy objects from the bottom of a lake or the ocean need to pay careful attention to Boyle's Law. The human lung is also a flexible container, which we fill with air as we breathe.

Suppose now that a diver is breathing on a scuba system at 99 fsw. The demand regulator delivers air at 99 + 33 fsw =4 ata of pressure; this makes breathing effortless at this depth. Suppose now that the diver fills his lungs with air, and begins to ascend, *while holding his breath*. As the ambient pressure decreases, the air in the lungs expands. Since the diver is holding his breath, the air has nowhere to go, and so the flexible lung must expand in volume. If he were to hold his breath until reaching the surface, the lungs would have to expand to 4 times their original volume. Of course, this does not happen — instead, the lungs rupture, which can be an exceedingly dangerous injury. This leads to the number one principle drilled into divers when they train:

> **The Number One Rule of Scuba Diving**
> *Breathe continuously, and never, never hold your breath.*

Under panicky conditions at depth, divers tend to hold their breaths, and also try to ascend very rapidly. Rapid ascents lead in turn to rapid decrease in pressure, and rapid lung inflation if the airway is not kept open. Even if a diver runs out of breathing air at depth, the airway still needs to be kept open while ascending to avoid lung-overexpansion injuries. This principle is incorporated in the emergency drill training agencies call the *Controlled Emergency Swimming Ascent*. If a diver has run out of air, and has no buddy nearby to provide air, he should maintain a continuous controlled exhalation while swimming steadily to the surface.[10]

Boyle's law is also important for ordinary ascents. For a routine ascent after a scuba dive, a diver should maintain neutral buoyancy and *swim* to the surface. Consequently, when ascending a diver frequently lets air *out* of her inflatable buoyancy vest, to avoid becoming positively buoyant, and potentially losing control of the ascent. Rapid and uncontrolled ascents are among the most dangerous situations in scuba diving, both because of the danger of lung injury, and also because of the dangers of decompression sickness, as we discuss in Chapter 8.

Boyle's Law has another practical consequence for scuba divers. Imagine again our diver at 99 fsw. Because the regulator is delivering air at 4 ata of pressure to make breathing effortless, this means that the air the diver is inhaling is 4 times as dense. In other words, each breath takes in 4 times as many molecules as the corresponding breath would at the surface. That is, scuba divers use up their air supply *at a faster rate when the dive is deeper.*

Let's now compute how long a typical scuba tank might last. To do this, let's start by considering a human being breathing at the surface. Physiologists report that a typical human respiration at rest might take in about 500 cubic centimeters of air; on average a human being might take between 12 and 20 breaths per minute.[11] If we suppose a 500 cc breath, 15 times per minute, we obtain about 500 × 15 = 7500 cubic centimeters of air consumed per minute, or 7.5 liters per minute.[12]

As we've seen, a typical scuba tank contains 80 cubic feet of air, at 1 ata. It turns out that there are 28.3 liters in a cubic foot, and so a scuba tank contains about 80 × 28.3 = 2264 liters of air. For our diver at the surface, this air will then last 2264/7.5 = 302 minutes, or 302/60 ≈ 5 hours.

But let's imagine the same diver at 99 fsw. Now the air supply is being used 4 times as fast, and so the same tank would last only 5/4=1.25 hours. And this calculation assumes that the diver breathes the tank down to the last sip, which is probably not a good idea![13] In addition, even experienced divers may have a slightly elevated rate of breathing on account of workload or anxiety. Indeed, air consumption for new divers is much worse than these figures.

Divers pride themselves on bettering their air consumption. They do this by taking deep, slow regular breaths, and avoiding the fast, shallow breathing anxiety induces. Fast shallow breaths are not efficient, because such breaths do not adequately expel the air that remains from previous exhalations. This stale air is oxygen-poor and laden with carbon dioxide, which in turn stimulates more rapid breaths.

There is a standard numerical way to compare air consumption rates for divers, which factors out of the effect of depth. This is called *Surface Air*

Consumption, or SAC. It measures the number of cubic feet of air per minute the diver would have consumed, *had he been breathing at the surface.* As a simple example, suppose that a diver consumed 39 cubic feet of air out of the 80 cubic feet available in a standard tank. The diver was on a 30 minute dive to 66 feet. Since the pressure is three times as great at 66 feet, the diver would expect to use only 13 = 39/3 cubic feet at the surface. This means that the diver's SAC is

$$\frac{13}{30} = .43\frac{\text{ft}^3}{\text{min}}$$

We can compare this value to our diver at rest discussed above. That diver consumed all 80 cubic feet of air in 302 minutes, and so would have a SAC of 80/302 = .26 ft^3/min.

Of course, divers don't usually think directly about how many cubic feet of air at one atmosphere that they consume on a dive. For a more typical example, suppose that a diver starts with a tank whose pressure reading at the start of the dive is 2895 psi, and is 1254 psi at the end of the dive. The dive was to 73 fsw for 29 minutes. What is the SAC for this dive?

We first observe that the pressure dropped 1641 psi over the course of the dive. This means that the diver consumed (1641/3000) × 80 = 43.76 cubic feet of air at 1 ata. But at 73 fsw, the ambient pressure is 106/33 = 3.21 ata. Consequently, the diver would have expected to consume 43.76/3.21 = 13.63 cubic feet at the surface. This yields a SAC of

$$13.63/29 = .47\frac{\text{ft}^3}{\text{min}}$$

Of course, the computation of surface air consumption is more complicated on typical dives, where the depth varies over time. There are dive computers that do this computation for their operators.

Ears and mask

Besides the lungs, there is another important air space in the body of a scuba diver that is affected by Boyle's Law. This space is the *middle ear*. The eardrum is a slender membrane at the end of the outer ear passage; sound makes the eardrum vibrate, and these vibrations are processed, first mechanically and then electrically, to create hearing. The area beyond the eardrum is called the middle ear. It is still air-filled (to allow the mechanical sound processing to take place), and this region is connected to the nose via passages called the Eustachian tubes. The electrical processing takes place in the inner ear, which is fluid-filled.[14]

When a scuba diver descends, pressure begins to build on the eardrum. In order to continue without pain or damage, this pressure must be counterbalanced by a corresponding pressure in the middle ear. However, ordinary respiration of pressurized air through the scuba regulator will not necessarily convey this needed pressure into the middle ear. Consequently, a typical diver must take action to push the pressurized air into these tiny spaces. Sometimes just a movement of the head and neck is enough to stretch the Eustachian tubes and bring this newly respirated (and pressurized) air into the middle ear. More often, however, the diver finds it necessary to perform a *valsalva* maneuver. This is an action many of us have taken to counteract the change of pressure experienced when flying in an airplane. Namely, you merely need to block the nostrils, and gently blow; the ears then "pop". This is enough to move the air from the mouth and nostrils up the Eustachian tubes and into the middle ear. Scuba divers call this process *equalization*, for obvious reasons. You are taught in scuba training to equalize early, and often!

Occasionally, a diver has difficulty equalizing, usually because of congestion in the Eustachian tubes and other nasal passages. The extra pressure in the outer ear causes pain first, and then perforation or rupture of the eardrum, if a diver were foolish enough to continue descending. Many a scuba dive has been canceled on account of a head cold!

New divers often blow more forcefully than necessary to equalize when descending. This forceful blowing can rupture capillaries in the nasal passages, resulting in a bloody nose. If gentle blowing does not clear the ears, the diver should go to a shallower depth and try again. Only a couple of feet can make the difference between successfully clearing the ears or not, especially when near the surface. If you can't clear your ears, you must abort the dive.

When ascending, the situation is reversed. Now the pressurized air behind the eardrum encounters less pressure in the outer ear, and in a

similar way, this pressure must be equalized. Fortunately, this expanding air behind the eardrum almost always finds its way out of the Eustachian tubes automatically, with no action necessary on the diver's part. Divers call the very rare problematic case a ***reverse block***.

There is one more air space that must be equalized — the space between the mask and the eyes.[15] It is for this reason that a diver's mask includes the nostrils. The diver need only exhale through the nostrils into the mask in order to introduce the denser air with the appropriate pressure she has been breathing from the regulator. If the diver does not equalize this air space, the increasing ambient pressure at depth will push the mask ever tighter against the face, eventually causing bruising,[16] and this pressure can even burst tiny capillaries in the eyes, making them bloodshot. But after only a little experience, divers equalize this air space almost without thinking about it.

Exercises

1. A large scuba tank called a 120 contains 120 cubic feet of air, when it is pressurized to 3000 psi. How much does the air weigh in such a tank, when it is full? What is the actual volume of this tank?

2. A scuba cylinder containing compressed air weighs 5 extra pounds when it is filled. The operating pressure of the tank is 2250 psi. How many cubic feet of air were compressed from 1 ata to fill the cylinder? What is the actual volume of the cylinder?

3. A balloon is inflated with 1 liter of air at the surface. We now take it on a dive to 45 fsw. What is the volume of the balloon? How dense is the air in the balloon now (give your answer in lb/ft^3)?

4. A spherical balloon at 72 fsw is inflated so that its diameter is 4 inches. It is tied off and released. When it reaches the surface, what is its diameter (assuming it has not burst)?

5. Repeat the previous exercise, assuming that the balloon is inflated in 72 feet of fresh water.

6. An inverted glass is held vertical, and submersed in a swimming pool (filled with fresh water). What happens to air in the glass? What is the exact situation, when the glass is on the bottom of the pool at 10 feet?

7. Take the same inverted glass to the bottom of the pool, and fill it with air from a scuba regulator. What happens as you carefully ascend with the glass?

8. A scuba tank contains air pressurized to 200 bar. It is brought to a depth of 100 fsw. What is the pressure of the air in the tank now?

9. A diver takes an air-filled balloon on a dive in the ocean. How deep should she take the balloon to ensure that the balloon has only 40% the volume that it had at the surface?

10. A scientist has a cylindrical tank containing oxygen; the tank is 50 centimeters in diameter. The scientist can change the height of the cylinder by means of a mechanically-operated piston at the top. The current height is 75 centimeters, and the pressure in the cylinder is 85 kiloPascals. To what height should the scientist adjust the height, if she wishes to achieve a pressure of 125 kiloPascals?

11. A free diver (without scuba) takes in a deep breath; his lung capacity is 6 liters.[17] He immediately does a breath-hold dive to 85 fsw. What is the volume of his lungs now (we assume for simplicity that he has not yet exhaled any air)? Is this diver in danger of lung overexpansion as he returns to the surface? Explain why or why not.

12. Sometimes free divers make breath-hold dives in the same vicinity as scuba divers, and they have been known to "cheat", by taking a breath from a scuba diver's alternate air source. Why might this be dangerous?

13. A scuba diver does a dive of 35 minutes to 60 fsw. He uses 2500 psi of air out of a standard tank. What is his surface air consumption? Suppose he now breathes from another such tank at the surface, and again uses 2500 psi. How long will this take? Suppose instead he breathes at the surface until the tank is empty (using all 3000 psi of air). How long will this take?

14. A scuba diver does a *multilevel dive.*[18] That is, she does 15 minutes at 70 fsw, and 25 minutes at 40 fsw. When she is done, she realizes that she has used 1800 psi of air out of her tank. What was her SAC for this dive (you may assume she is breathing at the same rate at both depths)?

15. A scuba diver knows that she can breath a certain supply of air for 2 hours when at 35 fsw. When she descends to 70 fsw, her breathing rate increases by 20%, on account of increased anxiety. How long will her air supply last under these conditions?

16. Under ordinary conditions in a town at sea level, the barometer reads 760 mmHg (1 atmosphere of pressure). Under certain weather conditions, the barometer now reads 780 mmHg. How much denser is the air than it once was (give your answer as a percentage increase)?

17. A full scuba tank is pressurized to 200 bar. After finishing his dive, a French diver notices that his tank gauge reads 50 bar. Give a proportion comparing the density of the air in his tank at the end of his dive to that at the beginning.

18. Explain why a scuba diver cannot wear swim goggles while diving.

19. A vacationer purchases a sealed bag of potato chips in Los Angeles; the bag contains 1 pint of air. She drives to Colorado Springs, where the ambient pressure is approximately .8 ata.[19] What happens to the bag (provide a quantitative description)?

20. Many scuba divers keep delicate equipment such as cameras in rigid boxes that can be sealed air-tight. What problem might a diver face, when she takes this sealed box on a flight from Colorado Springs (with ambient pressure .8 ata) to the Bahamas for a diving vacation?

Endnotes

1 Of course, we can't expect a tank to be rigid under all conditions. If a tank is pressurized beyond the limits to which it is engineered, it may actually be deformed, or rupture in catastrophic fashion. Scuba tanks are closely regulated in the United States by the Department of Transportation, and must be tested once every five years. They are placed under pressure beyond that to which they are rated (usually to 5/3 or 3/2 times the rated pressure). If they deform beyond very tight limits under this pressure, they are taken out of service.

2 The other way to increase the pressure is to make the molecules move faster. This precisely means to increase the *temperature* of the gas. We will explore this idea in the next chapter; for now, our implicit assumption in this chapter is that we are holding the temperature of the gas fixed.

3 It would actually be better to use units measuring *mass* per unit volume, and so grams per cubic centimeter would be a good choice. In this book we are mostly using imperial units however.

4 Actually, the typical aluminum 80 contains about 77.4 cubic feet of air at 3000 psi; for computations in the book, we will make do with the nominal figure.

5 Of course, we could have directly obtained this number by computing $(80\ ft^3)(.081\ lb/ft^3)=6.4\ lb$.

6 Actually, the pressure (and density) inside the balloon are *slightly* greater, to counteract the inward force of the elasticity of the balloon; we will disregard this in the discuss that follows.

7 For readers who just can't wait, we will observe now that *k* will depend on the temperature of the gas, and also on the amount of gas (measured in essence by the number of molecules) that is present.

8 We are obviously assuming here that the balloon has sufficient tensile strength to withstand inflation to this size.

9 When we explore the general gas law in the next chapter, where we will have further interpretation for the constant *k*, units will become important — and SI units are more desirable.

10 Of course, it is much better to monitor your air supply, and not run out of air at depth! Furthermore, safe recreational divers are trained to stay close enough to a buddy diver, so that the buddy could provide air to the out-of-air diver, using the extra second stage called the alternate air source.

11 The size of the respiration is call *tidal volume*. Both tidal volume and respiration rate vary widely, depending on many factors: size, age, state of health, anxiety, work level, etc.

12 A liter is exactly 1000 cc.

13 As we shall see, there are important adverse health effects that may arise if a diver spends an hour or more deeper than ninety feet. We shall return to this topic in future chapters.

14 Because the inner ear is filled with fluid rather than air, its workings are unaffected by scuba diving.

15 In a later chapter we will inquire more about vision underwater, and why we need to wear a mask at all to see clearly.

16 Divers call the resulting raccoon effect around the eyes a "mask hicky"!

17 This is a typical value for lung capacity. Serious athletes often have much larger lung capacities than this.

18 We'll learn more about multilevel dives later.

19 In a future chapter, we will learn how to compute this number.

CHAPTER 4
IDEAL GAS LAWS

Boyle's Law gives a mathematically elegant relationship between the pressure a quantity of gas is under and its volume. But as we've hinted earlier, this is not the full story, and as scuba divers we need to understand another important variable: temperature.

TEMPERATURE

For physicists, *temperature* is a property of a physical object that determines the direction of heat flow between objects. Objects with a higher temperature conduct heat to objects with a lower temperature. Here *heat* is a particular kind of *energy*, which arises from the continuous motion of the molecules that make up the object possessing the heat. Once again, nature seeks *equilibrium* — heat flows from hotter objects to colder ones, in such a way as to equalize the temperature of the entire system.

This transfer of energy is particularly well understood for ideal gases, which we can think of as consisting of unimaginably many tiny particles of the same size and mass that are in continuous, rapid and random motion. When this system of particles receives more energy, the particles move more rapidly.

It was the eighteenth-century French physicist Jacques-Alexandre Charles[1] who first made the observation that increasing the temperature of two equal volumes of two different gases in flexible containers will increase the volume in a regular and predictable way.

In the middle of the nineteenth century the British physicist Lord Kelvin formulated Charles' law as an elegant mathematical equation $PV = kT$ extending Boyle's Law; *pressure times volume is directly proportional to temperature*. But Kelvin realized that this simple law only held if temperature was measured on an *absolute* scale. This means that temperature needs to be measured from ***absolute zero***, a theoretical state at which all molecular motion stops.[2] In the Celsius scale, absolute zero occurs at about -273.15°. The law $PV = kT$ requires that we measure T in degrees above absolute zero. We call this unit K for Kelvin. A temperature of x degrees Celsius is equal to $x + 273.15$ Kelvin. Alternatively, we could use the Fahrenheit scale instead.[3] Absolute zero occurs on the Fahrenheit scale at -459.7°. The corresponding absolute scale is called the Rankine scale: a temperature of x degrees Fahrenheit is equal to $x + 459.7$ degrees Rankine. We can now state Charles' Law formally:

> **Charles' Law** *Suppose a given quantity of a well-mixed gas has temperature T (measured on an absolute scale), volume V, and the gas exerts a pressure P. Then these variable quantities satisfy the equation $PV = kT$, where k is a constant. Equivalently, the quantity PV/T remains constant as the variables change.*

Since in practice we don't know or care what the value of the constant k is, we will usually make our computations taking advantage of the fact that PV/T remains constant.[4]

Suppose then that a balloon is inflated at sea level in Cozumel during the middle of the day, to a volume of 1 quart. The temperature is 100 degrees Fahrenheit. The balloon is taken on a dive to 40 fsw in the 80 degree water. What is the volume of the balloon at this point of the dive?[5]

To solve this problem, we need to choose a system of units to measure pressure, volume and temperature. We will use atmospheres, quarts and degrees Rankine, respectively. These are the natural units given by the description of the problem. The only important constraint is that we must choose an absolute temperature scale. At the surface of the water, $P_1 = 1$ ata, $V_1 = 1$ quart, and $T_1 = 100 + 460 = 560$ degrees Rankine. At depth the pressure is $P_2 = 1 + 40/33 = 2.21$ ata, and the temperature is $T_2 = 80 + 460 = 540°$ R.

We then have

$$\frac{1\times 1}{560} = \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{2.21\times V_2}{540}$$

A bit of arithmetic gives us the new volume as $V_2 = (540/560)/2.21 = .436$ quarts.

What answer would we get if we had disregarded the effect of temperature? Using Boyle's Law, we would then have

$$1 \times 1 = 2.21 \times V_2$$

A little arithmetic gives us $V_2 = .452$ quarts. Notice that the change in temperature in the problem makes for a small but measurable additional decrease in volume. Obviously, this effect becomes more important as the relative change in temperature gets larger.

The General Gas Law

We now understand how the quantities pressure, volume and temperature for a quantity of a gas are related: $PV = kT$. However, this excludes the reasonably obvious observation that if we introduce more molecules of the gas into our quantity at a fixed temperature, this should increase either the pressure or volume or both. Indeed, in the previous chapter we have argued that pressure in a scuba tank can be used to infer how much gas is present in the tank. In our present context, we are asserting that the purported constant k should really be a variable proportional to the amount of gas present.

How should we measure that quantity of gas present? Chemists and physicists consider as their basic unit a ***mole*** of the gas, and this is related to the ***molecular weight*** of one molecule of the gas. The molecular weight, in turn, is essentially a count of how many protons and neutrons make up the atoms in the molecule. Oxygen, for example, consists of molecules built of two oxygen atoms linked together (such molecules are often abbreviated as O_2). According to the periodic table of elements, each oxygen atom weights 16 atomic units, and so each molecule weighs 32. We then define a mole of oxygen as 32 grams of oxygen gas, by mass. It turns out that at 0 degrees Celsius and 1 atmosphere of pressure, one mole of oxygen will fill 22.4 liters of volume. Furthermore, there will be present in this 22.4 liters of volume about 6.02257×10^{23} molecules of oxygen. This amazing number is called *Avogadro's number.*

But the really amazing thing about Avogadro's number is not its size. For consider the gas nitrogen instead. One atom of nitrogen weighs 14 atomic units, and nitrogen molecules also occur as pairs of atoms. Consequently, 1 mole of nitrogen will consist of 28 grams of nitrogen. Under the same conditions, 1 mole of nitrogen will then fill up 22.4 liters of volume and have the same number of molecules!

There are the same number of molecules of oxygen and nitrogen here, but the oxygen molecules have more mass. Consequently, we would expect the heavier oxygen molecules to exert more pressure, but they don't. What really happens is that the heavier oxygen molecules move at a slower rate. Indeed, what is called the *kinetic energy* of the particles is measured by mass times velocity, and these two factors exactly counterbalance one another to create the same pressure.

We can now state the full version of the ideal gas law:[6]

The Ideal Gas Law *Suppose that n moles of a well-mixed gas are contained in volume* V, *at absolute temperature* T, *and pressure* P. *Then* $PV = nRT$, *where R is a constant described below.*

The constant R is called the *universal gas constant.* It has the value

$$8.3149\frac{\text{newton} - \text{meter}}{\text{Kelvin} - \text{mole}}$$

if we measure pressure in pascals (newtons per square meter), volume in cubic meters, n in moles, and T in Kelvin.[7] For our purposes the actual value of the constant is not important; the crucial point is that this law holds for any gas!

Air

We can now consider what happens with a gas like air, which is a mixture of various gases. Indeed, air is composed of about 21% oxygen, 78% nitrogen, and 1% various trace gases (mostly argon).[8] For mammals like ourselves, the oxygen is the important part: we breathe air into our lungs, and the oxygen

is extracted from air and carried by red blood cells to the cells of our body, so that they can do their work.[9] The remainder of air is *physiologically inert.* Any nitrogen in our lungs when we inhale is expelled without any change (while some of the oxygen is replaced by the waste product carbon dioxide). It consequently will do us no harm in what follows to assume that air consists of 21% oxygen and 79% nitrogen.

But how do we measure this percentage? One way to think of it is as a count of molecules! So to fill up 22.4 liters of volume at 0° Celsius and 1 ata of pressure, we should take 21% of Avogadro's number as oxygen molecules, and 79% of Avogadro's number as nitrogen molecules. These two different molecules are in practice moving independently and are on average far enough apart that the pressure exerted by this mixture will exactly add up to the 1 ata[10] — the smaller nitrogen molecules moving a little faster to counterbalance their lesser mass. So equivalently, we can (and in practice do) think of these percentages in terms of volume.

Notice that this calculation is *not* in terms of mass! Since 79% of the molecules weigh 28 atomic units and 21% weigh 32 atomic units, this means that the average molecule weighs

$$.79 \times 28 + .21 \times 32 = 28.8$$

atomic units. This means that the oxygen takes up $(32 \times .21)/28.8 = 23.3\%$ of air by mass.

Suppose we were to form a mixture in the proportion of 21 to 79 by mass. If the total mass of the gas is M, n is the number of nitrogen molecules, and m is the number of oxygen molecules, we would have $28n = .79M$ and $32m = .21M$. By dividing one of these equations by the other, we obtain that the ratio of the number of nitrogen molecules to the number of oxygen molecules is $n/m = 4.30/1$. But we know that the ratio in air is $.79/.21 = 3.76$; that is, the new mixture would be

$$\frac{1}{1+4.30} = 19\%$$

oxygen by volume (or molecule count). We thus would need relatively more of the lighter nitrogen molecules.

Consider filling up a rubber balloon with helium. A helium molecule consists of a single helium atom, which weighs 4 atomic units. Consequently at 1 ata the helium will weigh only 4/28.8 = 14% as much as the corresponding volume of air — the buoyant force upward of the helium counteracts the small weight of the balloon, and it floats upward!

John Dalton, the nineteenth-century British physicist, first formulated the general principle we have used above, that in a gas mixture we can think of the different gas molecules as behaving independently:

> **Dalton's Law** *In a mixture of gases the constituent gases exert the same pressure as they would if they were separate, in the same volume.*

As we have seen, in air 21% of the pressure is due to the drum beat of the larger oxygen molecules and 79% is due to the nitrogen molecules. We thus see that the *partial pressure* due to oxygen in air at 1 ata is .21 ata. In abbreviated form, we might write this as pp O_2 = .21ata. As we shall see, Dalton's law is of considerable practical significance to scuba divers.

Consider a diver in 33 fsw. She is experiencing a total ambient pressure of 2 ata. But then the partial pressures due to the oxygen and nitrogen are doubled: .42 ata partial pressure due to the oxygen, and 1.58 ata due to the nitrogen.

Oxygen toxicity

It is easy to forget that oxygen is an incredibly corrosive substance; it is *oxidation* that causes rust and decay. We need oxygen so that our cells can use their fuel for energy, and the hemoglobin in our bloodstream does a marvelous job of providing each of our cells just the right amount of oxygen, even if the partial pressure of oxygen being breathed is quite far from the usual 21%.[11] However, too much oxygen is actually dangerous — it becomes toxic at high levels of pressure.

There are two kinds of oxygen toxicity. The first is called *pulmonary* oxygen toxicity. It affects individuals who breathe a gas mixture with an elevated partial pressure of oxygen (in excess of 50%), over a long period of time. The lungs become irritated, and the person affected may experience burning in the chest, or a cough. For the most part, this first type seldom affects scuba divers.

Concern for this type leads hospitals to schedule patients on 100% oxygen therapy with regular breaks on less rich mixtures.

The second type is called *central nervous system* (or *CNS*) oxygen toxicity. It occurs at higher partial pressures of oxygen—1.4 ata or above. The onset is unpredictable and sudden, and involves involuntary twitching or trembling, or more serious CNS effects, such as epileptic-like convulsions or unconsciousness. In the case of the latter symptoms, a diver with a conventional regulator would undoubtedly drown; this is why dive professionals take the risk of CNS oxygen toxicity so seriously.

There is considerable debate about what level of partial pressure is safe. Conservative practitioners recommend staying at 1.4 ata or below, while others recommend 1.6 ata as a safe upper bound.

So at what depth in the ocean would a diver encounter a partial pressure of oxygen this high, while breathing on a tank of ordinary compressed air? To answer this question, we use a proportionality argument: the ratio of the partial pressures of oxygen at depth and at the surface should equal the ratio of the total pressures. If p is the total pressure at depth, we then have $1.6/.21 = p/1$; we conclude that $p = 7.6$ ata. Translating this into a depth gives us $6.6 \times 33 = 218$ fsw. This is well below the ordinary limit of 130 fsw which certifying agencies recommend for recreational scuba divers, and so is not a serious limitation.[12]

Non-divers often talk about scuba divers and their "oxygen tanks" — but notice that if you were breathing with a tank of pure oxygen (1 ata at the surface), you would reach the danger zone of 1.6 ata in only $.6 \times 33 = 20$ fsw! This is why divers always include a substantial portion of a physiologically inert gas (such as nitrogen in air) in their breathing mixtures. The easiest way to do this is of course to breathe ordinary compressed air, but we will eventually learn why some divers use more exotic mixtures of oxygen and other physiologically inert gases.

Carbon monoxide

For another example of the importance of partial pressures, consider a scuba tank contaminated with carbon monoxide. Carbon monoxide (CO) is a pollutant that results from incomplete combustion; it is often found in fumes from internal combustion engines and cigarette smoke. When inhaled into the lungs, carbon monoxide attaches itself to the hemoglobin in the blood, at a rate 200 to 300 times more readily than oxygen. This interferes with blood transport of oxygen to the cells of the body, and can be fatal in large enough doses. Unfortunately, CO is tasteless, odorless and colorless, and can in principle be introduced into a scuba tank (especially if a diesel engine supplying power to the air compressor is not properly vented). But human beings can and do survive partial pressures at the surface of up to 2%. However, if such a concentration is put into a scuba cylinder, the diver at 99 fsw (or 4 ata) now suffers from 4x.02 = 8% partial pressure, a level that could have serious consequences.

Exercises

1. Suppose that a scuba cylinder is pressurized to 3000 psi at 60° Fahrenheit. After lying in the sun all day, the air inside the cylinder reaches a temperature of 100°. What is the pressure inside the cylinder now?

2. A balloon at sea level and 0° Celsius is filled to a volume of a half a liter with nitrogen gas. How much does the gas in the balloon weigh? Does the balloon float into the air or sink to the floor? (What assumptions must you make about the weight of the balloon to conclude this?)

3. Repeat the previous exercise, assuming that we fill the balloon with oxygen instead. How about hydrogen (a hydrogen molecule has atomic weight 2)?

4. A scuba tank is filled with a mixture of 32% oxygen and 68% nitrogen (this mix is called *Enriched Air Nitrox*, or EANx32 for short). At what depth in seawater will the partial pressure of oxygen reach the dangerous level of 1.6 ata? What about the more conservative limit of 1.4 ata?

5. Water freezes at 0° Celsius and 32° Fahrenheit. Water boils at 100° Celsius and 212° Fahrenheit. The relationship between the temperature C in Celsius and F in Fahrenheit is *linear*. Determine the straight line equation that describes this relationship; express C in terms of F and F in terms of C.

6. Using your answer to the previous exercise (if necessary), obtain the linear function expressing temperature R in Rankine in terms of K in Kelvin. Why is this formula simpler than the answer to the previous exercise?

7. Argon is a gas with atomic weight 40. How much does 1 liter of argon weigh (at 0° Celsius and 1 ata)?

8. Trimix is a diving gas mixture that consists of oxygen, helium and nitrogen. A commonly-used mixture has 21% oxygen, 50% helium and 29% nitrogen. What is the proportion of the three elements in this trimix, measured in terms of *mass*?

9. An aluminum 80 scuba tank is filled to 3000 psi with the trimix from the previous exercise. How much does the compressed gas in this tank weigh?

10. A malevolent tank filler inserts .5% of a deadly poisonous gas into a scuba tank. This poison takes effect once the partial pressure of the poison is at least 2.2%. At what depth in seawater will the poor diver using the tank be poisoned?

11. A scuba diver at Blue Hole, New Mexico fills her scuba tank with 200 bar of compressed air; the air in the tank is 85° Fahrenheit. What does her European pressure gauge read, after she submerses her tank into the 64° water of Blue Hole?

12. At sea level a balloon filled with air has volume 1.3 liters, and temperature 30° Celsius. By immersing the balloon into some cold water and then removing it, we succeed in reducing the temperature of the enclosed air to 22° Celsius. What is the volume of the balloon now?

13. A scuba tank is filled with an unknown gas that does conform to the ideal gas law. The tank is pressurized to 150 bar, and the temperature of the gas inside the tank is 23 degrees Celsius. The tank's volume is 11 liters. How many moles of the unknown gas are in the tank? How many molecules of this gas are there in the tank?

ENDNOTES

1 Another of Charles' interesting practical experiments made him the first person to ascend into the air in a hydrogen-filled balloon!

2 Notice that according to the equation, if the temperature of a gas at a given pressure is reduced to absolute zero, then the volume of the gas must similarly reduce to zero—the gas molecules are motionless and shrunk entirely together! This is clearly a theoretical limit that cannot be reached in the laboratory.

3 You may well know that if the temperature in the Celsius scale is C degrees, then the corresponding temperature in the Fahrenheit scale is $F = 9/5 \times C + 32$. You can explore this relationship in one of the exercises.

4 Later in this chapter we will inquire a bit into the value of k.

5 Notice that our intuitive justification for Charles' law depends on the assumption that the gas consists of many particles of the same size and mass. Air, on the other hand, is actually a mixture of both oxygen and nitrogen molecules (and some other molecules as well); as we shall see when we discuss Dalton's law, the general gas law actually also applies to air, as well as to gases consisting of only one kind of particle.

6 It is amazingly the case that most gases at reasonable temperature, volume and pressure conditions obey this law; most gases are 'ideal'. This mathematical model is not perfect, however; helium, for example, actually compresses a bit more than would be expected from the ideal gas law.

7 A newton-meter is actually a unit of energy, called a *joule*; note that when referring to temperatures using the Kelvin scale, the word "degrees" is not used.

8 For more details about the composition of air, see Appendix C.

9 This extraction mechanism is not very efficient; a good bit of oxygen is still exhaled; that's why rescue breaths given to a non-breathing person can still be effective.

10 We will formalize this observation below as *Dalton's Law*.

11 See more about this in Chapter 7.

12 As we shall see, there are important reasons why a scuba diver breathing ordinary compressed air should not venture as deep as 218 feet, even though the danger of oxygen toxicity is not one of them.

CHAPTER 5

WATER

The physical properties of water have direct effects on the way a scuba diver perceives the underwater world. In this chapter we will explore how a diver sees, hears, and senses heat underwater. In addition, we'll describe the typical water movements which affect divers.

COLD

We recall that *temperature* is a scale that measures whether or not two systems are in *thermal equilibrium*. That is, if one body has a larger temperature than another in which it is in contact, energy in the form of heat will be transferred from the first body to the second. This energy takes the form of the motion of the molecules that make it up. When a warm scuba diver immerses herself in cold water, she loses energy into the water.

Water has an unusually large *heat capacity*: it takes a lot of heat to raise its temperature a given amount. Indeed, it takes about 3200 times as much heat to raise a *volume* of water one degree Celsius as it does to raise that same volume of air one degree. Because water is so much denser than air, this comparison seems unfair. So recall that a cubic foot of fresh water weighs 62.4 pounds, while a cubic foot of air (at sea level) weighs .081 pounds. Thus, water is 62.4/.081 = 770 times as dense as air, and so it still takes 3200/770 = 4.2 times as much heat to raise a given mass of water one degree as it does to raise the comparable mass of air that one degree. One consequence of water's large heat capacity that is vital for life on earth, is that the temperature of ocean water is relatively stable, because it would take too much solar energy to change the water temperature a lot.

Now how is heat transferred? We obtain energy from the sun by *radiation*: the energy is carried from the sun to us by electromagnetic waves. While if the objects are in direct contact, then heat is transferred by *con-*

duction: hotter and more excited molecules directly transfer their energy to their less-excited neighbors. Metals are great conductors, as anyone burned by a spoon in a hot cup of coffee can attest.

By comparison to metals, water is not such a good conductor, but it does at least conduct heat 20 times better than air. And this effect is considerably accentuated by *convection*. As the water molecules pick up energy from the diver's body, they rise, and are replaced by a new set of colder water molecules. There is so much water in comparison to the diver that this process continues indefinitely.

Convection can and does happen in the air too; this is especially true on a windy day, when the moving air continually replaces the air molecules that have been warmed by the body with cold ones: this is the origin of the concept of *wind chill*. However, the larger conductivity of water makes this effect more powerful in water than in the air. The practical consequence of this is that a diver in 80° air can happily survive indefinitely. Our body continues to produce heat while turning food into energy. Some of this heat is lost to the surrounding air; indeed, we would quickly become overheated if we did not dissipate some of this energy. But in water of that same temperature more heat is lost than our body produces. The result is that the diver gets colder and colder. Indeed, a diver continually immersed even in warm tropical waters will eventually suffer from hypothermia. This is one of the major problems for divers lost at the surface.

How do we prevent this chilling effect? On the surface we wear clothing to keep warm. This functions in two ways. First, clothing tends to be made of *insulating* material; this just means that it is a poor conductor of heat. Less heat is lost to the air molecules surrounding the person, because the clothing conducts less heat than does the bare skin. In addition, body heat warms the air molecules that are trapped between the skin and the clothing; they are not drawn away from the body by convection. The person then is living within a micro-environment of air that is warmer than the ambient air. In the case of a warm human being nestling down into a cold bed, our prospective sleeper reaches a point where the bed no longer seems cold, even though the molecules of the bed covers far enough away from the person are still at the temperature they started at.

Unfortunately, most clothing underwater becomes saturated with water, and consequently loses most of its insulating value. Furthermore, the clothing now adheres to the skin and eliminates the trapped air warmed by the body. Therefore divers need a different sort of thermal protection underwater.

The classic solution is the *wetsuit*. A traditional wetsuit is made of neoprene, a rubber filled with closed tiny gas bubbles.[1] These gas bubbles continue to provide some insulation even underwater. Furthermore, the suit is snug enough that a small quantity of water is trapped between the suit and the skin. This trapped water is warmed by the body, and the insulating value of the neoprene keeps this warmth close to the body.[2]

There are some practical problems with the wetsuit. First of all, because of the trapped gas bubbles in the rubber, a neoprene wetsuit has relatively substantial positive buoyancy. A diver wearing such a suit consequently has to counteract that buoyancy by additional lead weight. This additional weight can be quite cumbersome out of the water.

But furthermore, as the diver descends, the trapped gas bubbles in the neoprene shrink in size, according to Boyle's Law. Consequently, the buoyancy of the suit (and its insulating value) substantially decreases with depth. Divers using such equipment must make frequent buoyancy adjustments by adding air to their vests in order to maintain neutral buoyancy.

The alternative is the *drysuit*. Such a suit is equipped with airtight seals at the neck and wrists (and also the ankles, if the feet are not included inside the drysuit); the diver keeps his torso dry. A diver with a dry suit can then wear conventional clothing with insulating value underneath the suit. In order to keep the suit from growing ever tighter when descending, a drysuit is equipped with a separate valve that enables the diver to inflate it with air to keep the suit comfortably away from the body.

The disadvantages of a drysuit are numerous. Such a suit is quite expensive and delicate to maintain. Small leaks are almost inevitable, and larger leaks eliminate much of the suit's thermal protection. Also, the air around the body inside the suit provides considerable additional positive buoyancy, which must be counteracted with more lead. In addition, a diver with a drysuit now has to be careful to achieve neutral buoyancy, since he must worry about the air in his buoyancy control vest and also in the suit itself. Extra training is necessary to avoid possible uncontrolled ascents.

There is another important factor for recreational divers, especially those who might be doing three or more dives each day.[3] Divers find that their susceptibility to cold increases as the dive day goes on – a diver after a second dive may think he has fully warmed up from the earlier dives, but will discover that he gets cold earlier on the next dive. Such a diver has not really returned his core body temperature to what it was at the beginning of the dive day. More intense dive schedules tend to require more thermal protection.

The molecular structure of water

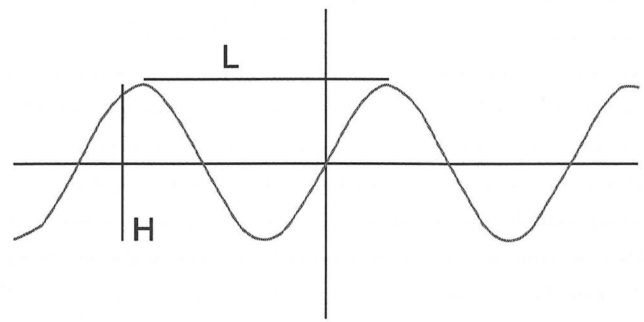
Water molecules are composed of one large oxygen atom, attached to two small hydrogen atoms. This structure is asymmetric, with the hydrogen atoms both attached to one side of the oxygen atom. This means that a water molecule has a positively charged end (where the hydrogen atoms reside), and a negatively charged end. Consequently, water molecules tend to align with one another. This makes liquid water an unusually *cohesive* substance. We see evidence of this in the way water beads up rather than dispersing. The cohesiveness of the water molecules also means that water has enough *surface tension* that in an appropriate temperature range liquid water remains that way; other liquids much more freely dissolve into the air. This increases the stability of the world's system of oceans: not only does the temperature stay relatively stable, but the water for the most part remains liquid.[4] The cohesiveness of water also has relevance to wave formation, as we shall see in the next section of this chapter.

As water gets colder, the molecules align ever more closely together, but only until the water *freezes*: the crystalline structure of solid water (ice) actually takes up more room than it does in liquid form! That is, ice floats. This is a very unusual circumstance: usually the solid form of a substance is more dense than its liquid form. This means that frozen water in the oceans is at the surface; if the physics were different, the bottom of the oceans would be permanently encrusted with ice, with radically adverse consequences for life on earth.

The increasing density of water with temperature does tend to drive colder water deeper. This effect is especially striking in smaller bodies of water, where there is less water movement to mix waters of different temperatures. Indeed, in most freshwater lakes and reservoirs the water temperature tends to be stratified, with relatively abrupt transitions to colder water as we descend deeper. Such transitions are called *thermoclines*; they can be a shock to the scuba diver not anticipating them!

Surface waves

On larger bodies of water scuba divers encounter *waves*. In deep water, a wave creates the characteristic sinusoidal shape on the surface of the water, as illustrated in the diagram below. The high points are called the *crests* of the wave, and the low points are called the *troughs*. The *height* H of the wave is the vertical distance from crest to trough; the *length* L of the wave is the distance from crest to crest.



Waves begin when energy from wind is transferred to the water. They start as so-called *capillary* waves, which may be an inch high or less; as more energy is transferred, the height of the wave gradually increases. The size and speed of a wave is effected by the speed of the wind, the area over which the wind is blowing, and how long the wind blows.

The motion of the wave is in a direction perpendicular to the *wave front* – the horizontal line on the surface of the water marking the crest of the wave. The front moves across the water in a fairly regular way.

We are describing here the typical behavior of waves on the ocean, or other large bodies of water. It is of course possible to have wave fronts which are not the horizontal lines perpendicular to the direction of the wind – the classic example is the ripples formed when a pebble is dropped into a smooth pool, where the wave fronts are circular. But the important waves in lakes and oceans have linear wave fronts.

The most surprising thing to understand about waves in deep water is that the water itself is *not moving in the direction of the wave*. The wave is actually *transferring only energy*: the water itself is only moving up and down in a periodic way, from crest to trough to crest.[5] We can (and do) talk about the *speed v* of a wave, but this measures the rate of energy transfer, and not water movement. The *period t* of a wave is the amount of time that elapses between two crests at a given location. To compute the speed v we thus need only divide the length by the period: $v = L/t$.

Another number often associated with waves is the frequency f. This is the number of full oscillations per unit time. Since the period t is the amount of time for one full oscillation, we have that $f = 1/t$.

Sailors call a wave with a period of 3 to 8 seconds *chop*; with a greater period, we have *sea*. Notice that longer waves tend to move faster; eventually a wave will outrun its wind, and the waves will gradually sort themselves out by length (and speed). On the ocean, waves can and do travel hundreds of miles, perhaps far outrunning the local conditions that created them.

The sinusoidal shape of the water in a wave arises because of the strong surface tension that water possesses. But eventually, a wave becomes high enough that the surface tension can no longer hold the shape. We say that the wave *breaks* – the tops of the crests break off and are no long smooth. We now have *white water*. Waves tend to break when the ratio of height to length exceeds 1/7.

By definition, a *deep water wave* is one for which the total depth of the water is at least $L/2$ – half the length of the wave. As a wave approaches shore, a wave will eventually pass this limit, and the wave will "feel" the bottom. When this happens, the friction of the bottom will slow down and steepen the wave. Eventually, the surface orbits of the water will be going faster than the speed of the wave; the wave breaks, and we have *surf*, where the water is actually moving towards shore. Our familiarity with surf at the seashore is why most people suppose that waves actually move water horizontally in open water.

When a diver is close to shore, and shallow enough to feel the effects of a surface wave, she may experience *surge*. This is a periodic surge forward in the direction of the shallow wave. This can be disconcerting to the diver, and it takes practice and timing to swim easily in surge conditions. If a diver is close enough to a rocky shore, surge can be dangerous, because it might thrust the diver up against rocks or other obstacles. Divers near the surface can be caught in the surface orbits; this vertical roll can be fun, but for many divers carries the peril of seasickness.

As a wave leads to surf at the shore, the extra water brought into the beach by a breaking crest flows rapidly out to sea, awaiting the next crest. Sometimes, the shallow underwater topography may include narrow cuts through which this seaward water must return. This leads to the potentially dangerous seaward currents called *rip currents*.

Waves can be a complicating factor for divers at the surface following a dive, especially if they come to the surface far from the dive boat; many scuba accidents actually occur at the surface, rather than underwater. The diver's most important step for safety is to achieve and maintain positive buoyancy at the surface. Inflation of the diving vest is usually sufficient, but in adverse conditions it may be necessary to drop weights. Scuba weight systems are designed so that this is easy to do, by the diver himself, or by a buddy, if necessary. If seas are heavy enough, divers may need to use their regulators rather than a snorkel to keep their airways clear. If the height of the waves is substantial, it may be very difficult for the boat crew to see divers at the surface of the water. In such conditions, divers may carry inflatable tubes in bright colors for signalling, which when held vertical are four or even six feet high; however, in heavier seas even such signalling devices are often below the crest of the waves. Mirrors and loud air horns can also be used, and there are also radio beacon signals that divers can wear.

SOUND

Sound comes to us as a transfer of mechanical energy through the air molecules surrounding us. The original source of the sound produces a local *condensation* (or compression) of the air molecules—the density of the air has locally increased. A little further away from the source, there is a corresponding *rarefaction*, where the density of the air has locally decreased. The motion of these molecules is energy, and this energy is transmitted through the air with successive condensations and rarefactions. We thus can and should think of sound as a *wave* moving through the atmosphere. Barring obstructions, the sound wave travels in all possible directions – each condensation is shaped like a sphere, centered at the source of the sound. These spheres are the *wave fronts* of the sound wave. A sound wave is consequently a *three-dimensional wave*, in contrast to a two-dimensional wave on the surface of a body of water. Because the source of the mechanical energy is at a single location, a sound wave can be better thought of as a three-dimensional version of the circular ripples formed when a pebble is dropped in a pool, rather than a surface wave arising from a wind blowing in a single direction over a large area.

Just as with a surface wave, it is important to realize that although the air (or water) molecules in a sound wave are moving locally, individual molecules are not carried along by the sound wave from source to ear. Once again, energy is transferred, but not individual molecules.

When a sound wave strikes the eardrum, its energy is conducted to the drum, which begins to vibrate. As we've discussed before, these vibrations are processed mechanically in the middle ear, and then translated in the inner ear into electrical signals the brain can process.

A louder sound conducts more energy, and this is reflected by the magnitude of change from the condensations and rarefactions of the molecules conducting the sound. This difference is analogous to the height of a surface wave on the ocean; it is the *amplitude* of the sound wave.

One big difference between surface waves and sound waves is that sound waves travel at a fixed rate of speed, which depends on the medium (like air or water) through which the sound is conducted.[6] As we've seen, surface waves can (and do) travel at different speeds. Sound waves travel through the medium of air at 1130 feet per second. We can easily translate this into miles per hour by the following computation:

$$\frac{1130\text{ ft}}{1\text{ sec}} = \frac{1130\text{ ft}}{1\text{ sec}} \times \frac{3600\text{ sec}}{1\text{hr}} = \frac{4068000\text{ ft}}{1\text{ hr}}$$

$$= \frac{4068000\text{ ft}}{1\text{ hr}} \times \frac{1\text{ mi}}{5280\text{ ft}} = 770\text{ miles per hour}$$

This sounds quite fast, but the delay between seeing a lightning strike and hearing the clap of thunder shows that it is easily detectable by humans. We see the lightning (essentially) instantaneously, while the sound of the thunder takes some time to get to us.[7]

Why is it that we can determine the *direction* a sound is coming from? It turns out that this is because we have two ears. The brain is trained to interpret the very small delay between when a signal arrives at one ear and when it arrives at the other as a clue about the direction of a sound. In one of the exercises you will compute the length of time this delay amounts to!

Now consider sound underwater. Although Jacques Cousteau wrote of the "silent world", sounds are actually quite detectable and indeed even loud underwater. But sound travels 4625 feet per second in fresh water. This is because water is denser and more elastic than air. Salt water is even denser, and so sound travels at a rate of 5084 feet per second in this medium.

The four-fold increase in the speed of sound in water as compared to air means that our brain cannot as effectively detect the direction sounds come from! Hearing a boat motor overhead but being unable to determine the direction it is coming from is a common and frustrating experience for scuba divers. Scuba divers often carry rattles or other noise makers to attract the attention of a buddy or other nearby divers—one must often look in all directions before spotting the diver making the sound.

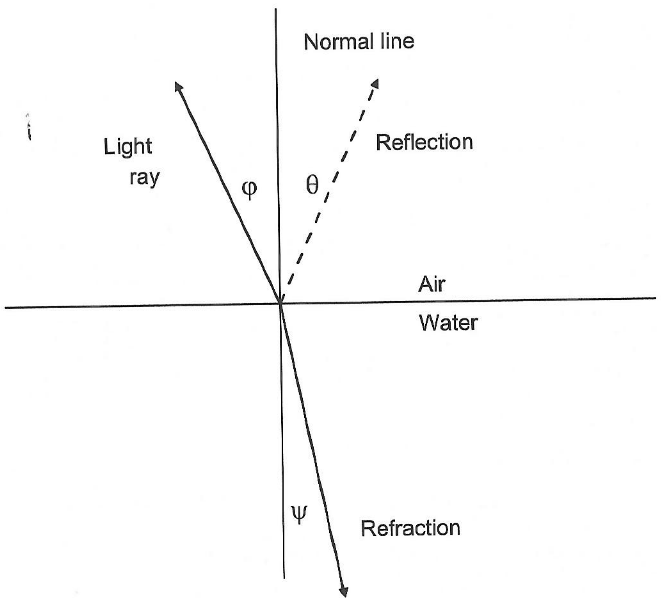
Sounds that originate above water are particularly problematic, because sound waves propagating through air lose a lot of energy when they encounter the air-water boundary.

Scuba divers who have tried discover that they cannot speak clearly underwater. Our vocal apparatus is quite delicately attuned to producing exactly the right vibrations in the air to make language intelligible. These subtle vibrations do not translate to the denser medium of water. With practice, divers can succeed in making loud inarticulate sounds underwater.

SIGHT

We see because our eyes are able to interpret signals in the unimaginably rapid transmission of light through the air. Light travels in a straight line[8], at about 186,000 feet per second! It is one of the truisms of modern physics[9] that the speed of light is fixed, under all circumstances. But this is a little misleading. That fixed velocity is only through a given medium — the figure given above is for the speed of light in a vacuum, like outer space. Actually, like sound, light travels at *different speeds through different media.* Generally speaking, the speed is slower through denser media.[10] To the level of accuracy we care about, this means that the speed of light through air is essentially the same as it is in a vacuum. Through water, however, light moves about 25% slower. If we denote the speed of light through air by v_a, and the speed of light through water v_w, then $v_w = .75v_a$; this is about 140,000 feet per second. Equivalently, $v_a/v_w = 1/.75 = 1.333$. This value is called the *index of refraction* of water.[11]

When light passes from one medium to another, the difference in speed means that the light changes direction! This is called *refraction.*[12] The situation is depicted in the diagram below. We may suppose that a light ray strikes the line of intersection between the two media (in this case air and water) at an angle φ from the *normal* (or perpendicular) line to the line of intersection between the media.



An effective way to measure the extent to which the direction of the light ray is *refracted* is to measure the angle ψ. In the case of a light ray passing from air into water, the angle ψ is smaller than the angle φ. More generally, when the light ray passes from a less dense medium into a more dense medium, the light ray bends *towards the normal line.* On the other hand, if we consider instead the light ray starting in the water (the more dense medium), the light ray will bend *away from the normal line* in the less dense medium.

This principle is made mathematically precise in one of *Snell's Laws,* which makes it possible to compute the amount the light ray will bend. This law asserts that the ratio of the sines[13] of the angles φ and ψ is equal to the ratio of the speeds of light in the two media. In our case,

$$\frac{\sin\varphi}{\sin\psi} = \frac{v_a}{v_w}$$

where the latter quantity we have called the *index of refraction* of water.

Now consider a scuba diver underwater, looking at the world through a scuba mask. We first note that our eyes have evolved with the capacity to focus on objects in air. When we are submerged in water, our eyes consequently are unable to bring objects into focus. This is why we need to wear a scuba mask to see clearly. But the glass of a scuba mask marks a boundary between the air inside the mask and the water outside of it.[14] Light rays passing through this boundary are bent. Since air is less dense than water, the light ray from an object in view is bent away from the normal direction — that is, they are bent away from the direct line of sight. This means that an object looked at either appears *larger* than it actually is, or else *closer* than it actually is. This effect is in the ratio of the refractive index of water: namely, in the ratio of 1.333 to 1. We can thus say that familiar objects appear 33% larger than they do at the surface. To rephrase this, we can say that the size of an unknown object underwater should be reduced by 25% to obtain its actual size (because 25% of 1.333 is .333).[15]

New divers who see a familiar object (like a coin) underwater are often startled to see that it appears larger than it actually is. But they often forget that this applies also to objects whose size they do not know. This is why that 12 foot shark a diver sees was actually only 9 feet long!

Another thing happens when light from the surface strikes the surface of the water. Part of the light does not enter the water at all, but is instead *reflected* back. The reflection is represented in the picture above by the angle θ. Another of *Snell's Laws* asserts that φ = θ: *the angle of incidence* (φ) *equals the angle of reflection* (θ).

This means that not all of the light energy enters the water. When the sun is directly overhead (at high noon in the tropics), about 4% of the light bounces off the surface rather than entering the water. But as the sun goes lower and lower, a higher and higher percentage of the light reflects off the surface. Indeed, at lower angles near sunset, almost 100% of the light is reflected. Consequently, the best visibility for scuba divers occurs near the middle of the day, when the sun is almost directly overhead.

Another property of the white light we encounter is that it is actually composed of a *spectrum* of lights of various colors; experiments with a prism reveal this. Physicists tell us that light actually behaves like a wave through space (called an *electromagnetic wave*). These colors actually tell us the length of the light wave — the more energy the shorter the wave.[16] The visible spectrum can be remembered by the mnemonic ROY G. BIV: red, orange, yellow, green, blue, indigo, violet; the red end of the spectrum has less energy. As light enters the water, the light energy is absorbed more and more as we go deeper,[17] until finally no light remains. In this process the lower energy light disappears first. Objects that are actually red look green or black at depth. In clear water, the blue light persists the longest; this is why we have the "deep blue sea". The startling colors of coral and sponges do not appear that way to the diver. Instead, the underwater world appears suffused with tinctures of blue. When a diver uses a dive light, or uses an underwater flash on a camera, these colors are abruptly restored to the diver's vision.

There is one other effect of water on the light passing through it. The light tends to be dispersed and softened. The sharp shadows at the surface on a sunny day appear no longer.

Finally, the rippling curved surface of the water at the surface can at moments cause a lens effect, concentrating more light than usual in one spot, while putting less light on a nearby spot. Scuba divers experience this as ripples of light and dark on the bottom, especially when on shallow dives during the middle of the day.

The world ocean

The world ocean covers about 71% of the earth's surface, with an average depth of over 12 thousand feet; there are approximately 285 million cubic *miles* of water in the oceans! Because of the three-dimensional nature of the oceans, biologists estimate that over 99% of all "living space" on earth can be found in the oceans.

Ocean water consists of pure water, with various dissolved salts (mostly table salt sodium chloride), which in solution dissociates into positive chloride ions and negative sodium ions. The salinity of the ocean is measured in *parts per thousand* (*ppt*) by weight, and is on average about 35 ppt. Salinity actually varies a lot in the oceans of the world, ranging from a high of 40 ppt in the Red Sea, and a low of 18 ppt in the Baltic Sea. Variations tend to occur more at the surface and near coastlines, while deep water tends to be more uniform.

For scuba divers, this has practical consequences. As we have already seen, salt water is denser than fresh water, and so we are more buoyant in salt water than in fresh water. But the relatively more saline water in Hawaii than in the Caribbean might mean that a diver should add a little more weight to maintain neutral buoyancy.

Scuba divers sometimes encounter relatively abrupt changes in salinity; this can be caused by surface fresh water entering the ocean, or by encountering currents bringing waters with different salinity into contact with one another. Such an abrupt boundary in salinity is called a *halocline*; scuba divers recognize this as shimmering boundary region with limited visibility.

Currents

There are a number of different water movements in the ocean that have important effects on scuba divers. The first are the major surface currents throughout the world, caused by the prevailing winds in the world's atmosphere; these currents do live on the surface of the ocean, and have little effect deeper than 300 feet or so.

If we look at it from above the north pole, the earth spins on its axis in a counterclockwise direction. This leads to something called the *coriolis effect*: in the northern hemisphere, objects headed north tend to deflect to the right. This sets up surface currents in the form of large elliptical cells called *gyres*. In the northern hemisphere these surface currents tend to run clockwise, while in the southern hemisphere, the corresponding effect leads to gyres running counterclockwise.[18]

These surface currents have important consequences for scuba divers. Because the clockwise California current tends to bring colder water from the northern Pacific to the coast of California, the water temperature when diving off San Diego tends to be much colder than the latitude of San Diego would suggest. On the other hand, Bermuda, which is at a similar latitude in the Atlantic, enjoys the relatively warmer water brought from the tropics by the Gulf Stream.

The subject of ocean currents is actually a very complicated one, and is well beyond the scope of this text. Local topography and weather have important effects on local currents, and how they vary over time. There are some general comments to make regarding the effect of currents on divers.

Some dive locations have reasonably predictable local currents, and divers often take advantage of them. In a *drift dive* divers leave an untethered dive boat, and let the current take them as it will. The boat captain's job is to follow the divers, so that when they surface they can easily get back on the boat. It is usually quite easy for someone at the surface to follow the bubbles of the divers. For additional safety, the divemaster working for a drift dive operator may also bring along a float on a rope, which is easily visible on the surface and marks the location of dive group.

Even reasonably dependable currents can change abruptly and without warning. Commercial dive operators often send a diver down to check the current before allowing their patrons to enter the water.

When diving from a tethered boat, divers are taught to begin their dive *against* the current. That way, they can expect some help from the current when they return; on the return trip they may well be tired, and also worried about air supply. Obviously, such a dive plan is only possible if the current is moderate enough that the divers are able to swim against it; it takes careful

judgement and experience to decide whether a current is modest enough that divers can make headway against it.

If a diver is caught in a current taking him in a direction he does not wish to go, he should never try to swim directly into the current. This leads to exhaustion and a rapid depletion of air supply. In such a situation, the diver should swim in a direction *perpendicular* to the current; in most cases, the current will dissipate, and the diver then is in a position to find help. This is true at the surface, too; the notorious *rip currents* are strong seaward currents that can lead to panic and disaster if a diver or swimmer fights directly against them.

Neophyte divers understandably prefer still, clear water, like that found in many locations in the Caribbean. More experienced divers look forward to diving in currents, and part of that is the natural tendency in any sport to enjoy situations more challenging than that found at the introductory level. But the major reason why many divers like currents, is that moving water tends to bring the sea life! Without the strong currents at Cocos Island, divers would be unlikely to see the great schools of hammerhead sharks, which frequent that location. It is an amazing experience to be swimming alongside a 35 foot whale shark in the Galapagos – the shark swims into the current, precisely because the moving water brings to it the plankton it feeds on. Because such a dive is into the current, the whale shark, with the most casual of fin strokes, can quickly outswim the diver working hard against the current.

TIDES

An additional movement of water in the ocean is the *tides*. This is a slow change in water level which occurs over a period of about 12 hours; indeed, tides can be thought of as very long waves. The operation of the tides was connected in ancient times to the moon, but it was Isaac Newton who first explained the tides as a gravitational effect: the gravitational attraction of the moon causes a bulge in the waters of the ocean to form, when that side of the earth is facing towards the moon. Because the earth spins on its axis once every 24 hours, this leads to the regular periodic behavior of the tides.[19] Tides are so predictable that maritime locations have *tide tables* that sailors and divers make use of. Navigators need to know whether the accessibility of a site is affected by whether the tide is high or low; divers traditionally try to dive at slack tide – the lack of water movement usually makes the conditions better.

Exercises

1. Why is it important for a wetsuit to fit snugly?

2. A diver in a drysuit neglects to attach the hose used to add air to the drysuit. What consequences could the diver suffer?

3. It takes one calorie[20] of energy to raise a gram of water one degree Celsius. How many calories would be required to raise one gram of air three degrees Celsius?

4. How many cubic meters of water are in the world's oceans? How many kilograms does this seawater weigh?

5. A fully geared diver weighing 200 pounds is neutral in average seawater. How many pounds positively buoyant would he be, in the Red Sea?

6. Is the Great Barrier Reef off the northeast corner of Australia likely to be warmer or colder than its latitude would suggest? Explain.

7. A ocean wave has height H =2 feet and length L = 20 feet. Suppose the wave is traveling 10 miles per hour. What is its period? What is its frequency? At what depth would it begin to "feel" the bottom?

8. What is the speed of light in a vacuum, in meters per second? How about miles per hour?

9. The index of refraction of a certain pane of glass is 1.5. How fast does light travel through the glass?

10. A light ray enters the glass in the previous exercise from air, at an angle of 45°. At what angle does the light ray leave the glass? Draw a picture that shows which direction the light ray bends.

11. A naive diver claims that a manta ray he saw had a wing span of 10 feet. What is the likely actual wing span of the ray?

12. A diver using her knife to cut a rope feels a twinge, and then sees a milky green cloud. Explain what happened.

13. The sun at a certain dive location and a certain time is 50° degrees above the horizon. At what angle from the surface of the ocean does a sunbeam bend under the water?

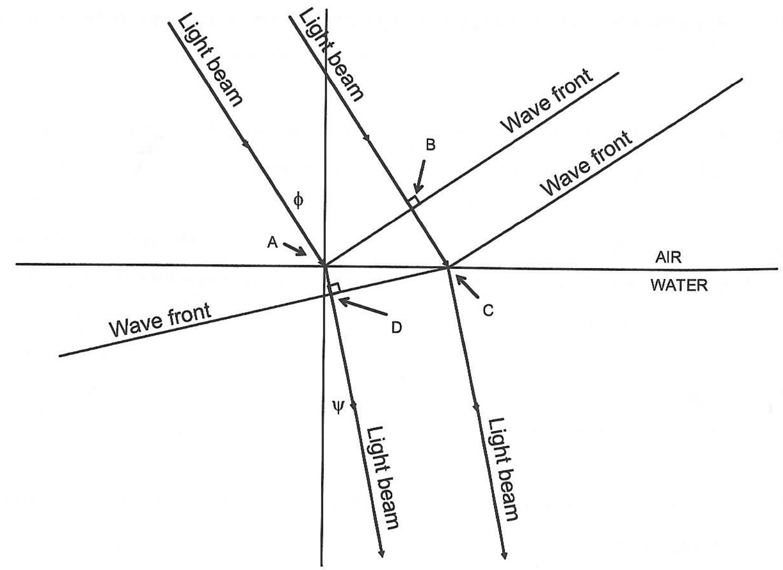
14. You are standing on the beach. An explosion occurs a mile away in air. How long before you hear it?

15. You are diving underwater. An explosion occurs a mile away in the ocean. How long before you hear it? How long would it take, if you were diving in Lake Michigan?

16. A sound comes from your right. The sound wave strikes your right ear first. How much delay is there before the sound wave strikes your left ear, if you are in air? How much delay is there, if you are in seawater? (You may assume that your ears are 5 inches apart.)

17. In this problem, you will explore the reason why light changes direction, when it passes from one medium to another; we will in fact obtain Snell's law. We assume that light can be modeled as wave of energy in the medium it is passing through. We assume further that light proceeds in a straight line, and its direction is perpendicular to its wave front. For the purposes of this problem we also assume that we have a wave of *coherent* light, with a single wavelength.

    Consider the picture of this situation. Express lengths BC and AD in terms of the period t, and the velocities v_a,v_w. Use the diagram to show that Snell's law holds. In particular, note that angle ψ is smaller than angle φ.



Endnotes

1 The gas used is nitrogen.

2 If you dive in a wetsuit in cool water, and then step onto dry land, you can *feel* the warm water running out of your suit.

3 Divers who take trips on liveaboard dive boats have the advantage of being right at the dive sites; under such circumstances, it is not unusual to do three or four (or five!) dives each day. With good conditions and good air consumption, each such dive might be an hour or more long.

4 Of course, the cycle of water in and out of the atmosphere in the form of precipitation is an essential process for life on earth. But the amount of water involved is modest as compared to the volume of the oceans.

5 Actually, the water particles are moving in small circular paths called *surface orbits*, as they bob up and down.

6 The fixed speed of a sound wave through a given medium should be compared to the fixed speed of *light waves* through a given medium; we will discuss light waves later in this chapter.

7 Knowing the speed of sound and taking note of this delay can even make it possible to estimate how far away the lightning strike took place.

8 This is actually not really true, according to general relativity, but for our purposes we can assume this is true.

9 And the basis for special relativity!

10 It is interesting to note that sound travels faster through denser media, while light travels more slowly. This has to do with the fact that sound is a transmission of mechanical energy, while light (which can also be considered a wave) is a transmission of electromagnetic energy.

11 Technically speaking, we should replace v_a by the speed of light in a vacuum; to our level of accuracy, there is no difference.

12 In a homework exercise, you will look at an argument for why this is true; it requires that we think of light as a wave!

13 This is the ordinary sine function of elementary trigonometry. We won't actually need to know much about trigonometry, but only that the sine function associates a number (between -1 and 1) for each angle.

14 Actually, the light must pass from the water to the glass, and from the glass to the air. But the refractive index of the glass is close enough to that of water, and the glass is thin enough, that we can disregard the effects of the glass.

15 A professor is moved from a 9 month salary to a 12 month salary. He has received a 33% increase. Later, he is reduced to a 9 month salary. He has consequently suffered a 25% decrease. Despite the different numbers, he ends up with his original salary. The difference is in the denominator used. Whenever percentage increases or decreases are described, it is really important to make sure you understand what the denominator is!

16 Because the wave-length $L = vt$, where v is the fixed speed of light, and t is the period, we can phrase this in a number of different ways: higher energy light has a shorter *wave-length*, a shorter *period*, and a higher *frequency*.

17 In the next chapter we will model this process mathematically.

18 An urban legend suggests that the water in a toilet bowls spins clockwise in the northern hemisphere, but counterclockwise in the southern hemisphere. Actually, the coriolis effect is so tiny for a body of water of this size that any such minute tendency is overwhelmed by the design of the plumbing.

19 Because the earth is moving around the sun too, the tide period is a little more complicated than exactly 12 hours.

[20] This is the definition of the calorie; one calorie equals about 4.186 joules. We are referring here to the so-called *small* calorie; the Calorie used when measuring the energy content of food is 1000 times larger than this.

Chapter 6
The Exponential Function

We have up to now been able to understand a good bit about pressure and diving, using only elementary arithmetic and proportions. But we now must face the task of understanding some more sophisticated mathematical machinery. Namely, we need to understand the concepts of *exponential growth* and *exponential decay*. It turns out that many phenomena in physics, biology and economics are well-modeled by exponential growth and decay, and some understanding of this is necessary for us to comprehend important physiological effects on scuba divers.

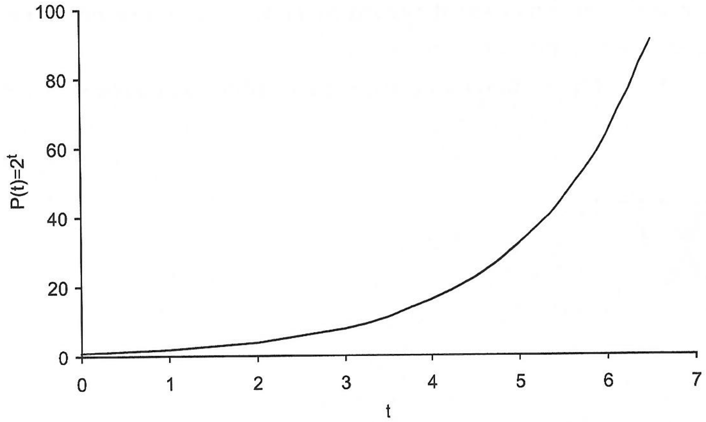
Exponential growth

A good example of an exponential growth function is $P(t)=2^t$. Our variable t occurs as the *exponent* in this formula, while the *base* of the exponentiation is 2 in this example. Let's imagine for the moment that $P(t)$ represents the population of a certain colony of bacteria at time t (to be specific, let's suppose that t is measured in days).

The *initial* population of bacteria occurs at time $t = 0$, and that population is $P(0) = 2^0 = 1$ bacterium! Since bacteria reproduce asexually by cell division, 1 individual is in principle enough to get our population started. After each day the population doubles:

$$P(1) = 2^1 = 2,\ P(2) = 2^2 = 4,\ P(3) = 2^3 = 8,\ \ldots.$$

We could imagine some species of bacteria that subdivides once a day — under this assumption this function seems to be a plausible model for the population. The ever more rapid increase of the function $P(t)$ is characteristic of exponential growth. We can represent this growth well in a graph of the function.



We have drawn this curve *continuously*, without any gaps. And indeed, we are happy to interpret our function at times other than the beginning of a new day. For example, $P(5.5) = 2^{5.5} = 45.25$ (where we have used a calculator to compute this value). It of course makes no sense to talk about .25 bacteria, but this is after all only a *mathematical model*, which is intended to provide a good estimate for the actual biological data. If this is a good model for our bacteria population, we should indeed expect 45 or 46 bacteria in the colony at the middle of the sixth day.[1] Note that even if our imaginary species of bacteria does indeed subdivide exactly once a day, the model reflects the fact that these one-day subdivision cycles are unlikely to be synchronized: individual subdivisions might be occurring at any time of the day, and so the population increases take place at any given moment.

What's really interesting about our exponential function $P(t)=2^t$ is that the population will double after one day, *starting at any given instant.* For example, let's compute the bacteria population exactly one day after $t = 5.5$:

$$P(6.5) = 2^{6.5} = 2^{5.5+1} = 2^{5.5} \times 2^1 = 45.25 \times 2 = 90.5$$

where we have done some basic manipulations regarding exponents.[2] The *doubling time* for this exponential function is exactly one day. It doesn't take too much mathematical imagination to see that the extra factor of 2 in the calculation above will arise at *any* starting time: $P(t+1) = 2P(t)$. It actually

turns out that all exponential growth functions will have a doubling time; we'll see many examples of this below.

Many biological populations do tend to exhibit exponential growth, at least while they grow under favorable conditions (enough room to spread out, and enough nutrients to support the population). But to model such behavior, we need to change our function in two ways: First, our initial population might not be 1 organism; we need to adjust for that. Second, the doubling time for the population might not be such a nice number as exactly 1 day (or 1 year, or whatever other unit of time we wish to use).

The first of these problems is easy. Suppose the initial population is the constant P_0. Then the function $P(t) = P_0 2^t$ has the appropriate *initial condition* $P(0) = P_0 2^0 = P_0$, and it will still double once a day.

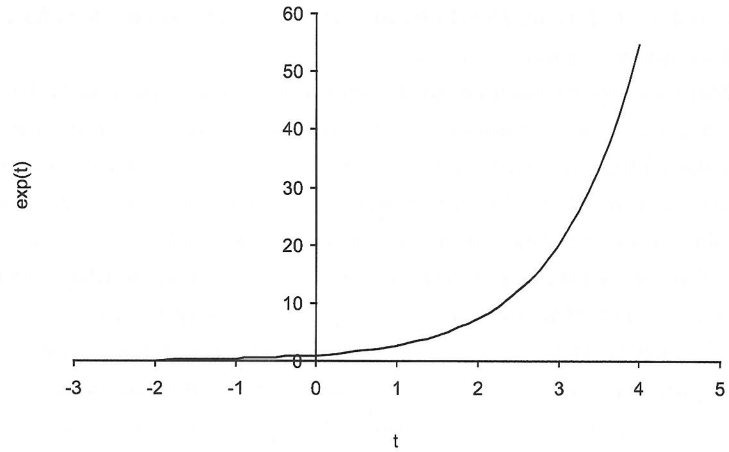
To change the doubling time requires us to change the *base* of the exponential function. For example, the new population function

$$P(t) = P_0 8^t = P_0 (2^3)^t = P_0 2^{3t}$$

now has a doubling time of 1/3 day instead. And notice that in this case we have in fact expressed this function in terms of the original base 2.

In practice, mathematicians and scientists choose a single base to express all their exponential growth functions, and adjust the doubling time by inserting a constant multiple on the time t like the 3 in the expression above. Unfortunately, that base is not the integer 2, but instead the famous number $e = 2.71828 \cdots$. Indeed, on almost any calculator, you will find the exponential function e^t prominently displayed.[3]

It is worth graphing here the exponential function e^t. It clearly has a strong family resemblance to 2^t graphed above. It just grows a bit faster, because the base is a bit bigger. Notice that in this picture we have also drawn the graph for $t < 0$. If we evaluate this function at a negative number like −5, we have $e^{-5} = 1/e^5 = .0067$.[4] Thus for large positive t, e^{-t} gets very close to zero, as the picture shows:



Now how did we change 8^t into a function related to base 2? We used 2^{3t}, and this is precisely because $8=2^3$. In other words, we needed to find the exponent on 2 to get 8. This was easy in this case. But to express 2^t in terms of base e, we need to find a number k for which $e^k=2$. This exponent we call the *natural logarithm* of 2, and we ordinarily write $k = \ln 2$. You will also find this function on any calculator, and by that means we discover that $\ln 2 = .6931$. Consequently, $2^t = (e^{\ln 2})^t = e^{(\ln 2)t} = e^{.6931t}$. We can use this trick to change any exponential base to an expression involving base e.

If you are not used to the exponential function e^t and the corresponding natural logarithm function $\ln t$, now might be a good time to play a little bit with these functions on a calculator.[5] If you compute e^3, you obtain 20.086. But if you then compute ln(20.086) you should get back to 3.

On the other hand, if you first compute ln(5) = 1.609, this is exactly the exponent we need to put on e to obtain 5; that is[6], $e^{1.609}=5$. Indeed, on many calculators, the exponential function e^t (sometimes called exp(t)) is operated using the very same button as the ln(t) function, only preceded by an "inverse" button. Using the jargon of mathematicians, the functions e^t and $\ln t$ are *inverse functions* — they exactly undo one another!

So our most general exponential growth function looks like $P(t) = P_0e^{kt}$, where P_0 is a constant giving our initial function value, and k is a constant

that affects the rate of growth of the function. Here is an important and illuminating example.

Suppose that we have \$1000 to invest. A bank reports that it will pay 5% yearly interest, continuously compounded. It turns out that this is an exponential growth function, given by $P(t) = 1000e^{.05t}$, where P is of course measured in dollars, and t is measured in years. After one year, we will have $P(1) = 1000e^{.05} = \$1051.27$ in the bank. Notice that 5% of \$1000 is just \$50, and we have actually made \$51.27 on our investment. This is because the interest is *continuously compounded.* We not only get interest on the principal, but also on interest that is paid during the year. Admittedly, this doesn't make much difference in this case, but every little bit helps![7]

What is the *doubling time* for this exponential growth function? We are looking for the length of time after which our \$1000 investment has turned into \$2000. We can do a little algebra to figure this out. We want

$$P(t) = 1000e^{.05t} = 2000.$$

But dividing both sides by 1000 gives us $e^{.05t}=2$. To get our hands on the exponent on the left side of the equal signs, we should take the natural logarithm of both sides. From our discussion above, the natural logarithm precisely gives the exponent on e. We thus have

$$.05t = \ln(e^{.05t}) = \ln 2 = .6931.$$

Therefore $t = .6931/.05 = 13.86$. Our investment will double after 13.86 years. Furthermore, this investment will continue to double after each such interval of time.

Now we could change this growth function to a base other than e. Namely,

$$1000e^{.05t} = 1000(e^{.05})^t = 1000 \times 1.05127^t.$$

But we would argue that using base e with the interest rate 5% appearing in our expression is actually much more illuminating than dealing with the peculiar base 1.05127.

After how many years will our investment at 5% triple? To solve this, we need only consider the equation $1000e^{.05t} = 3000$. We then obtain $e^{.05t} = 3$, or $t = \ln 3/.05$. Recourse to a calculator reveals that $\ln 3 = 1.0986$, and so the tripling time for this investment is $1.0986/.05 = 21.97$ years.

The constant k in the generic exponential growth function $P(t) = P_0e^{kt}$ determines the rate of growth of the function, *as a proportion of the current value of the function.* Your investment grows at a rate *proportional to the amount of money you have invested.* Similarly, a biological population experiencing exponential growth grows at a rate *proportional to the number of organisms currently present.* So if a country has a initial population of P_0, and a relative growth rate of 10%, then the growth function will be $P_0e^{.1t}$. Notice that the doubling time in this case is only $\ln 2/.1 = .6931/.1 = 6.9$ years!

So, for a population with a growth function $P(t) = P_0e^{kt}$, the rate of growth of the function is actually kP, where the units would be individuals per unit time. Or for our financial example, the units would be dollars per year. In the particular case of an investment at 5%, we would say at any given instant that we would expect growth of $.05P$, that is, five percent of the money in the bank at that instant. Over the course of the following year we will actually do better than that, because the growth rate is actually increasing, as the amount of money in the bank grows!

It is important to distinguish between the *relative growth rate k*, and the *rate of growth* of the function. If you invest a million dollars at $k = 5\%$, and I invest a thousand dollars at $k = 5\%$, your money will be growing at a rate of $50,000 per year, while mine will only be growing at a rate $50 per year, even though we are paid the same interest rate (have the same relative growth rate)!

This can be illuminated by looking at the units on the constant k. In the population example, the units on *kP are individuals per unit time, and the units on P are individuals. The units on k* itself is *percent per unit time.* The calculation thus looks like this:[8]

$$k\frac{\%}{\text{unit time}} \times P \text{ individuals} = kP\frac{\text{individuals}}{\text{unit time}}$$

Mathematicians represent the rate of growth[9] of the function $P(t)$ by the notation $\frac{dP}{dt}$. This notation suggests to us that to find a rate of growth,

we should take a small interval of time dt, and divide that into the resulting increase in the principal dP. This fraction then suggests the units: number of dollars per unit time. If instead we are considering a population growth model, the units would be number of individual organisms per unit time.

In general, we have argued that the rate of growth of the exponential growth function $P(t) = P_0e^{kt}$ is given by

$$\frac{dP}{dt} = kP \qquad (6.1)$$

— the larger the population, the faster it grows, and the larger the relative growth rate k is, the faster it grows![10] Equations like 6.1 are called *differential equations*; such growth rate equations occur in all areas of applied mathematics.

For a specific example, suppose that a certain biological population has an initial population of 1200 individuals, and it has a relative growth rate of 3%. How fast is this population growing after 7.5 years? To compute this, we proceed as follows:

$$\frac{dP}{dt}(7.5) = .03P(7.5) = .03(1200e^{(.03)(7.5)}) = .03(1502.8) = 45.1$$

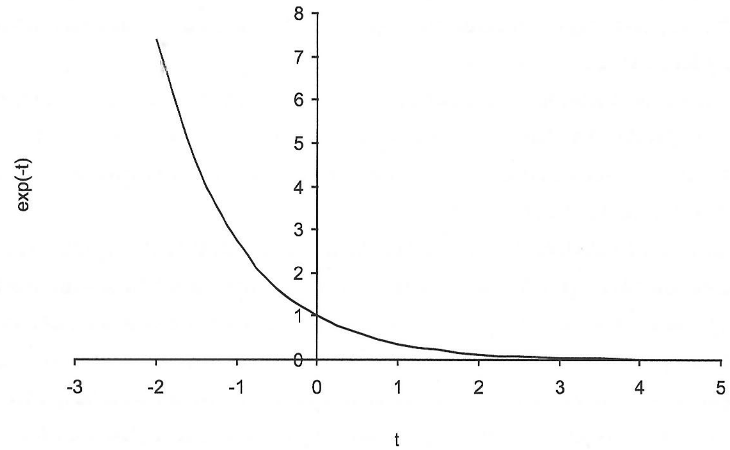
At this moment in time, the population is growing at a rate of about 45 individuals per year. Note that embedded in this calculation is the value $P(7.5)$ of the actual population at this time, which is about 1503 individuals.

There is one more thing to say. Consider the situation when $k = 1$ — that is, the relative growth rate is 100%. Then the exponential growth function is $P(t) = e^t$ (assuming that $P(0) = 1$). Then the total growth rate of our exponential function is exactly P itself. The prettiness of the equation $\frac{dP}{dt} = P$ is a part of why mathematicians prefer base e to any other![11]

Exponential decay

Now that we have described exponential growth, *exponential decay* is actually quite easy to understand. Instead of the function $P(t) = P_0e^{kt}$, where k is positive, we now have $P(t) = P_0e^{-kt}$. As we saw in the previous section, as t

grows large, $-kt$ grows very negative (for any positive k), and so in the long run $e^{-kt} = 1/e^{kt}$ grows close to zero. Here is a picture of e^{-t}:



In this case, the rate of change of the decay function is $\frac{dP}{dt} = -kP$. The negative sign means that the function $P(t)$ *decreases* as t increases. This makes sense if we think of the rate of change as a small change in t, divided into the corresponding small change in P. When P is a decreasing function, a small positive change in t leads to a corresponding negative change in P.

Since an exponential decay function is decreasing, we have a *halving time*, rather than a doubling time. It is more customary to call this the *half-life* for the function. For example, consider the exponential decay function $P(t) = P_0 e^{-.3t}$. What is the half-life for this function? We would like to have

$$P(t) = 1/2P(0) = 1/2P_0.$$

This leads to the equation $1/2P_0 = P_0 e^{-.3t}$. We can divide both sides by P_0, and take the natural logarithm of both sides. We obtain $\ln(1/2) = -.3t$. But[12]

$$\ln(1/2) = -\ln(2) = -.6931,$$

and so

$$t = \frac{-.6931}{-.3} = 2.31$$

The exponential decay function will reach half the original value at time $t = 2.31$. In fact, the value of the function will halve from any given value after this amount of time has elapsed (the argument for this being identical to the argument for the doubling time for the exponential growth function described above).

The classic example of a function which exponentially decays is the radioactive decay function: the amount of radioactivity in a given substance decays over time, and eventually approaches zero. You can explore these ideas in Exercise 10 at the end of the chapter.

But for scuba divers, there are other more important examples. Our first such example is the decrease in *atmospheric pressure*, as we ascend in altitude. We can use the idea of exponential decay to make precise Pascal's observation that air pressure should decrease as we ascend in altitude. We all know that if we get far enough from the surface of the earth, there is no atmosphere left, and consequently no remaining air pressure. How is this decrease modeled mathematically?

In our previous examples of exponential decay (or growth), the process described evolves over time. In our new example, the process described evolves as we ascend above the earth. In the earlier examples, our function depends on *time*; in this example our function will depend on *altitude*.[13]

Let's denote the pressure of the ambient air at altitude h by $p(h)$, where we measure h in feet, and $p(h)$ in ata (atmospheres). Consequently, the initial condition in this case is the air pressure $p(0) = 1$ ata at sea level. It seems plausible to assume that at any given altitude, another foot of altitude would diminish the pressure by a fixed percentage k of the pressure there present. That is, the rate of change is $\frac{dp}{dh} = -kp$. Note the units suggested by the notation: this rate of change is measured in atmospheres per foot (of altitude).

As we've seen above, our function is consequently of the form $p(h) = e^{-kh}$, where k is a positive constant that needs to be determined. The constant $k > 0$ controls how rapidly decay takes place — the larger the value, the faster the decay. It is possible to determine the constant k by measuring the pressure of air at some height greater than sea level. (You will explore this in one of the exercises.) It turns out that the value of the constant k is .0000383 (or .00383%). Consequently our exponential decay function modeling air pressure as a function of altitude [14] is $p(h) = e^{-.0000383h}$.

Now recall that for a gas its density is directly proportional to its pressure; an increase in the number of molecules leads to a proportional increase in pressure. Consequently, we can also think of the *density* of air $d(h)$ at altitude h to be an exponential decay function, with the same rate of decay as the air pressure function. If we measure density in pounds per cubic foot, and remember that at sea level air weighs .081 pounds per cubic foot, then $d(0) = .081$, and so we have $d(h) = .081e^{-.0000383h}$.

Santa Rosa, New Mexico is a small town on historic Route 66, and it is at an elevation of about 4600 feet. This means that the atmospheric pressure there is

$$p(4600) = e^{-.0000383 \cdot 4600} = .838.$$

Or to say that in another way, in Santa Rosa the atmospheric pressure is only 83.8% of that at sea level; alternatively, the air is only 83.8% as dense as that at sea level. It is interesting to note that human beings can quite easily live under such conditions (although sometimes it takes some acclimation).

But now consider the top of Mount Everest, where the altitude is approximately $h = 29000$. In this case

$$e^{-.0000383 \cdot 29000} = .33.$$

With air only 33% as dense, it is not surprising that most climbers require supplemental oxygen at this elevation.

So what is the half-life of this function? That is, at what elevation is the atmospheric pressure equal to half of that at sea level? We need to have $-.6931 = \ln(1/2) = \ln(e^{-.0000383h})$. In an exercise below, you will compute this altitude.

Now consider a dive in the Blue Hole, a spring-filled natural well located in the town of Santa Rosa. Let's suppose that we have a balloon filled with 1 quart of air at the surface of the water, and we descend to 34 feet. What is the size of the balloon now? By Boyle's Law, we clearly need only calculate the total pressure on the balloon.[15] The 34 feet of fresh water contribute 1 atmosphere of pressure. But the air above only contributes .838 atmospheres, because of the elevation change. This means that the total pressure on the balloon at depth is 1.838 ata, while the total pressure

at the surface is only .838. This means that the size of the balloon is given by .838/1.838 = .46 quarts. Compare this to the volume if this dive had been done in a fresh water site at sea level: 1/2= .5 quarts. The balloon is smaller in Santa Rosa because the 34 feet of water makes for a larger change in pressure, relatively speaking, because of the thinner air in New Mexico. Pressure changes relative to surface pressure occur more rapidly with changes in depth when we scuba dive at higher altitudes; as we shall see, scuba divers need to be aware of this effect.

Another important example of exponential decay for scuba divers is the attenuation of light as we penetrate deeper into the ocean. To express this mathematically, we must measure the *intensity* of light at a given depth. We will call this function $I(h)$, where h is the depth in the water. It again seems plausible to assume that the light intensity will exponentially decay as we go deeper and deeper. That is, for each foot of additional depth, we should expect to lose a fixed percentage k of the light intensity there present. That is, $\frac{dI}{dh} = -kI$, and so $I(h) = I_0 e^{-kh}$, as before. Once again, if we accept this mathematical model, we need only determine empirically the numerical constant k. We would expect that this percentage might vary, depending on whether the water is clear or turbid.

To actually carry this out, we would really need to decide on how light intensity could be measured, and what units we should use. Since the physics is a little complicated, we will consign this discussion to a footnote.[16] It turns out that the value of k varies from 1.8% per foot in clear Caribbean water, to 3.6% or higher.[17]

We can carry this discussion one step further. As we observed in the previous chapter, white light is actually made of a spectrum of colors (light of varying wavelengths). And the different colors actually have *different* values for the constant k – larger values for the less energetic light at the red end of the spectrum.

For the Mathematically Inclined Only!

Let's consider again air pressure at sea level. In the next few paragraphs we'll provide an argument that shows how our figure of 14.7 psi at sea level comes from the air density function $d(h)$ we discussed earlier. The argument that follows amounts to doing a little calculus, and so can be a bit tricky. Fortunately, the argument is not important for what follows. If you've had a little calculus, forge ahead!

The air pressure at sea level (in psi) should be the total weight of the air in a skinny box extending directly above an area of 1 square inch, extending through the entire atmosphere. But since our density function is expressed in terms of feet, we will actually find it easier to imagine our skinny box with a base of 1 square foot. Once we've done our calculation, we'll divide by 144, because a square foot is actually $12^2 = 144$ square inches.

Let's take a small slice of our skinny box at altitude h, with height Δh.[18] The volume of this little slice is the thickness of the slice times its cross-sectional area: $\Delta h \times 1\ \text{ft}^2 = \Delta h$. The weight of this little slice in pounds is given by density times volume. This weight is

$$d(h)\Delta h = .081e^{-.0000383h}\,\Delta h$$

where we have of course used our formula for the density $d(h)$ of air at altitude h.

To calculate the total weight of air above our base of 1 square foot requires us to add up all the weights of these small slices, for all altitudes $0 < h < \infty$. Such a sum is known to calculus students as an *integral*, which is written like this:

$$\int_0^\infty .081e^{-.0000383h}\,dh$$

If you've had calculus, this is not a difficult integral to compute, but in any case, we obtain .081/.0000383 = 2114.9 pounds. If we divide this by 144 to obtain the weight over 1 square inch instead, we get 2114.9/144 = 14.7 pounds. This is of course the value 14.7 psi for air pressure that we have discussed earlier.

Newton's law of cooling

Suppose that the temperature of a cup of coffee is 100° Fahrenheit. It is set on a desk in a 70° room. Nature as always seeks *equilibrium* — heat flows

from the coffee, and its temperature gradually approaches the temperature of the room.[19] Isaac Newton proposed that the rate of this temperature change should be proportional to the *difference in temperature* between the coffee and the ambient air. This is called *Newton's Law of Cooling*. This sounds a lot like the rate of change description of exponential decay.

To make this situation more mathematical, let's suppose that $T(t)$ gives the temperature of the coffee at time t (measured in minutes). Then $T(0) = 100$. Newton's observation about the rate of change of the function T can then be expressed as the following differential equation:

$$\frac{dT}{dt} = k(70 - T) \qquad (6.2)$$

In our example the coffee temperature T is greater than 70, and so $70 - T$ is negative. With a positive constant k, this means that this rate of change is negative: the coffee's temperature is decreasing.

The close resemblance of this equation to the $dP/dt = kP$ equation describing exponential growth (or decay) makes it believable that its solution[20] is $T(t) = 70 + 30e^{-kt}$. This function clearly starts with value $T(0) = 100$, and decays exponentially, but to the equilibrium value $T = 70$, instead of to 0. In fact, it is quite illuminating to recognize that Equation 6.2 above says that there is no change in the coffee temperature if it is *already* at this equilibrium value of $T = 70$. Of course, to actually make use of this function, we would need to determine the value of the relative rate of change k. This is easy to do if we know the temperature of the coffee at some other time (see the exercises).

One way that is frequently used to specify the value of k is reminiscent of our calculations above with exponential growth and decay — with an e^{-kt} function involved, we still have a *half-life*. In the case of cooling, that would be the time after which the coffee has cooled to a temperature halfway between its starting temperature and the ambient temperature. For example, in our situation above, let's suppose that this half-life is 10 minutes. The halfway point between 70 and 100 degrees is 85 degrees, and so this would mean that after ten minutes the coffee temperature is $T(10) = 85$. Plugging this information into our general equation gives us that $85 = 70 + 30e^{-10k}$.

Exercises

1. The following expressions are easy to simplify if you understand the relationship between the exponential and logarithm functions. Perform these simplifications, without the use of a calculator:

   (a) $e^{\ln 4}$.
   (b) $\ln (e^{27})$.
   (c) $e^{-\ln 4}$.
   (d) $\ln 1$.
   (e) $\ln e$.
   (f) $\ln (e^2)$.
   (g) $\ln\left(\frac{1}{e}\right)$.
   (h) Recall that $\ln 2 = .693$. Compute $\ln(2e)$.
   (i) Suppose that $e^a=3.2$. Compute $\ln(3.2e)$, in terms of a.

2. After how many years does an investment at 8% (continuously compounded) double?

3. Suppose a bank wishes to promote an investment that doubles after 14 years. What continuously compounded interest rate should they advertise?

4. A certain biological population can be accurately modeled using exponential growth. We know that the initial population is 1543 individuals. After 4 years, the population is 2844 individuals. Determine the exponential growth function $P\ (t)= P_0 e^{kt}$ which describes this population. What is the population after 7.2 years? What is the rate of growth of this population after 7.2 years?

5. At what elevation in feet is the air pressure 50% of what it is at sea level? To rephrase this, we are asking for the *half-life* for the exponential decay function describing air pressure.

6. What is the atmospheric pressure (in ata) in Colorado Springs, Colorado? The elevation is 6035 feet.

7. How deep would you need to dive in a freshwater lake in Colorado Springs, in order that the ambient pressure is exactly twice what it is at the surface?

8. One pint of air is put into a balloon at the bottom of Blue Hole in New Mexico. The water is fresh, the depth is 80 feet, the altitude is 4600 feet and the water is 64° from top to bottom. The balloon is taken to the diving platform at 25 feet deep in the water. What is the volume of the balloon now?

9. A scuba tank is filled with 5% oxygen and 95% physiologically inert gas. At what depth on a seawater dive will the partial pressure of oxygen equal the partial pressure of oxygen you would find on Pikes Peak (altitude 14110 feet)?[21]

10. Carbon-14 is a radioactive isotope of the element carbon that decays to carbon-12 as time passes. Suppose that $R(t)$ is the function that gives the amount of a sample of carbon-14, where t is measured in years, and R is measured in some appropriate units. Suppose that $R(0) = 1000$. The half-life of this radioactive isotope is 5780 years — that is, half of the radioactive material has decayed over that time. Express $R(t)$ as an exponential decay function, including a numerical value for the constant k. How much radioactivity is present after 10,000 years?[22]

11. Consider the exponential decay function $P\ (t)= e^{-kt}$, where k is an arbitrary positive constant. Determine the half-life T of this function (in terms of k), and show that $P\ (t + T)=1/2P\ (t)$, for any time t.

12. A balloon with a volume of .75 liters is filled at sea level with air; the air temperature is 25° Celsius. The balloon is driven to an elevation of 5000 feet, where the temperature is 10° Celsius. What is the volume of the balloon now?

13. Recall that for clear Caribbean water, the relative rate of decay of light intensity is .018 (where we are measuring depth in feet). At what depth will the light be half as intense as it is at the surface? At what depth will there we essentially no light (use the standard that the light intensity is only 1% of what it is at the surface).

14. (If you know calculus.) Check by differentiation that $T(t) = 70 + 30e^{-kt}$ and $T(t) = 70 - 30e^{-kt}$ both satisfy the differential equation 6.2. More generally, check that the solution given by equation 6.3 satisfies equation 6.2.

15. (If you know calculus.) Solve the differential equation 6.2, using the method of separation of variables – you should get the answer described by 6.3.

16. (If you know calculus.) Actually compute the improper integral cited in the text, which computes the total weight of the air above a square foot, thus verifying the number given in the text.

17. Suppose that a cup of 100° coffee is brought into a 70° room. Suppose that we also know that $T(15) = 80$. Determine the value of k in this case. What is the temperature of the coffee after 20 minutes?

18. Suppose a cold glass of fruit juice is removed from refrigerator, and brought out on a picnic on a warm summer day. The juice is 40°, while the air temperature is 89°. Suppose the half-life for the corresponding exponential function is 12 minutes. After how many minutes will the juice reach 75°?

19. Cookies are removed from an oven, and placed on a rack to cool. At the beginning of the cooling process, the cookies are 120°, and the surrounding air is 68°. After 5 minutes, the cookies are 92°. You wish to consume them when they are 85°. How many minutes should you wait, from the time they were removed from the oven?

ENDNOTES

1 Constructing and evaluating mathematical models is the important task of applied mathematics. The model allows us to predict the future, but it is also important to analyze how accurate we expect these predictions to be. These are not simple questions, and it requires an understanding of *statistics* to carry out such an error analysis. In this book, we will present many models, but avoid these more delicate questions of error analysis.

2 See Appendix D for a discussion of the rules of exponents.

3 Later in this chapter we'll see some informal justification for why this peculiar choice of base is actually best; the full story comes only from a course in calculus.

4 See Appendix D for the rule of exponents we have used in this calculation.

5 You might also want to read Appendix D.

6 We are of course reporting the values given by the calculator to only a few decimal places; consequently, the equalities to which we refer are only as accurate as our approximate input. On the other hand, such equations as $e^{\ln 5} = 5$ are *exactly* true.

7 You might see this interest rate advertised as "5% compounded continuously, with an *effective annual yield* of 5.127%."

8 The percentage here is a pure number without units.

9 Mathematicians call this rate of growth the *derivative* of the function $P(t)$.

10 Some readers may recognize that we have sneaked a little calculus into this discussion. Fortunately, we will not find it necessary to do much more than this, since in this book we are only concerned with the rate of growth for exponential functions (and their close relatives).

11 You are forgiven if this observation leaves you cold — rest assured that we will make no use of it in the future; we will regularly perform our calculations using base e, however!

12 $e^{-\ln 2} = 1/e^{\ln 2} = 1/2$, and so $\ln(1/2) = -\ln 2$. See Appendix D for more about rules for the logarithm function.

13 Mathematicians' jargon: in the earlier examples the *independent variable* is time; now the independent variable is altitude.

14 This function is a little easier to look at and work with if we measure altitude in *thousands of feet y*; then it takes the form $p(y) = e^{-.0383y}$. But 'thousands of feet' can be a confusing unit, and so we will stick with the formula in the main body of the text.

15 We are assuming the temperature is fixed, and so we do not have to use the general gas law.

16 Light is really a form of energy, and physicists use the *joule* (or newton-meter) to measure that. But we should really be considering how much energy the sunlight is supplying per unit time. This leads to the unit of *power* called the *watt* (one joule per second). But this still doesn't capture the light intensity, since we should consider the area over which the light is spread. Consequently, the correct scientific unit for $I(d)$ should be watts per square meter!

17 Of course, to use the units described in the previous footnote, depth should really be measured in meters; these empirical constants translate to between 6% and 12% per meter.

18 We are here using the notation Δh is stand for small *difference* in the height h; Δ is the Greek letter 'D', and stands for 'difference'.

19 Actually, we would expect that the temperature of the room will rise a little too, but in this example, the volume of gas molecules at 70° is so vast that we can disregard any minute increase in room temperature.

20 We will leave it as a exercise below for students of calculus to verify this.

21 Note of course that this oxygen mixture would be too lean to breathe on, until the diver has reached a depth at which the partial pressure of oxygen is at least at a level like that in this problem. Technical divers often breathe different mixes at different depths.

22 Calculations of this kind are used by archaeologists to date artifacts.

Chapter 7

Modeling Nitrogen Absorption

Now that we have acquired some knowledge of exponential growth and decay, we are ready to return to the physiological effects of breathing compressed air at depth.

When we inhale, the air in our lungs is brought into contact with the bloodstream with the incredibly tiny structures known as the *alveoli*. Here is where the hemoglobin cells pick up the oxygen molecules from the fresh inhaled air, and the carbon dioxide molecules, which are the waste product of the oxidation process of the cells, are carried off.

Approximately 79% of the air molecules are nitrogen, which does not play any role in this physiological process.[1] These nitrogen molecules dissolve into the bloodstream, until the partial pressure of nitrogen dissolved into the blood is equal to the ambient pressure of the nitrogen in the surrounding air. The nitrogen dissolves into the bloodstream in such a way as to equalize pressure: the pressure exerted by the molecules of nitrogen dissolved in the blood should be exactly equal to the pressure exerted by the nitrogen molecules in the inhaled air. This is a general principle, worth emphasizing:

> **Henry's Law** *Suppose that a gas is in contact with a fluid. The gas dissolves into the liquid at a rate proportional to the relative gas pressure—that is, the difference between the ambient pressure of the gas, and the pressure of the gas already dissolved into the liquid.*

Under ordinary circumstances, this equalization of nitrogen partial pressure in the bloodstream and the ambient air has no physiological effects. If we fly in a commercial airplane, the cabin is pressurized to only 8000 feet of altitude, which is $e^{.0000383\times 8000} = 73\%$ of normal. Consequently, under these circumstances, some of the nitrogen dissolved in our bloodstream is released into the ambient air, in order to equalize this pressure. Our bodies are able to accommodate this change in pressure quite easily.

However, if we breathe pressurized air on scuba at depth, the relative change in nitrogen partial pressure can be much larger than any relative change we might encounter by changing altitude. According to Henry's law, equilibrium in pressure still takes place. So, suppose we dive to a depth of 33 fsw; the *total* pressure is now 2 ata = 66 fsw. The *partial* pressure of nitrogen is now 2 × .79 = 1.58 ata (or 66 × .79 = 52 fsw), and upon reaching equilibrium we will have twice as much nitrogen dissolved into the bloodstream as we did at the surface.

This additional nitrogen dissolved into the blood is not a problem. Indeed, aquanauts have successfully lived for weeks at time at 60 fsw or more, where the partial pressure of the physiologically inert nitrogen dissolved into the blood is two or three times that experienced at the surface at sea level.

The problem arises when such a diver ascends to the surface, and the nitrogen dissolved into the bloodstream leaves, in order to maintain equilibrium. If the rate at which the nitrogen leaves is too fast, bubbles of nitrogen can form in the bloodstream, with potentially disastrous results. Consequently, we must inquire into the *rate* at which the nitrogen leaves the blood.[2]

The rate of on-gassing and off-gassing

Henry's law asserts that the rate a gas dissolves into a liquid with which it is in contact is proportional to the *relative* gas pressure. To express this in mathematical terms, suppose that the partial pressure of nitrogen dissolved into the blood is given by the function $P(t)$, and that the partial pressure of nitrogen in the breathed air is P_a. Then

$$\frac{dP}{dt} = k(P_a - P)$$

This differential equation looks exactly like *Newton's Law of Cooling*. We thus have that the partial pressure of nitrogen dissolved in the blood is described by the equation

$$P(t) = P_a + (P0 - P_a)e^{-kt}. \qquad (7.1)$$

This is the direct analogue of equation 6.2. As time passes, this function grows ever closer to the ambient pressure P_a.

For example, suppose that a diver begins a dive at sea level.[3] Then the initial partial pressure P_0 of nitrogen dissolved into the bloodstream is .79 × 33 = 26 fsw. If the diver descends to 66 fsw, this means that the ambient partial pressure of nitrogen increases by .79 × 66 = 52 fsw, to a total of 26+52 = 78 fsw. Consequently, the pressure as a function of time is given by

$$P(t) = 78 + (26 - 78)e^{-kt} = 78 - 52e^{-kt}.$$

As time passes, this function clearly approaches the ambient pressure 78 fsw.

If the ambient pressure is greater than the current pressure, the diver is *on-gassing* nitrogen; if the ambient pressure is less than the current the pressure the diver is *off-gassing* nitrogen. To understand either of these processes in a more than qualitative way, we must clearly inquire into the relative rate of decay: namely, we need to determine the value of the constant k.

Haldane's model

It was the eminent British physiologist John Scott Haldane who first made use of the model for nitrogen on-gassing and off-gassing that we have described in the section above. His goal in understanding these rates was to help divers avoid the serious health effects of overly rapid decompression. We will discuss this carefully in the next chapter. For now, we will only consider Haldane's observations about the rates of on-gassing and off-gassing.

Haldane observed that various tissues in the body should be expected to have different rates of nitrogen absorption. A liquid (like blood) should be expected to very rapidly absorb nitrogen under pressure brought into contact with it. But a solid tissue like bone might be expected to absorb extra nitrogen at a much slower rate. Haldane consequently built a mathematical model to describe this. He assumed that body tissues could be put into five categories he called *compartments*, with different rates of absorption. In the context of equation 7.1, this amounts to assuming that different tissue compartments have different values for the constant k.

Haldane's method of specifying these values for k should be familiar to us. He did not directly give the value for k, but instead specified the *half-life* of our exponential decay. In his complete model for nitrogen absorption, he assumed that the half-lives for nitrogen absorption for his five theoretical tissue compartments were 5, 10, 20, 40 and 75 minutes. You can check that these correspond to the following k values:

Compartment	*k value*
5	ln(2)/5 = .1386
10	ln(2)/10 = .0693
20	ln(2)/20 = .0347
40	ln(2)/40 = .0173
75	ln(2)/75 = .0092

When a tissue compartment has reached ambient nitrogen pressure P_a, we say that the compartment is *saturated* with nitrogen — it has absorbed all that it can. Of course, the mathematical function $P(t) = P_a + (P0 - P_a)e^{-kt}$ that *models* this process will *never* reach the value P_a, because for arbitrarily large t, the quantity e^{-kt} is always at least a tiny bit above zero. In practice, we can consider a tissue compartment to be saturated after six half-lives, because by that time our function predicts that $P(t)$ will be within

$$1-\left(\frac{1}{2}\right)^6 = \frac{63}{64} = 98.4\%$$

of P_a. Technical, scientific and military divers sometimes spend long enough in pressurized underwater environments to saturate all their tissue compartments; this is called *saturation diving*. Using the original Haldane model and applying this standard for saturation, a diver would then have saturated all his tissue compartments to the ambient nitrogen partial pressure after 6 × 75 = 450 minutes, or 7.5 hours. Of course, this same time period would need to elapse in order for a diver returning to the surface to off-gas all the excess nitrogen in his 5 theoretical tissue compartments.

The same mathematical model can be applied to describe the rate of on-gassing and off-gassing for any physiologically inert gas in the mix being breathed by the diver. However, we should expect that the rates (or equivalently,

the half-lives) would be different. Not surprisingly, gases consisting of smaller molecules dissolve into tissues more rapidly, and symmetrically leave the body more rapidly too. This is one of the reasons that technical divers sometimes include the very light molecule helium in their dive mixes.

Let's consider a couple of examples of these calculations. Suppose that a diver descends to 60 fsw, and remains there for 30 minutes. What is the resulting partial pressure of nitrogen in her 75 minute compartment?

The initial partial pressure of nitrogen is $P_0 = (.79) \times 33 = 26$ fsw. The total ambient pressure is 93 fsw, and so the ambient partial pressure due to nitrogen is $P_a = (.79) \times 93 = 73$ fsw. For the 75 minute compartment, k = .0092. Consequently, according to equation 7.1 we have this model for the on-gassing of nitrogen: $P(t) = 73 - 47e^{-.0092t}$. The nitrogen partial pressure after 30 minutes is thus

$$P(30) = 73 - 47 \times (.76) = 37 \text{ fsw.}$$

Notice that the absorption has only gone about 24% of the way to full saturation (because 1 - .76 = .24).

Now consider a diver whose 20 minute compartment has a nitrogen partial pressure of 43 fsw. She ascends directly to the surface, and waits there for 8 minutes. What is the nitrogen partial pressure in that compartment after this time? Notice that in the previous example, the diver was on-gassing; in this example she is off-gassing. In this case, the initial partial pressure is P_0 = 43 fsw, and the ambient partial pressure is P_a = 26 fsw. For the 20 minute compartment the value for k is .0347. The function describing the off-gassing is then $P(t) = 26 + 17e^{-.0347t}$. Evaluating this function at $t = 8$ gives $P(8) = 39$ fsw.

The important general observation to make is this: *on-gassing and off-gassing are described by the same function.* If $P_0 > P_a$ we are off-gassing, and if $P_0 < P_a$ we are on-gassing. In other words, the sign of $P_0 - P_a$ in the function determines which is which.

THE ABSORPTION OF OXYGEN

As a diver descends into the water, the partial pressure of the oxygen in the breathing mix increases in exactly the same way as does the partial pressure of the nitrogen (or other gases present). You would consequently expect Henry's law to apply to oxygen just as well as to nitrogen. And oxygen does indeed obey Henry's law, if we were to conduct a laboratory experiment designed to dissolve oxygen under increasing pressure into a beaker of fluid. But in the human body, things are very different.

The hemoglobin in our blood binds to the oxygen molecules in the air encountered in the alveoli, and carries those molecules to the cells that need it. But hemoglobin does not merely deliver whatever oxygen is encountered in the breathing gas. It also *regulates* the amount of oxygen released to the cells of the body, maintaining the partial pressure of oxygen encountered by the cells within relatively tight levels. So if you are breathing oxygen-lean air at the top of Pikes Peak, the hemoglobin compensates for the relative lack of oxygen in the breathing air by releasing more oxygen to the cells. Indeed, this delivery system works well over a wide range of oxygen partial pressures, including those encountered by divers at recreational diving depths. Consequently, when we discuss in the next chapter the dangers of rapid off-gassing and formation of gas bubbles in the blood, we will not need to worry about oxygen, but only about the physiologically inert gases in the breathing mix, which means under ordinary circumstances the nitrogen in the air.

EXERCISES

1. Liquid water and pure helium are enclosed in a container, with the helium at a pressure of 1 bar. At this pressure, .2 grams of helium has dissolved in the water. When the pressure is increased to 2.4 bar, how much helium will eventually dissolve into the water?[4]

2. Consider the 20 minute compartment in Haldane's model. After how many minutes would we consider this compartment to be saturated?

3. The differential equation

$$\frac{dP}{dt} = k(P_a - P)$$

applies to both on-gassing and off-gassing. When the sign of dP/dt is positive, $P(t)$ is increasing, while if the sign of dP/dt is negative, $P(t)$ is decreasing. Explain clearly in a few words how this distinguishes between on-gassing and off-gassing. That is, what makes the rate of change positive or negative?

4. Six half-lives is considered enough to fully saturate a tissue compartment. Suppose someone insists instead that we must achieve 99.5% saturation, before we are willing to say that the tissue is fully saturated. How many half-lives must elapse before this level is reached?

5. Dive physiologists following Haldane have proposed tissue compartments with other half-lives than those used by him. Suppose we consider a tissue compartment with a 120 minute half-life. What is the value for *k* for such a tissue compartment?

6. A diver goes to 50 fsw from the surface. A certain tissue in her body begins absorbing nitrogen from the compressed air she is breathing.

   (a) What is the initial partial pressure of nitrogen in this tissue?

(b) What is the ambient partial pressure of nitrogen at this depth?
(c) Give the appropriate function $P(t)$ to describe the partial pres sure of nitrogen dissolved into the diver's blood at time t.
(d) Suppose further that this tissue has a 10 minute half-life. What further information does this give us about the function?
(e) What is the partial pressure of nitrogen in this tissue after 20 minutes? How about after 5 minutes?

7. On a dive to 50 fsw, the diver's theoretical 20 minute tissue compartment has absorbed 60% of the relative nitrogen pressure.

   (a) What is the partial pressure of nitrogen in this compartment at that instant?
   (b) How long did it take for this absorption to take place?
   (c) Suppose the diver now returns immediately to the surface. How long will it take for his 20 minute compartment to off-gas to 28 fsw partial pressure of nitrogen?

8. A diver descends to 87 fsw from the surface. She is breathing a EAN32 mixture (32% oxygen and 68% nitrogen). She stays at 87 feet for 27 minutes. What is the partial pressure of nitrogen in each of her Haldane compartments?

9. A diver descends to 104 fsw.

   (a) How long will it take until the partial pressure of nitrogen in her 75 minute tissue compartment reaches 46 fsw?
   (b) At this instant, what are the partial pressures for her other four Haldane tissue compartments?

10. Repeated computations using equation 7.1 quickly become tedious when using a calculator. One option is to program such a function into your calculator, if you have that capability. Another alternative is to use a spreadsheet like MicroSoft Excel. In this exercise you will explore how to do this. This exercise is by no means a complete tutorial in Excel, but it will be easy if you've had only a little experience with a spreadsheet (or know someone who does).

    A *spreadsheet* is basically a large array of numbered rows and lettered columns. Each individual entry is a *cell*. The advantage is that mathematical formulas can be entered in one cell, which use data from other cells.

    (a) To create a *Nitrogen Partial Pressure Calculator*, we will begin with a row with the five half-lives according to Haldane. We will place these numbers in cells D1:H1.
    (b) Let's compute the relative rate k for each of these. In cell D2, we can type =LN(2)/D1. This calculates $k = .138629$. For the other k values, we can merely grab the lower right corner of cell D2 and drag this formula across the cells E2:H2. Excel *autofills* these formulas.
    (c) In the next row D3:H3, we would like to enter our starting nitrogen partial pressure in fsw. For now, we will enter 26 in each of these cells; this corresponds to a diver at the surface. (You can use autofill to avoid typing 26 five times.)
    (d) We now wish to consider a dive of 15 minutes to 100 fsw. Let's enter these data into cells A4 and B4.
    (e) We now need the ambient partial pressure of nitrogen at 100 fsw. This can be computed in cell C4 as =.79*(33+B4).
    (f) In the cells D4:H4 we wish to compute the nitrogen load in each of the Haldane tissue groups; we use formula 7.1. In cell D5 we type: =$C4+(D3-$C4)*EXP(-D$2*$A4). We can now drag this formula into cells E4:H4. You can probably guess that EXP is just the Excel command for the e^x function. The dollar signs in this formula are an important Excel technicality. They tell Excel that these are *absolute references*: the time will always occur in column A, the ambient partial pressure will always occur in column C, and the k value will always occur in row 2.

(g) With this setup, you can now drag the formulas in C4:H4 into the cells C5:H5 (or into further rows after the 4th row). This will mean that you can add additional stages to your dive.

(h) As an example,[5] suppose that we follow up our 15 minutes at 100 feet with 20 minutes at 50 feet and 5 minutes at 15 feet. You should obtain a row of partial pressures for the tissue compartments like this:

53, 60, 58, 49, 41.

(i) (Optional) You might wish to add some labeling in empty nearby cells, so that you can remember what these various numbers represent. Excel has lots of formatting capabilities you might explore. You can specify how many digits are displayed for the numbers – recall that we are really only calculating partial pressures to the nearest foot. You can change fonts, and width of columns. You could color code cells, to remind which numbers you need to enter, and which numbers are computed by Excel.

11. Feel free to use your *Nitrogen Partial Pressure Calculator* to check your answers for some of the earlier exercises in this section!

Endnotes

1 Because of this, we will sometimes say that nitrogen is *physiologically inert.* This is not literally true, however, as we shall discover when we consider the phenomenon of *nitrogen narcosis.*

2 Note that tissues of the body other than blood also absorb extra nitrogen when under pressure. Bubbles can then form when the ambient pressure decreases. However, we here place our primary emphasis on bubbles in the bloodstream, because these bubbles seem to be the most problematic.

3 With this example, and in the next several chapters, we will consistently use fsw as our unit of pressure, computed to the nearest foot.

4 We are assuming here that helium strictly follows Henry's Law.

5 This example really amounts to a multi-level dive, followed by a long safety stop; we'll discuss such dives formally in Chapter 12.

Chapter 8

The Bends

In this chapter we will examine the history and symptomatology of decompression sickness, and begin to understand Haldane's efforts to prevent it.

What is decompression sickness?

As we saw in the previous chapter, when a diver spends time at depth breathing compressed air, his body absorbs additional nitrogen, according to Henry's Law. This process is not instantaneous, but can be described mathematically using a modified exponential decay function. Because the nitrogen is physiologically inert, the process of on-gassing is harmless.

Now as the diver ascends and reaches lesser ambient pressures, the reverse process begins. Ideally, the nitrogen that has dissolved into the tissues of the body is now carried back through the bloodstream to the lungs, where it is harmlessly exhaled. However, if the pressure gradient is too great, the nitrogen in the bloodstream may form tiny bubbles which may be carried through the arteries to all parts of the body. In addition, bubble formation is possible in other tissues of the body too.

A person with any of these various adverse effects to the body caused by nitrogen coming out of solution, is said to be suffering from *decompression sickness* (DCS for short); as we discuss below this affliction is more popularly known as *the bends*.

One reason why decompression sickness is so difficult to diagnose is that the symptoms of the disease vary greatly, depending on where these bubbles lodge. Bubbles can cause mechanical effects. They may restrict blood flow in tiny vessels, causing damage and pain. They may compress nerves, which might cause tingling, numbness or even paralysis.

Symptoms can occur immediately after the dive, but may take up to 36 hours before they manifest. The symptoms vary greatly depending on the severity of the case, and where the bubbles have lodged in the body. They tend to continually get worse until medical intervention, which is usually recompression therapy in a hyperbaric chamber.

A hyperbaric chamber is basically a large metal tank, which medical personnel can pressurize. The patient is placed in the chamber, and pressure is introduced. The idea is that the pressurized environment will drive the problematic bubbles back into solution; with careful depressurization protocols, the nitrogen off-gassing can then be controlled in a safe way. Patients treated in a hyperbaric chamber are often administered pure oxygen. In larger models patients can be accompanied by a technician in the chamber.

Dive medicine practitioners distinguish between Type I and Type II DCS, depending on the severity and type of the symptoms. Type I DCS often involves dull aching pain in the joints or limbs, as the bubbles put pressure on the tendons and ligaments connected to the joints. Another Type I symptom is a characteristic red rash, as bubbles come out of solution in the capillaries of the skin.

Type II DCS involves more serious, life-threatening neurological effects, as bubbles exert pressure on nerves, and may also block blood flow to vital organs. Tingling and numbness can escalate into paralysis.

Divers suspected of DCS should immediately discontinue diving, and be treated with the circle of care for general first aid. They should be administered pure oxygen if it is available; the breathing of oxygen helps to flush the nitrogen out of the system. Medical help is necessary, preferably from personnel familiar with dive medicine. Hyperbaric therapy will almost undoubtedly follow.[1]

Fortunately, reputable dive operators always have oxygen on hand, and experienced divers are trained to administer it. The Divers Alert Network (DAN) maintains a free 24 hour medical hot-line for questions and referrals to nearby help. Most experienced divers carry supplemental insurance to cover the extraordinary costs involved in emergency evacuation and hyperbaric therapy. But most important of all, divers can *drastically decrease the likelihood that they get decompression sickness*, merely by following the safe diving protocols developed out of the mathematics we are beginning to explore.

Other barotraumas

There are a number of other moderate to severe pressure-related injuries possible that may occur while ascending from a dive, which are not related to nitrogen off-gassing. Such injuries are called *barotraumas*. For the most part, these other injuries are related to rapid ascents or breath-holding during ascent.

The *alveoli* are the tiny sacs in the lungs in which the oxygen transfer to the blood takes place. If some of them are ruptured on account of lung over-expansion, this can conceivably permit air bubbles to enter the tiny capillaries in the lung. These can then pass through the heart and out into arterial system. Such bubbles can then obstruct blood flow if they become lodged in a small enough vessel. This is called an *arterial gas embolism*, or AGE for short. Depending on which blood vessels are effected, the outcome can be catastrophic. If the embolism blocks blood flow to the brain, a stroke is possible. Such effects can be sudden and dramatic. Notice the difference between AGE and DCS. In both cases bubbles are the culprit, but the bubbles themselves emerge for different reasons. In DCS, the bubbles are nitrogen gas, and they arise because nitrogen in the bloodstream comes out of solution too rapidly. In AGE, on the other hand, the bubbles are air, which have passed into the bloodstream from the lungs on account of physical injury to the lung tissues.

Over-expansion of the lung could also lead to a rupture of the lung and its resulting collapse. This is called *pneumothorax*. Sometimes air leaking from the lungs can put pressure on the heart and decrease its function — leading to faintness and shortness of breath; this is called *tension pneumothorax*. Sometimes the air leaking from the lungs lodges in the soft tissues at the base of the neck, leading to "skin crackling".

Dive medicine practitioners subsume all of these effects (including decompression sickness) under the general term *decompression illness* (or DCI).

One reason for this is that one such injury might well suggest that others are present. Also, it is difficult on the scene of a dive accident to determine whether a diver is suffering from decompression sickness, or an air-overexpansion injury, or both.

In practice, a detailed diagnosis is not necessary on the scene of a dive accident. Any afflicted diver should be treated in the same way with good basic first aid (checking the ABCs of airway, breathing and circulation), treating for shock, administering oxygen, and seeking medical treatment. It is important to gather information about the dives the victim has made and the symptoms afflicting him, so that qualified medical personnel can later formulate a sound diagnosis.[2]

To avoid the barotraumas discussed in this section, divers should always ascend slowly, and keep the airway open at all times. To avoid decompression sickness (the bends), divers should follow the restrictions on depths and times that we will begin to explore in this chapter.

Historical background

The illness we call *decompression sickness* (or *the bends*) was first reported in the scientific literature in France in 1845, when a mining engineer named Charles-Jean Triger described the limb pain suffered by coal miners; he called their difficulties *caisson disease*.[3] The mine had been filled with pressurized air to prevent ground water from entering the passages. Consequently, the miners had been working all day while breathing compressed air. By 1854 the French physicians Pol and Waltelle had published a clinical description of the illness, and noted explicitly that "one pays only upon leaving" — that is, the symptoms only occurred upon leaving the pressurized environment; furthermore, they noted that the symptoms could be alleviated with a return to the pressurized environment.

The construction (1869-1883) of the Brooklyn Bridge in New York was one of the engineering marvels of the nineteenth century. Massive caissons were constructed, including one as deep as 75 feet. Many workers suffered from caisson disease, including not only limb pain, but also paralysis and even death. The chief engineer for the project was stricken by the disease and remained paralyzed for the rest of his life. It was during this period that the illness became popularly known as *the bends*, in reference to the exaggerated bending of the back workers attempted in a futile effort to avoid the pain. The name compared the posture of the bridge workers to the "Grecian Bend", a posture adopted by fashionable women of the day.

In the 1870s the French physiologist Paul Bert studied the illness, and conjectured the role of nitrogen bubbles caused by off-gassing upon leaving the pressurized environment. In 1878 he published a massive and important book entitled *Barometric Pressure* that summarized his findings.

The British Royal Navy was concerned about the incidence of the bends among their hard-hat surface-supplied divers, and commissioned J. S. Haldane to conduct a study of the illness. In 1908 he (and his co-authors Damant and Boycott) published a book-length paper in the *Journal of Hygiene* that described their experiments with goats, which led to their dive tables. These dive tables were protocols involving staged decompression at shallower and shallower depths, designed to help divers avoid decompression sickness. The basic idea is this: once a diver has spent enough time at depth, and consequently absorbed excess nitrogen, the rate this nitrogen is off-gassed must be kept under control, by gradual ascents involving what are called *decompression stops*, at various intermediate depths. We describe Haldane's strategy in more detail in the next section.

Haldane's protocols

Haldane and his colleagues simulated dives by keeping goats in pressurized chambers for a time long enough that they believed that they were saturated with nitrogen. The goats were depressurized in stages and examined to see whether they were suffering from the symptoms of decompression sickness. Their empirical evidence suggested that the goats were safe from DCS if at any stage they were moved to a lesser pressure in ratio no greater than 2:1. For example, if the goats were pressurized to saturation at a total pressure of 100 fsw, Haldane believed that it would be safe to immediately depressurize them to no less than 50 fsw. Because the percentage of the air that is nitrogen is at all levels 79%, we can equivalently compute the ratio of tissue nitrogen to the ambient nitrogen, and make sure that ratio is no more than 2:1.[4]

Haldane compiled tables that described safe exposures based on this principle. He based his calculations on his five tissue compartments, and built his tables on the basis of *decompression stops* at 10 foot increments. By a decompression stop, we mean a pause in the diver's ascent at a given depth for a given time, which is designed to allow enough off-gassing that it is safe to ascend to a higher level. Typically, a diver might need to make several decompression stops in order to avoid violating the 2:1 ratio.

The Haldane procedure is cumbersome. Furthermore, it recommends dive protocols that by modern standards are sometimes overly conservative, and on the other hand sometimes unsafe. Consequently, we will not fully explore Haldane's tables, but instead provide examples that will well illustrate the ideas. In the next chapter we will turn to some modern dive tables that by now have a large body of empirical evidence to testify to their safety.

For a given tissue compartment, we know that the nitrogen off-gassing formula is given by equation 7.1:

$$P(t) = P_a + (P_0 - P_a)e^{-kt},$$

where P_0 is the starting nitrogen pressure in the tissue, P_a is the ambient partial pressure of nitrogen, and k is the relative rate of decay for the exponential function; we computed these values for Haldane's half-lives in the previous chapter.

Suppose to begin with that a diver descends to 60 fsw, and stays there for 60 minutes. The total ambient pressure at that depth is 60+33 = 93 fsw, and the nitrogen partial pressure is .79×93 = 73. The diver begins the dive at sea level, and so as before the initial partial pressure of nitrogen is 26 fsw.

We will now compute the resulting nitrogen load in each of the five compartments. This is easy to do with a calculator, or with the *Nitrogen Partial Pressure Calculator* in Excel, discussed in the exercises following Chapter 7. For example, for the 20 minute compartment, $k = .0347$, and so the nitrogen load in this tissue after sixty minutes is

$$P(60) = 73 - 47e^{-.0347(60)} = 68 \text{ fsw}.$$

We can in a similar way compute the other four numbers. We leave these verifications to you:

Tissue	5	10	20	40	75
N_2 load	73	73	68	57	46

Since 60 minutes is 12 half-lives for the 5 minute tissue, it is completely saturated. At 6 half-lives, the 10 minute tissue is essentially saturated too. The other tissue compartments are not saturated.

Our diver now wishes to return to the surface. But the nitrogen partial pressure at the surface is 26 fsw, and four of the five tissue compartments are loaded to a level more than double this. Haldane thus believed that this diver cannot ascend directly to the surface, but must instead ascend to a shallower depth for some decompression. The biggest nitrogen load is in the 5 minute compartment, with 73 ata. Consequently, our diver can safely ascend to a depth where the ambient nitrogen pressure is no more than 73/2 = 37 fsw. The total pressure at this safe depth would thus be 37/.79 = 47 fsw. Remembering the 1 ata of pressure due to the atmosphere, we discover that Haldane's diver can safely ascend to 47 − 33 = 14 feet. Haldane always placed his stops at 10 foot increments, and so he would recommend a decompression stop at 20 fsw. By his protocol, he would recommend that the diver stay at 20 fsw until the nitrogen partial pressures in all compartments allow him to ascend to the next 10 foot level upward (at 10 fsw).

Our next calculation is to determine how long Haldane's diver needs to stay at 20 feet. It will be most efficient for us to do this calculation once in general.

So we are now assuming that a diver has just begun a decompression stop; we wish to compute how long he must stay at the stop, before ascending to the next one. For each compartment, we have a value P_0, the nitrogen partial pressure load, and the value P_a, the ambient nitrogen partial pressure. We then determine P_M, the nitrogen partial pressure our diver must achieve in order to safely go to the next higher decompression stop.[5] We would then like to compute how long it takes for the ambient pressure P_a to reduce the tissue compartment pressure to no more than the *goal pressure* P_M. In a typical situation, some tissue compartments may already be at the goal pressure, and we need not worry about those compartments. We will consequently only be concerned about the computations below for $P_0 \geq P_M \geq P_a$.

So, pick a particular relevant compartment with half-life T. Its relative rate of decay is $k = \ln 2/T$. We now want to know the time t it takes for our compartment to off-gas from P_0 to P_M, given that the ambient pressure is P_a. We thus have

$$P_M = P_a + (P_0 - P_a)e^{-kt}.$$

Our goal is to solve this equation for t. We will now do some algebra, and use the logarithm to get rid of the exponential function:

$$P_M - P_a = (P_0 - P_a)e^{-kt}$$

$$\frac{P_M - P_a}{P_0 - P_a} = e^{-kt}$$

$$\ln\left(\frac{P_M - P_a}{P_0 - P_a}\right) = -kt$$

We can now solve[6] for t:

$$t = -\frac{1}{k}\ln\left(\frac{P_M - P_a}{P_0 - P_a}\right)$$

$$t = \frac{1}{k}\ln\left(\frac{P_0 - P_a}{P_M - P_a}\right)$$

$$t = \frac{T}{\ln 2}\ln\left(\frac{P_0 - P_a}{P_M - P_a}\right) \qquad 8.1$$

We can now apply formula 8.1 to our Haldane diver who has spent 60 minutes at 60 fsw. Recall that the shallowest permissible ten-foot level he can ascend to is 20 feet. This means that the ambient partial pressure of nitrogen is .79 × (20+33) = 42 fsw; this is the constant P_a. The partial pressure of nitrogen at ten feet is .79 × (10+33) = 34 fsw. Our diver must then stay at 20 fsw until the partial pressure of nitrogen in each of his five tissue compartments is no more than 2 × 34 = 68; this is the value of the pressure P_M. The value of P_0 depends on which compartment we consider. But the three slower compartments are already at this level, and so we need only compute the number of minutes it takes to achieve this goal for the 5 and 10 minute compartments. For the 5 minute compartment we have that P_0 = 73, and so by formula 8.1 we get that

$$t = \frac{5}{\ln 2}\ln\left(\frac{73-42}{68-42}\right) = 1.4 \text{ minutes.}$$

Similarly, for the 10 minute compartment we have that P_0 = 73, and so

$$t = \frac{10}{\ln 2}\ln\left(\frac{73-42}{68-42}\right) = 2.4 \text{ minutes.}$$

So, rounding to the nearest minute, we will assume that our diver spends 3 minutes at 20 feet. The diver will be off-gassing in all five compartments, because the ambient nitrogen pressure is 42 fsw, which is less than the tissue loads reported above.

We can now compute the result of our off-gas of three minutes at 20 feet in each of the five Haldane compartments, using equation 6.1. For example, for the 10 minute compartment, we have

$$P(3) = 42 + (73 - 42)e^{-.0693(3)} = 67.$$

If we do the remaining off-gassing calculations[7] we obtain the following:

	Tissue	5	10	20	40	75
60 ft	60 min	73	73	68	57	46
20 ft	3 min	63	67	65	56	46

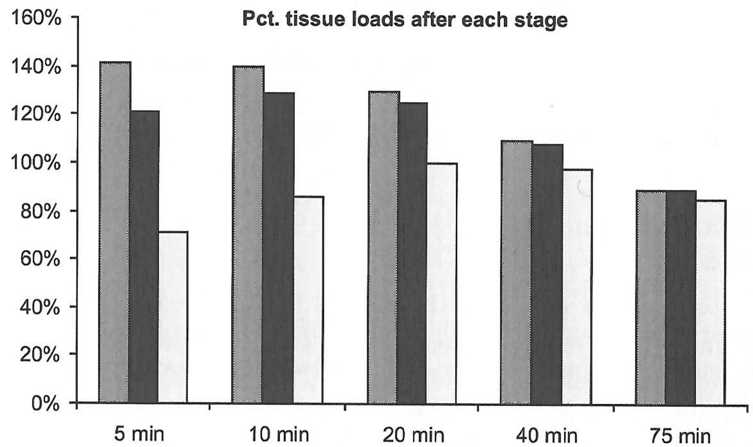
For our diver to ascend directly to the surface, we would need for these nitrogen partial pressures to be less than twice the nitrogen partial pressure at the surface; as we've already seen, that pressure is 2 × 26 = 52; four of our five compartments do not meet this standard. We can now use formula 8.1 compute the time necessary to bring each of the first four tissue compartments to this pressure, just as we did at the previous decompression stop.

Tissue	5	10	20	40	75
Time Needed	3.4	8.7	15.7	11.4	0

Our diver must now take 16 minutes at 10 fsw (which gives us P_a = (10+33)×.79 = 34 fsw of partial nitrogen pressure). We can now complete the dive, because all tissue pressures are now less than 52 fsw (double the nitrogen partial pressure at the surface):

	Tissue	5	10	20	40	75
60 ft	60 min	73	73	68	57	46
20 ft	3 min	63	67	65	56	46
10 ft	16 min	37	45	52	51	44

We can graphically represent this data with a bar chart. Instead of graphing the numbers in the table, it is instead more illuminating to compute percentages. For each tissue we will compute the percentage the load is of the maximum allowed load of 52 fsw, before the diver can go to the surface. It is precisely because the third bar for each tissue is loaded at less than 100% that the diver is ready to end his dive!



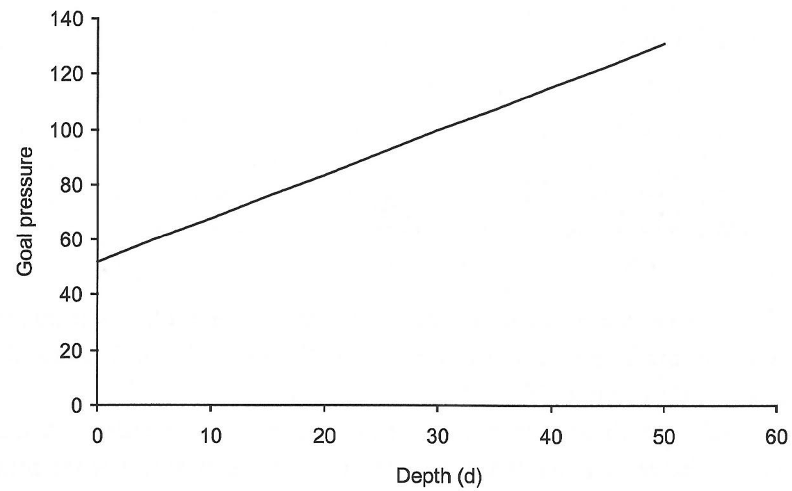
It is quite interesting to note that after the original dive, the compartment with the largest nitrogen load was the 5 minute tissue. But after the decompression stop at 20 feet, it was the 10 minute tissue that was most advanced. At each stage we will speak of the *controlling compartment.* Our example shows that the controlling compartment may well change from stage to stage of Haldane's recommended decompression. This will also be the case for more modern dive tables.

In this case the diver spent 19 minutes off-gassing. By modern standards this is a *very* conservative protocol, as we shall see in the next chapter.

Let's now review the Haldane decompression procedure, from a slightly more general perspective. To ascend directly to the surface, the nitrogen partial pressure in each of the five compartments needs to be no more than twice the nitrogen partial pressure there, or $2 \times .79 \times 33 = 52$ fsw. But similarly, to ascend to a depth of ten feet, each compartment loads most be no more than twice the pressure there, or $2 \times .79 \times (33+10) = 52+16 = 68$ fsw; a similar calculation can be made for twenty feet as well. In fact, if a diver wishes to ascend to d feet, then the compartment loads must be no more than

$$2 \times .79 \times (33 + d) = 52+1.58d \text{ fsw}.$$

In other words, the *goal pressure* P_M needed in each compartment to ascend to d feet is a *linear function* of d. We write this as $P_M(d) = 52+1.58d$. If we graph this straight line, we get the following, which in principle will work for any target depth (even though Haldane only used target depths which are multiples of ten).



Exercises

1. A diver spends 32 minutes at 74 feet. Compute the nitrogen load on each of the five Haldane tissue compartments as a result of this dive.

2. A diver's nitrogen load in the 10 minute tissue compartment is 72 fsw. According to Haldane's standards, can the diver ascend to the surface? To 10 feet? How about 20 feet?

3. A diver is decompressing at ten feet. His controlling compartment is the 40 minute tissue. His nitrogen load in that compartment is 57 fsw. How long does he need to decompress before ascending to the surface?

4. A diver begins at the surface. Her starting nitrogen load is thus $P_0 = .79 \times 33 = 26$ fsw. She descends to a depth d and stays at that depth s minutes. Compute the nitrogen load on all five Haldane tissue compartments (in terms of d and s). Explain why the controlling compartment at this point of her dive is always the five minute tissue.

5. In this exercise you will use Excel to construct a *Haldane Calculator*, which will compute the amount of time required at a given depth to reach a given nitrogen partial pressure goal.
   (a) Begin as with the *Nitrogen Partial Pressure Calculator*, by entering a row of half-lives, and the computations for the k values.
   (b) Immediately below these k values you can enter the starting partial pressures in these compartments (in practice, these might well be copied from the *Nitrogen Partial Pressure Calculator*). These are the P_0 values.
   (c) In the next row, you will need to compute the ambient partial pressure P_a (depending on the entered depth of the decompression stop) and the goal partial pressure P_M (depending on entered depth of the next higher stop). You will then need to enter formula 8.1 to compute the time needed for each compartment;

this depends on k, P_a, P_0 and P_M. Notice that you will probably have the P_a and P_M values in fixed columns, and so you will need to use a \$ to tell Excel this.
   (d) You will then need to pick out the largest positive time in this row. You can then take that time at that depth, and compute the next stage, using the *Nitrogen Partial Pressure Calculator.*

6. In practice, you would never use the *Haldane Calculator* unless your goal pressure P_M is greater than the ambient pressure P_a, for otherwise your off-gassing goal could never be achieved. But for a given compartment and a given initial partial pressure P_0, there are several cases that might occur; these cases can be understood in terms of the ratio $\frac{P_0 - P_a}{P_M - P_a}$ inside the logarithm function:
   (a) Describe qualitatively what is occurring for a compartment with $P_0 > P_M > P_a$. What sort of number do you get in this case when you take the logarithm of $\frac{P_0 - P_a}{P_M - P_a}$?
   (b) Describe qualitatively what is occurring for a compartment with $P_M > P_0 > P_a$. Will you ever reach your goal pressure? Does this matter? What sort of number do you get when you take the logarithm of $\frac{P_0 - P_a}{P_M - P_a}$?
   (c) Describe qualitatively what is occurring for a compartment with $P_M > P_a > P_0$. Will you ever reach your goal pressure? Does this matter? What happens when you try to take the logarithm of $\frac{P_0 - P_a}{P_M - P_a}$?
   (d) Explain why we are thus only interested in computing the time to goal in case that $\frac{P_0 - P_a}{P_M - P_a} > 1$.
   (e) (Optional) Explore how to modify the *Haldane Calculator* by using an IF command in Excel to record zero minutes needed for a given compartment, if $\frac{P_0 - P_a}{P_M - P_a} < 1$.

7. Use your Excel files to figure out a Haldane decompression protocol for a dive to 80 fsw for 28 minutes. What were the controlling compartments, at each decompression stop?

8. Repeat the previous exercise, for a dive to 54 fsw for 78 minutes.

9. Recall our diver who spent 60 minutes at 60 feet. How long would he have to stay at 20 feet, in order to safely go to surface (without a stop at 10 feet)?

10. Recall our diver who spent 60 minutes at 60 feet. Why can this diver safely ascend to 15 feet? How long would he have to stay at 15 feet, in order to safely go to surface (without any other stops)?

11. A superstitious diver who believes in the Haldane protocols prefers decompression stops ending in eight: 8, 18, etc. What would his decompression schedule be for an 80 foot dive for 28 minutes?

Endnotes

1 There is a temptation for divers with DCS symptoms to attempt recompression in the water, but this is a therapeutic option that is difficult to manage within the context of recreational diving. Military and commercial divers have the infrastructure to consider this option, although they often also have hyperbaric chambers on site.

2 Examination of the diver's equipment can often be helpful too, especially the diver's computer, if he has one. Modern dive computers can be downloaded to provide a minute-by-minute history of the dive. We'll have much more to say about dive computers in Chapter 12.

3 A caisson is a pressurized watertight chamber inside of which workers can help construct bridge footings and tunnels.

4 The partial pressure of nitrogen at 100 fsw total pressure is .79 × 100 = 79 fsw, and the ratio of nitrogen tissue pressure to safe ambient pressure is 79/50 = 1.58. In the dive literature Haldane's rule is sometimes described in these terms. In the rest of this book we will always calculate the ratio of the tissue nitrogen to the ambient nitrogen.

5 For Haldane, this pressure is no less than half of P_0. As we shall see in the next chapter, other dive protocols modify Haldane's rule.

6 The second step in this derivation requires the use of the logarithm rule that $-\ln(A) = \ln(A^{-1}) = \ln(1/A)$. See Appendix D if necessary.

7 We can of course easily generate these data using the *Nitrogen Partial Pressure Calculator*.

Chapter 9

The Navy Dive Tables

In this chapter we will explore one particular set of dive tables. These tables grew directly out of Haldane's work, but in contrast to Haldane's they are practical and safe tables, used by many divers today. These are the tables produced by the United States Navy.

Historical background

By the end of the last chapter we had understood mathematically how Haldane proceeded to generate his tables for safe diving. Not surprisingly, the U. S. Navy took an active interest in these developments, and beginning in 1913 began constructing their own dive tables based on Haldane's concepts. Having a ready corps of able-bodied volunteers, the navy was able to conduct a number of empirical experiments over the years. The results of these experiments led the navy to change Haldane's model in a couple of important ways. The actual historical evolution is a bit complicated, and took a long time before the standard version of the Navy Tables came out in the late 1950s.

First of all, the navy replaced Haldane's 75 minute tissue compartment with an 80 minute compartment. More importantly, they added a sixth, slower compartment, with a 120 minute half-life. Second, the navy's experiments led them to believe that Haldane's hard-and-fast 2:1 rule was not appropriate for all compartments. They concluded that the 2:1 rule was too strict for the fast compartments (5, 10, 20, 40), but not conservative enough for the slower compartments (80, 120). They thus introduced a sliding scale of ratios, called *M-values*. We will explore the full mathematical details in the next chapter. But before looking at this mathematics, it seems best to look at the finished product, and how divers in practice use them.

USING THE NAVY TABLES FOR A SINGLE DIVE

One thing that complicates the issue of nitrogen on-gassing and off-gassing is the issue of *repetitive dives*. If a diver commences a new dive before his tissues have returned to the original surface loads, the diver will end up with more nitrogen in his system than a diver who hadn't been diving recently. Recall that we can consider all tissues to be completely off-gassed after 6 half-lives. Because the slowest tissue compartment in the navy model is 120 minutes, we can thus assume that complete off-gassing occurs after 12 hours. We then call a dive taking place less than 12 hours after the end of the previous dive a *repetitive dive*, and we need to take the residual nitrogen into account when computing the nitrogen load for the diver in the six tissue compartments; we call the time between the end of one dive and the beginning of the next a *surface interval*. We will discuss repetitive diving using the navy tables in the next section. But for this section, we will first consider only dives taking place after at least 12 hours of surface interval.

The first table we need to examine is the *U. S. Navy Dive Table 3*. You will find simplified and truncated versions of the Navy Tables in Appendix E; should you wish to see the complete versions, they may be found in the *NOAA Dive Manual*, among other places.[1] For convenience of cross-reference, we will refer to these tables by the numbers the Navy itself uses.

Most of the information in Table 3 is only relevant for repetitive dives. The important information for our present purpose is contained in the first two columns. The first column gives the depth of the dive in feet. The second column consists of the *No-Decompression Limits* for those depths; we call these the NDLs for short. A NDL tells how long a diver can spend at the given depth and then ascend directly to the surface, *without making any decompression stops at all*. This is called *no-decompression diving*. Modern training for recreational divers recommends that they do only no-decompression diving. We'll explore further the evolution of recreational diving standards in a future chapter.[2]

For example, according to Table 3, a diver descending to 80 feet is reasonably safe from decompression sickness, as long as he stays no more than 40 minutes at that depth, and then ascends immediately to the surface, at a safe slow rate.

Particularly interesting is the NDL for 60 feet; it is 60 minutes. In the previous chapter we computed Haldane's protocol for a dive to 60 feet for 60 minutes; this involved two different decompression stops, lasting 19 minutes altogether. The navy tables, in contrast, allow this diver to proceed straight to the surface.[3] As we shall see, the reason the navy tables are so much more liberal is that for Haldane the controlling tissues were the fast 5 and 10 minute compartments. As noted above, the navy adopted a less strict rule than Haldane's 2:1 ratio for these compartments; we will look at this carefully in the next chapter.

Consider instead a dive to 20 feet. According to Table 3, *there is no* NDL for this depth. According to the navy tables, *any* dive to no more than 20 feet is a no-decompression dive.

Suppose now that our diver wishes to remain at 60 feet for 80 minutes. To understand this dive we must turn to the *U. S. Navy Dive Table 5*: the *Standard Air Decompression Table*. This table is broken up into a number of sub-tables organized by depth (from 40 feet to 300 feet).[4] The first column gives the depth planned, and the second column gives the bottom time, and in the middle are several columns that describe the mandatory decompression stops. For our 60 foot dive for 80 minutes, we discover that we have a single mandatory decompression stop for 7 minutes at 10 feet (the letter designation in the last column is only relevant for repetitive dives).

For another example, consider a 1 hour dive to 100 feet. In this case the diver is required to make two decompression stops, one for 9 minutes at 20 feet, and a second for 28 minutes at 10 feet. We should emphasize that this dive is well beyond the capability or training of recreational divers. In fact, a typical recreational diver would be unable to complete this dive on a single tank of air. In the military, the diver might be equipped with double tanks, or else supplemental air as needed would be brought to the diver at the appropriate depth. Some of the more extreme exposures described on the tables require hours of decompression![5] For example, a two hour dive to 120 feet would require four separate decompression stops at 40, 30, 20 and 10 feet, totaling nearly three hours.

For another example, consider a 58 minute dive to 68 feet. Since there is no 68 foot sub-table, navy rules require that we round up for conservatism. So we examine the 70 foot sub-table instead. Similarly, there is no 58 minute row, and so we also round up to the 60 minute row. The navy diver should consequently plan on a 8 minute decompression stop at 10 feet.

The complete version of the Navy Tables also includes precise time calculations for how long decompression might last, and when it should begin. For the navy (and for the recreational agencies that followed their lead) the *bottom time* for a dive technically begins at the moment the diver enters the water, and then ends at the moment the diver begins his ascent. Consequently, for our hour-long 100 foot dive, the hour ends at the moment the diver begins swimming upward to the first decompression stop at 20 feet. The latest navy tables recommend an ascent rate of no more than 30 feet per minute (or one-half foot per second). It consequently takes this diver 2 × 80 = 160 seconds (2 minutes and 40 seconds) to reach the twenty foot stop. The complete version of the navy table includes this figure.

Because the *bottom time* ends with the beginning of the ascent, this means that the diver will be off-gassing during all of the ascent time, and also during the stops. It will take the diver 20 seconds to rise from the 20 foot stop to the 10 foot stop, and so the total time for decompression for this dive is

$$2{:}40+9+0{:}20+28+0{:}20 = 40{:}20.$$

The complete navy table records this figure in a *Total Decompression Time* column. For our purposes these numbers are not particularly important, and so we have suppressed this column in Appendix E; such numbers are helpful for a navy diver making a careful and rigid dive plan as specified by the tables. What is important for us is the observation that ascents (either to the next decompression stop or to the surface), are required to be slow and controlled. In earlier versions of the Navy Tables, this ascent rate was restricted to 60 feet per minute[6], but recent studies have suggested that even slower is better; as we mentioned above, the current navy standard is 30 feet per minute.

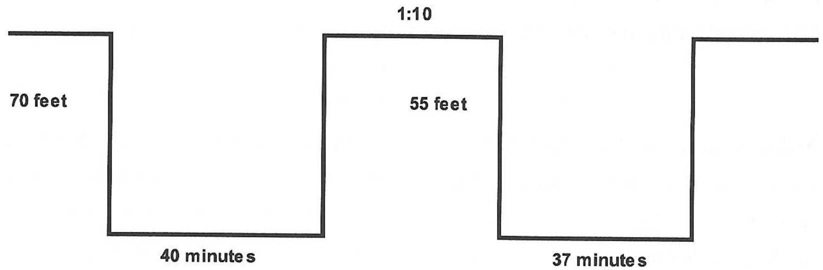
The last column in our version of the navy tables 3 and 5 gives a letter designation; this is concerned with repetitive dives, which we will explore in the next section.

USING THE NAVY DIVE TABLES FOR REPETITIVE DIVES

Recall that a dive is *repetitive* if it takes place less than 12 hours after the end of the last dive. It is repetitive because we must expect in this case that at least some of the six tissue groups may still have some nitrogen load. The navy pioneered

the use of letter groups as a way to easily understand this residual nitrogen load. The closer the letters are to the end of the alphabet, the more residual nitrogen is present. As we shall see in the next chapter, this tracking is actually only for the slowest compartment (the 120 minute tissue), the expectation being that the most residual nitrogen resides there. The navy uses the letters A—O and also Z for these *pressure groups*. If a diver has waited at least twelve hours since the last dive, he has no residual nitrogen and consequently does not have a pressure group; we say he is a *scratch diver*.[7]

Let's then explore an example of a two dive series. We shall suppose that the diver makes a first dive to 70 feet for 40 minutes. After the first dive she stays on the surface for 70 minutes; this is called a *surface interval*. Her second dive is 55 feet for 37 minutes. We can record this information efficiently in the following *dive profile diagram*:



When we examine Table 3 we discover that the first dive is a no-decompression dive, because 40 minutes is less than 50 minutes, the NDL for 70 feet. We will now use the interior part of this table. Finding 40 minutes in the 70 foot row, we discover that the diver leaves the first dive in pressure group H; we say that *she is an H diver*. This is merely a symbolic way of recording the amount of residual nitrogen the diver has in the 120 minute compartment.

During the surface interval the diver is off-gassing. The *U. S. Navy Dive Table 4* (or *Residual-Nitrogen-Time Table*) tells us the result of this process. Entering the table horizontally in the H row, we look for the time interval that includes our diver's surface interval 70 = 1:10; that interval is 1 : 07 to 1 : 41. Moving vertically downward, the table reveals that the diver has now off-gassed enough nitrogen to be an F diver.[8]

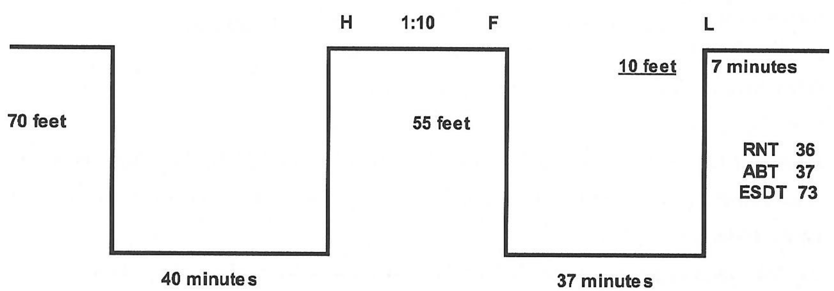
We now must compute the nitrogen for the second dive. By continuing to move vertically downward in the F column, we move to the depth row for the second dive. Since the second dive is 55 feet, we round upward to the first available depth of 60 feet. We then discover the number 36. *This number is the translation of the residual nitrogen into the number of minutes at 60 feet that we would need to dive to reach group F.* We can view this number as a penalty we suffer because we're making a repetitive dive. We call this number 36 the *Residual Nitrogen Time* (or RNT).

Consequently, to compute the result of the next dive, we must add the Residual Nitrogen Time to the *Actual Bottom Time* (or ABT), which for the second dive is 37. This gives a total of 73 minutes. To say this another way, doing the second dive with the residual nitrogen from the first dive is equivalent to making a first dive to 55 feet for 73 minutes. For this reason, we call this sum the *Equivalent Single Dive Time* (or ESDT).[9]

We can now return to Table 3, using the ESDT 73 instead of the Actual Bottom Time 37. Since there is no 55 foot row, we must round up to 60 feet. We then discover that this second dive is beyond the No-Decompression Limit of 60 minutes for a dive to 60 feet. This means that we must return to Table 5, to find the appropriate decompression schedule necessary for this dive. We do this by looking for an 80 minute dive (73 rounds up to 80) to 60 feet. We find that our diver must make a 7 minute decompression stop at 10 feet in order to be safe. After this dive (and the decompression stop) the diver is in the L pressure group.

We can now record all of this information in our dive profile diagram. In practice, we would of course record each of the appropriate numbers and letters step by step, as we obtain them from the dive tables.

In principle, we could string together any number of repetitive dives following this procedure. And recreational divers can and do three or more repetitive dives in a single day, especially on a liveaboard dive boat.

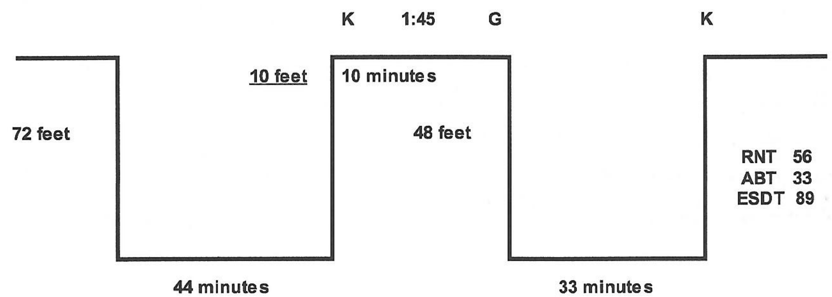


Before computing another example, it is well-worth emphasizing again the meaning of the *Residual Nitrogen Time* (RNT). In Table 4 we discover that if an E diver goes to 30 feet, his RNT is 70 minutes. This means the number of minutes a diver should spend at 30 feet to become an E diver; we can check this directly on Table 3. There we find that a 70 minute dive to 30 feet does indeed put us in the E pressure group.[10]

Notice that there are asterisks in the 10, 15 and 20 foot rows in Table 3. Now, there are no NDLs for these depths; the asterisks mean that dives of arbitrarily long duration will put the diver into the F, H and K pressure groups respectively.

Here is a second example of the entire process. Suppose that a diver descends to 72 feet for 44 minutes. At the completion of this first dive, the diver spends 1:45 on the surface. He then returns to the water at 48 feet for 33 minutes. You might want to draw a profile diagram, and compute the resulting pressure groups and decompression schedules.

You can check that the diver must do a ten minute decompression stop at ten feet after the first dive. He is then in Pressure Group K. After the surface interval, he is in Pressure Group G. For the second dive his RNT is 56, making a ESDT of 89 minutes. This dive does not require decompression. The final resulting Pressure Group is K. Our dive profile diagram looks like this:



SPECIAL CASES

Notice that in Table 5 there are some exceptionally long or deep dives for which no repetitive pressure group is given. The navy classifies such dives as *exceptional exposures*, and prohibits any repetitive dive following such long exposures. They are consequently not assigned a pressure group letter.

There are also some dashes and asterisks in the 10, 20 and 30 rows of Table 4. The dashes merely indicate that these shallow repetitive dives will not change the entering pressure group. The asterisks are a special case: proceed further as if the dive were to 40 feet.

Exercises

1. What is the No-Decompression Limit (NDL) for a dive to 93 fsw? How about for 120 fsw? What about 18 fsw?

2. In what pressure group will a diver be, who dives to 56 feet for 47 minutes? How about 97 feet for 18 minutes?

3. A F diver spends three hours on the surface. What pressure group is she now in? How long would an F diver have to spend on the surface, to become an E diver?

4. A D diver wishes to dive to 70 feet. What is his Residual Nitrogen Time (RNT)? How long can this diver spend at 70 feet, without needing a decompression stop?[11]

5. What is the shortest bottom time at 80 feet that requires a diver to make two decompression stops? How about 3 decompression stops?

6. Suppose that a diver is in the J pressure group. She is unwilling to make her next planned dive unless she is in the F pressure group. How long must she wait at the surface, before making her dive?

7. An E diver wishes to make a 40 minute no-decompression dive to 60 feet. How long must he wait before diving? *Hint:* First determine the pressure group he must be in in order to make the dive. Then determine the surface interval.[12]

8. A scratch diver makes a 42 minute dive to 55 feet. How long must this diver wait on the surface, if she wishes to make a 2 hour no-decompression dive to 40 feet?

9. A diver can breathe for 5 hours from a full tank at the surface. He wishes to make a 66 foot dive for 100 minutes, using a full tank. He

will of course do all necessary decompression. Does he have enough air to make this dive?

10. For the following dive series, draw a dive profile diagram, and compute all resulting pressure groups and decompression obligations:

    (a) A 48 minute dive to 65 feet; surface interval 1:15; a 37 minute dive to 49 feet.
    (b) A 55 minute dive to 70 feet; surface interval 2:47; a 48 minute dive to 40 feet.
    (c) A 20 minute dive to 100 feet; surface interval 1:55; a 33 minute dive to 48 feet; surface interval 2:10; a 48 minute dive to 37 feet.

11. For each of the dives in the previous problem, compute the *total off-gassing time required during ascent.* Recall that off-gassing begins with the start of the ascent, and ascents are computed at the rate of 30 feet per minute; furthermore, include all decompression stops as well.

Endnotes

[1] The truncated versions in the appendix are of the most recent edition of the Navy Tables, for the convenience of the reader. There have been some relatively minor changes to the tables since the standard tables came out in the 1950s. These changes will be for the most part of little consequence for us; we will refer to a couple of these changes below.

[2] The term *no-decompression diving* is a bit misleading, because a diver decompresses on *all* dives, in the sense that he on-gasses at depth, and then off-gasses as he moves to the surface. The term is actually shorthand for *no-decompression-stop-required diving*!

[3] The recreational tables used by PADI (the largest certifying agency for recreational divers) have a 55 minute NDL for 60 feet; thus by PADI standards the present dive would be considered a decompression dive, and therefore prohibited to recreational divers.

[4] In Appendix E we give partial versions of these sub-tables, only up through 130 feet.

[5] In addition to the logistical support necessary to supply divers adequate air to complete them, there is also the considerable problem of hypothermia — divers that long in the water are likely to get extremely cold, even if the water is tropically warm.

[6] This is the ascent speed that PADI recommends.

[7] He is starting from scratch.

[8] Note that in all rows in this table, a surface interval of more than 12 hours moves off the chart: the diver is completely off-gassed.

[9] PADI calls this the *Total Bottom Time* (or TBT); the navy's terminology more clearly explains what this number means.

[10] Actually, because of round-off errors, we will usually end up in the next pressure group when doing this with navy tables. The PADI tables report to the nearest minute, and so a check like this invariably works for them.

11 This last calculation give the *Adjusted No-Decompression Limit* (or ANDL) for this diver. Some versions of the navy tables give this number explicitly — but it can always be computed by a subtraction like you did to answer this question.

12 This is a called a *minimum surface interval* calculation.

CHAPTER 10

THE MATHEMATICS OF THE NAVY TABLES

In this chapter we will actually compute some of the numbers in the Navy Dive Tables, by applying what we have learned about the Haldane theory of nitrogen absorption. While we will not generate all of the numbers in the navy tables, we will do enough to understand this approach.

M-VALUES AND NO-DECOMPRESSION LIMITS

As we discussed in the previous chapter, the navy uses six tissue compartments, with half-lives of 5, 10, 20, 40, 80 and 120 minutes. For Haldane, ascending to the surface is only permissible if the partial pressure of nitrogen in each tissue is no more than twice the partial pressure of nitrogen at the surface. Since the surface pressure of nitrogen is $.79 \times 33$ fsw or 26 fsw, Haldane insisted that each tissue compartment be loaded to no more than $2 \times 26 = 52$ fsw. The navy introduced different levels for each tissue compartment, and called these the *M-values*.[1]

The navy determined the M-values they used by gathering empirical data on many dives conducted by naval personnel. The values they determined (and the pressure ratios they correspond to) are contained in the following table:

Half-Life T	M-value PM	Ratio $PM/26$
5	104	4.00
10	88	3.38
20	72	2.77
40	58	2.23
80	52	2.00
120	51	1.96

The ratios in the third column are in each case just the M-value divided by 26, the partial pressure of nitrogen at the surface. For Haldane, every entry in the second column would be 52, and the ratios would all be 2.00. We consequently see that the navy table is *less conservative* for the first four tissue compartments, and *at least as conservative* for the last two tissue compartments.

We can now do a little mathematics to compute the no-decompression limits (NDLs), according to these M-values. For example, consider a dive to 80 feet. We need to find how long we can stay at 80 feet, so that our six tissue compartments are each loaded to no more than the corresponding M-values in the table above. For if we do this, our diver will be able to *immediately proceed to the surface.* This is of course what we mean by *no-decompression diving.*

At 80 feet, the partial pressure of nitrogen is .79 × (33+80) = 89 fsw. The M-value for the 5 minute compartment is larger than this, and so our diver could become fully saturated in that compartment, and still proceed directly to the surface.

So now consider the 10 minute compartment. We have P_0 = 26, P_a = 89 and the pressure we wish to achieve is P_M = 88. In Chapter 8 we derived formula 8.1, which tells how long it takes to move from P_0 to P_M, with ambient pressure P_a. That formula was

$$t = \frac{T}{\ln 2} \ln\left(\frac{P_0 - P_a}{P_M - P_a}\right)$$

Applying this for T = 10, we obtain t = 56 minutes. In other words, our diver could spend 56 minutes at 80 feet without exceeding the M-value P_M = 88 in the 10 minute compartment.

But we have only considered two of the six tissues. To complete our calculation, we need to compute the time for each of the compartments. These calculations are tedious applications of the formula 8.1 above.[2] We obtain the following results:

5	10	20	40	80	120
no limit	56.4	37.5	40.7	61.1	87.0

We should now look for the *shortest* time in this chart. We discover that the 20 minute tissue is the *controlling compartment* for an 80 foot dive.

According to these calculations, a diver who spends no more than 37.5 minutes at 80 feet will have loaded all six compartments to no more than the corresponding M-values. Such a diver can thus return directly to the surface, *without decompression stops.* If we actually look at the Navy Table, we discover that the NDL there is set to 40 minutes. When generating tables, the navy tends to round numbers and chooses spacing that is easy to understand and remember. They consequently use 40 minutes rather than 37.5.

We can then repeat these calculations for all six tissue compartments at various depths. This is clearly a task for a computer rather than a human being with a calculator — the actual mathematics involves only repeated application of formula 8.1. When we do all of these calculations, we obtain the results reported in the table below. In that table the abbreviation NL means that the ambient pressure is less than the M-value, and so the given tissue does not place any time limit on a dive at that depth.

You should be able to verify any of the particular numbers in this table, by doing a little work with a calculator or Excel. You will do some of these calculations in the exercises.

For example, consider the 90 foot row. The partial pressure of nitrogen at this depth is .79 × (33+90) = 97, as the table asserts. Because this number is smaller than the M-value 104 for the 5 minute compartment, we have recorded NL at that spot in the table. But what about the 20 minute compartment? For that calculation, we have P_a = 97, P_0 = 26, and P_M = 72. This means we must perform the following calculation:

$$t = \frac{T}{\ln 2} \ln\left(\frac{P_0 - P_a}{P_M - P_a}\right) = \frac{20}{\ln 2} \ln\left(\frac{26-97}{72-97}\right) = 30.0$$

as we find in Table 1.

To find the NDL for any depth, we need only seek out the *shortest* time in the corresponding row. If we do this, we get the calculated numbers in Table 2. We compare these numbers to the actual numbers in the navy table, and see the close correspondence. The last column lists the controlling compartment. You should be able to generate this table yourself, by just looking at the previous table.

Table 1.

Depth	Tissue pp N_2	5	10	20	40	80	120
35	54	NL	NL	NL	NL	320.8	401.9
40	58	NL	NL	NL	NL	198.5	269.7
50	66	NL	NL	NL	95.4	123.5	173.0
60	74	NL	NL	100.3	64.7	91.6	129.5
70	81	NL	NL	51.3	49.8	73.2	104.0
80	89	NL	56.4	37.5	40.7	61.1	87.0
90	97	NL	29.6	30.0	34.5	52.5	74.9
100	105	31.0	22.1	25.2	29.9	46.0	65.8
110	113	16.4	18.0	21.7	26.5	41.0	58.7
120	121	12.5	15.3	19.1	23.7	37.0	53.0
130	129	10.3	13.3	17.1	21.5	33.7	48.3
140	137	8.8	11.9	15.5	19.7	30.9	44.3
150	145	7.7	10.7	14.2	18.2	28.6	41.0
	M-values	104	88	72	58	52	51

Table 2.

Depth	Table	Calc	Tissue
35	310	320.8	80
40	200	198.5	80
50	100	95.4	40
60	60	64.7	40
70	50	49.8	40
80	40	37.5	20
90	30	29.6	10
100	25	22.1	10
110	15	16.4	5
120	10	12.5	5
130	10	10.3	5
140	10	8.8	5
150	5	7.7	5

You will notice that we have not computed the NDL for dives to 25 and 30 feet, as reported in Navy Dive Table 3. Actually, in earlier versions of the Navy Table, there were no NDLs for dives this shallow: *any* dive to 25 or 30 feet was considered a no-decompression dive. And if you were to calculate the NDLs for these dives based on our discussion above, you would indeed have obtained no limit. What has happened more recently is the Navy has used very slow tissues (160 and 240 minutes) to consider exceptionally long exposures at 25 and 30 feet, and as a consequence has imposed NDLs on these depths; 20 foot dives remain without an NDL. For simplicity of exposition, we have omitted these special cases here.[3]

A quite significant observation we can make by examining the previous table is that *shallower dives are controlled by slower compartments*. This becomes important when we consider the modifications made to the navy tables for recreational divers.

We have thus shown how the navy table NDLs are generated from the M-values. In principle, we could continue to discuss decompression dives in the same way. However, the navy actually varies the M-values for decompression stops, making them generally more conservative. We will not pursue this subject in this book.

REPETITIVE DIVES

We now consider the navy approach to *repetitive dives*. After a reasonable surface interval, the slowest tissue compartment ($T = 120$) will be the tissue with the most remaining nitrogen. Consequently, to track residual nitrogen in the tissues of the body, we really need only track it in the 120 minute compartment. As we shall see, the navy tables encode this information in the pressure group letters we've discussed above.

Now, as we have observed earlier, a tissue compartment reaches equilibrium with the ambient pressure after six half-lives.[4] In the case of the 120-minute compartment, this amounts to 6×120 minutes, or twelve hours. This is precisely why the navy defines a repetitive dive as one that occurs after a surface interval of fewer than 12 hours. A diver who has waited at least

12 hours after a dive has no residual nitrogen (in any of his compartments), and consequently has no pressure group assigned: he is a *scratch diver*.

For really short surface intervals, the 120-minute compartment may not be the controlling one. The navy's rule is that a surface interval should be at least 10 minutes — if the surface interval is shorter, navy divers are instructed to consider the two dives as a single long dive. Notice that this 10 minute minimum surface interval requirement is built into the structure of Navy Dive Table 4.

The formal mathematical definition for the navy pressure groups is thus made in terms of the nitrogen load in the 120-minute tissue. Actually, this definition is customarily made in terms of the *total air pressure* $A(t)$ in that compartment, rather than the nitrogen partial pressure $P(t)$ we are accustomed to computing. To illustrate the navy's approach, we can easily compute $A(t)$ directly, using the equation

$$A(t) = A_a + (A_a - A_0)e^{-kt},$$

where A_a is the ambient total pressure, and A_0 is the initial load in the compartment. Here $k = \ln 2/120 = .00578$, because we are using the 120-minute compartment.

The pressure groups are now *defined* by intervals of total air pressure in the 120-minute compartment, of size 2 fsw: Pressure Group *A* means a total air pressure between 33 and 35 fsw, *B* is 35 to 37 fsw, *C* is 37 to 39 fsw, and so forth. We can record this information in the following table:

Letter	Total Pressure	Letter	Total Pressure	Letter	Total Pressure
A	33-35 fsw	F	43-45 fsw	K	53-55 fsw
B	35-37 fsw	G	45-47 fsw	L	55-57 fsw
C	37-39 fsw	H	47-49 fsw	M	57-59 fsw
D	39-41 fsw	I	49-51 fsw	N	59-61 fsw
E	41-43 fsw	J	51-53 fsw	O	61-63 fsw

Let's check this for a 35 foot dive (this choice is convenient, because according to Navy Table 3 a 35 foot dive is the only depth that leads to all

possible pressure groups with no-decompression dives). Let's compute the the total air pressure for the time periods given in the 35 foot row in the table. In this case

$$A_0 = 33, k = \ln 2/120 = .00578,$$

and

$$A_a = 35+33 = 68,$$

and so

$$A(t) = 68 - 35e^{-.00578t}.$$

We obtain the following results:

t	A(t)	Letter
5	34	A
15	36	B
25	38	C
40	40	D
50	42	E
60	43	F
80	46	G
100	48	H
120	51	I
140	52	J
160	54	K
190	56	L
220	58	M
270	61	N
310	62	O

We note that in each case we do indeed land in the appropriate pressure group, according to Table 3. Because of the navy's practice of making NDLs easy-to-remember numbers, we will actually often land in the next pressure group, rather than the mathematically correct one; this is of course builds an additional layer of conservatism in the navy tables.

Now it is easy to understand the Residual Nitrogen Time (or RNT): it is the time it takes for a scratch diver who dives to the depth under consideration to have their total air pressure in the 120-minute compartment rise to the interval corresponding to the letter designation.

Let's explore this with an example. Table 4 says that the RNT for a C diver at 40 feet is 25 minutes. And if we look to Table 3, we discover that a scratch diver will take 25 minutes to become a C diver! Instead of looking at the table, we can also verify this from our definition above, by computing the total air pressure on the diver after 25 minutes at 40 feet. This pressure is

$$(33 + 40) - 33e^{-.00578(25)} = 38.4,$$

which falls in the 37-39 fsw interval, which defines Pressure Group C. You will explore some more examples of this in an exercise.[5]

An apparently paradoxical fact about Navy Table 3 is that shallower dives to the NDLs lead to pressure groups *further* in the alphabet than do deep dives to the NDLs. But the reason for this is now clear: while pressure groups measure nitrogen buildup in the *slow* 120 minute compartment, deep dives are controlled by *fast* compartments. The diver on a deep dive runs out of no-decompression time in the fast compartments before she has a chance to build up enough nitrogen in the 120 minute compartment to land in a pressure group late in the alphabet. This leads to the triangular shape at the bottom of Table 3. (For why there is a triangular shape to this table at the top, you should think about Exercise 6 below.)

Exercises

1. In this exercise you will check by hand some of the calculations we reported above when determining the NDLs for the navy tables.

   (a) Consider a dive to 40 feet. Use formula 8.1 to compute how long a dive to 40 feet must last to reach the M-value in each of the six tissue compartments. Then determine the computed NDL for this depth, and specify the controlling compartment.
   (b) Repeat this exercise for a 90 foot dive.

2. In the previous chapter we did not compute the NDL for a Haldane diver. Let's do this now for a 60 foot dive, to compare the result to our navy table. For Haldane, the M-value is 52 feet for all tissue groups. We can thus use formula 8.1 for the 5 Haldane tissue compartments. What NDL do you obtain?

3. In this exercise you verify that several dives do in fact lead us to the correct pressure group, as described in the text.

   (a) Consider a 50 foot dive for 70 minutes. Compute the total air pressure in the 120 minute compartment for this dive. What is the pressure group that the table in the text predicts for this dive? Is this the same pressure group given by the navy table?
   (b) Repeat this calculation for a 100 foot dive for 20 minutes.
   (c) Repeat this calculation for a 70 foot dive for 30 minutes.

4. In this exercise you check that the RNT does in fact give the number of minutes at a given depth to reach the pressure group. (Actually rounding up when using the tables will often put the diver in the next higher pressure group, as some of the following examples show.[6])

   (a) What is the RNT for an E diver going to 30 feet? What is the pressure group for a scratch diver who dives for this many minutes at 30 feet?

(b) Repeat the previous exercise for an H diver going to 50 feet.
(c) Repeat the previous exercise for D diver going to 80 feet.
(d) Repeat the previous exercise for a B diver going to 100 feet.

5. A renegade dive physiologist decides to invent her own M-values, for a set of four tissue groups, with 8, 16, 32 and 48 minute half-lives. Her corresponding M-values are 98, 77, 80, and 60 respectively. Compute her NDLs for 40, 50, and 80 foot dives.

6. Explain why a no-decompression dive to 25 feet will never bring a diver to a pressure group higher than M. Hint: think about *total* pressure.

ENDNOTES

1 M stands for "maximum". The historical story is a little more complicated than what we are portraying here. The introduction of the concept of M-value was actually relatively late in the history of the tables; it was introduced by Robert Workman in the 1960s to efficiently describe the generation of the Navy tables. There have also been some variation in the M-values actually used by the Navy, and also some additional slower tissues added for certain profiles, as we will remark on below.

2 Alternatively, we could use a modest revision of our *Haldane Calculator* in Excel.

3 As we will see later, PADI computes all dives shallower than 35 feet as 35 foot dives, and thus also imposes NDLs on very shallow dives.

4 This is because $1 - (1/2)^6 = 98.4\%$.

5 Once again, you will find that because of the rounding up when using the navy tables, you will often actually end up in the next higher pressure group; this does not diminish the important conceptual understanding of the meaning of RNT which we discuss above.

6 There is less rounding in the PADI tables, and so this invariably works there.

Chapter 11

Variations on Table Use

The navy tables (and the PADI tables we will discuss qualitatively in the next chapter) are both based on *saltwater dives at sea level, on ordinary compressed air.* In this chapter we will look at how these calculations vary if we dive in fresh water, change our altitude, or change our breathing mix.

Fresh water

In our discussion of dive tables we have always assumed that the dives in question were in the ocean at sea level. But we learned in the first chapter that fresh water is a little lighter than seawater, and so pressure calculations in fresh water differ by a small amount. What is the effect of this on dive table calculations?

If we make a dive to 34 × 3 = 102 feet in fresh water (and at sea level), the total pressure experienced by the diver is 4 atmospheres. But in salt water we would reach this pressure at only 99 feet. Consequently, when computing nitrogen on-gassing, we should really interpret 102 feet of fresh water as 99 feet of seawater. We would thus use 100 feet on the navy (or PADI) table, rather than 110 feet, as rounding up from 102 foot would suggest.

Technically then, if you make a dive in fresh water, you should first multiply your depth by 33/34 = 97% before using a dive table designed for seawater diving. That is, we should slightly discount depths in fresh water, in order to obtain an *equivalent seawater depth.*

In practice, however, almost no-one makes this adjustment. In the example we looked at above, the small adjustment would actually make a difference on the navy dive tables, but usually the 3% difference would not have any significant effect. Using seawater tables for fresh water dives merely adds a small level of additional conservatism to dive calculations. This adds conservatism, because using fresh water depths on the navy (or PADI) table over-estimates the actual pressure the diver is operating under.

EFFECTS OF ALTITUDE

As we have already seen, changing altitude changes the atmospheric pressure, following the exponential decay function $p(h) = e^{-.0000383h}$, where we are measuring altitude h in feet and pressure $p(h)$ in atmospheres. We saw how we had to incorporate such effects in some of the Charles' Law problems we did.

But diving at altitudes significantly above sea level also has important effects on the absorption and off-gassing of nitrogen. The adjustment from fresh to salt water is not large enough to worry about, and using seawater tables for fresh water dives only makes the profile more conservative. However, divers who ignore adjustments for diving at altitude do so at their peril.

What is important for nitrogen absorption and off-gassing is the *relative change* in pressure, and if the surface pressure is smaller than 1 ata = 33 fsw, then the diver's tissues encounters larger relative changes.

The practical approach is to *translate* depths at altitude into *equivalent sea-level depths*. Divers traditionally use a table to perform this transformation, but the mathematical transformation is so easy that we will provide that instead of a table.[1]

For example, suppose that we are diving at 5000 feet of altitude. Then the atmospheric pressure is only

$$e^{-.0000383 \times 5000} = .83$$

atmospheres. Consequently, a diver will double his ambient pressure at only 34 × .83 = 28.2 feet of fresh water. To do this calculation in reverse, if we dive to d feet of fresh water at this altitude, we should consider that to be equivalent to a dive of $d/.83 = 1.2d$ feet of fresh water at sea level. For 5000 feet of altitude this means that for every 10 feet of depth, we should add an additional 2 feet, to obtain the *equivalent sea-level depth*.

More generally yet, if a diver is at altitude h feet, and does a dive to d feet, for the purposes of table calculations, we should consider it a

$$\frac{d}{e^{-.0000383h}} = de^{.0000383h}$$

foot dive.[2] In the exercises you will do some dive table problems for dives at altitude.

But there remains another problem with diving at altitude. If you travel to a higher elevation, *your body is not yet in equilibrium with the reduced ambient atmospheric pressure.* That is, your body already has a residual nitrogen load. This ascent to altitude can be considered a dive already! An easy way in principle to avoid this is to first get acclimated to the new pressure. According to navy standards, this would mean waiting until the 120 minute compartment is in equilibrium. This would take 12 hours.

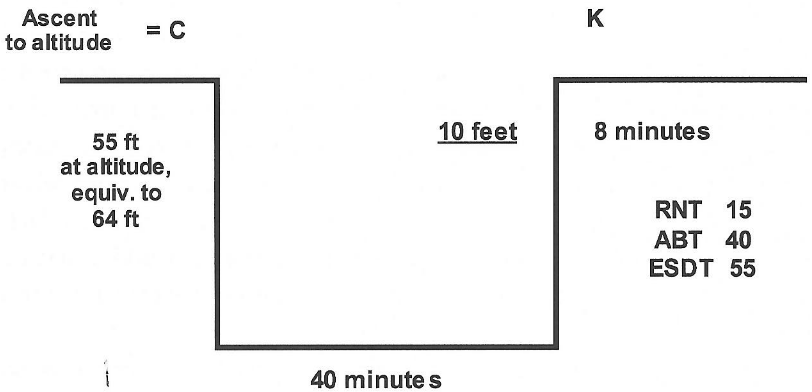
But sometimes we would prefer not to wait that long before diving. The alternative is to *actually compute what pressure group the rise in elevation puts us in.* This is again a determination that navy divers do by consulting a table, but the mathematics is easy enough that we will do it by calculation.

Remember that the letter pressure group designations are merely signposts to designate various total pressure loads in the 120 minute tissue. Suppose our diver has ascended to an altitude with pressure p. The sea-level total pressure load is 33 fsw. To translate we should divide 33 by p to determine the proportional total pressure load our diver newly arrived at altitude must have (in all his tissue compartments, and hence in particular in the slowest 120 minute compartment).

For example, if a diver ascends immediately to 4000 feet of altitude, then you can easily compute that $p = .86$. Consequently, his total pressure load is equivalent to 33/.86 = 38.5 fsw *at sea level.* If you check in the previous chapter, you will discover that this total pressure corresponds to pressure group C.[3]

Now suppose that the diver does a 40 minute dive to 55 feet at 4000 feet of altitude. The 55 foot depth corresponds to a depth of 64 feet at sea level. Since the diver is entering the water as a C diver, we need to obtain the RNT at 70 feet. This is 15 minutes. We thus have that the ESDT is 40+15 = 55. This makes this a decompression dive, because the NDL for 70 feet is only

50 minutes. We consequently will need an 8 minute decompression stop at 10 feet at the conclusion of this dive. Notice that this would have easily been a non-decompression dive, if we had disregarded the residual nitrogen coming from the ascent to altitude.



There is another important situation where we need to consider altitude effects while diving. This is the potential danger of flying immediately after diving. The cabin of a commercial airline needs to be pressurized, because the air at cruising altitudes of 30 thousand feet is too thin to safely breathe.[4] However, airlines pressurize their cabins to an nominal pressure equivalent to an altitude of 8000 feet (about 74% of 1 atmosphere). After all, such aircraft routinely land at elevations well above sea level. But we have just seen that an ascent to altitude is the operational equivalent of another dive. This means that we must be careful about flying too soon after diving, lest we in effect increase our nitrogen load, and court a case of the bends.

We would expect that a safe rule of thumb would be to wait until equilibrium would take place. For the navy this would be 12 hours, and for PADI (as we will see in the next chapter) the diver would need to wait for only 6 hours. However, the standard recommendations for this are usually much more conservative. This has been a controversial issue, with much study of the empirical data. Organizations like PADI and DAN have actually changed their recommendations more than once over the years. Presently, divers are encouraged to wait a minimum of 12 hours, with 18 or even 24 hours recommended.

But why should a diver wait until long after all the tissue groups in the mathematical model have reached equilibrium? The best answer is that accident reports gathered over the years suggest that a shorter interval is not safe!

One theory for what is happening is this. Safety from the bends in these circumstances actually involves some further tissue compartments, with very slow half-lives (from 240 minutes for the navy, and on up to as much as 635 minutes in other mathematical models). And while these slow compartments are not relevant for typical dives (especially for the recreational diver), they do become important when considering flying after diving. In addition, we noticed that M-values for slower tissues are more restrictive; we might expect that even slower tissues would have even more restrictive M-values. A.A.Buhlmann was a Swiss dive physiologist who was the world expert on diving and altitude, and his models included these very slow tissues. We will not pursue this subject quantitatively here.

The nitrogen load in these very slow theoretical tissue compartments would tend to increase over a typical week of recreational dives, with two or more repetitive dives each day. The 120 minute tissue would typically be almost entirely off-gassed overnight — slower tissues would not. This is why we could expect such a recreational diver to have a considerable backlog of nitrogen in these very slow tissues, which then would require additional time for off-gassing. Indeed, conservative dive practitioners even recommend a mid-week break from diving on a week-long recreational trip, in addition to a conservative interval after the trip before flying.

Hard-core recreational divers at tropical dive resorts often have to be reminded that there might actually be things to do and see above the water! A conservative and safe plan is to reserve the last day before flying for these activities, which thus increases the margin for safety for flying after diving. The dive resort industry would really prefer a less restrictive standard, and many of them recommend the 18 hour rule, which allows a diver to do some morning dives on the day before her airplane departure.

The effects of flying after diving are accentuated in the case of an air evacuation to a hyperbaric chamber for a diver suffering from suspected DCS. Such evacuations are usually made by helicopter. This makes it possible to stay at 1000 feet or less, where the change in altitude is unlikely to further worsen the victim's condition.

A note about depth gauges

The previous sections made the naive assumption that a scuba diver in fresh water (and perhaps at altitude) can always precisely determine his depth! To determine depth, a diver ordinarily looks at his depth gauge. Such gauges were originally analog devices, often with a needle on a dial. Modern depth gauges more often give digital readouts. But how accurate are such gauges?

Depth gauges either function by measuring the *absolute pressure*, and then subtracting off one atmosphere of pressure, or by sensing the *relative pressure change* from the pressure at the beginning of the dive. For a gauge of the first type, a diver in fresh water at sea level would obtain a reading which is *already converted* into feet of seawater: it would not really give the actual depth in feet in fresh water. For a gauge of the second type, the diver would obtain a reading *already converted* in terms of altitude. And especially with electronic digital gauges, most divers don't really know what the reading on their depth gauges actually mean, in fresh water and/or at altitude. And so while the mathematics of these calculations is straightforward, the actual implementation depends heavily on the technology used to measure the depth.

Nitrox

The absorption and off-gassing of the physiologically inert gas nitrogen is what causes the bends. Consequently, the natural approach to avoiding the bends would be to eliminate the inert gas in the breathing mix. But unfortunately, oxygen becomes toxic under pressure! Building an optimal breathing mix is consequently a compromise: we need to have as little inert gas as possible to avoid the bends, while keeping the oxygen content low enough that oxygen toxicity will not preclude dives to a reasonable depth.

Such considerations led to the invention of *enriched air nitrox* (EANx): a breathing mix with more than 21% oxygen. The two most common mixes are EANx32, consisting of 32% oxygen and 68% nitrogen, and EANx36, with 36% oxygen.[5]

Remember that the danger of oxygen toxicity requires that the partial pressure of oxygen in a breathing mixture should be restricted to no more

than 1.4 to 1.6 atmospheres of pressure. So how deep can a diver safely go, using EANx32 (with the more conservative limit of 1.4 ata)? If the *total pressure* (in atmospheres) on the diver at the limiting depth is p, then we want $.32p \leq 1.4$, or $p \leq 1.4/33 = 4.375$. If we then subtract off the 1 atmosphere of pressure due to the atmosphere, we obtain a limiting value of $3.375 = 33 \times 3.375 = 111$ fsw. And EANx36 has an effective pressure limit of $1.4 \times 33/.36 = 128$ fsw, and so a depth limit of $128 - 33 = 95$ fsw. These depth limits are shallower than the ordinary depth limit (130 fsw) for recreational divers, and so a recreational diver using nitrox needs to be careful to observe a more restricted depth limit.

The good news is that diving with nitrox provides additional bottom time, because the ambient nitrogen pressures are less while diving with a nitrox mix than with ordinary air. Although there are tables available for nitrox divers, the easiest way to understand a nitrox dive is to compute an *equivalent air depth* (EAD) for a dive to a given depth conducted with nitrox.

Suppose that a dive is made to d feet on a EANx32. The partial pressure of nitrogen at this depth is consequently $(d + 33) \times .68$, in fsw. At what depth x would we have this same partial pressure of nitrogen, if we were breathing compressed air? At x feet with ordinary air, the pressure would be $(x+33)\times.79$. We consequently need to have $(d+33)\times.68 = (x+33)\times.79$.

Solving for x, we find that the EAD is

$$\frac{(1-.32)\times(d+33)}{.79}-33$$

Replacing .32 by the oxygen percentage in any nitrox mixture would give the EAD in that case.

Suppose that a diver descends to 108 feet for 25 minutes, while breathing EANx32. This would be a decompression dive on air. But the EAD for this dive is

$$\frac{(1-.32)\times(108+33)}{.79}-33=88$$

Consequently, the richer mix has made this dive a no-decompression dive, because the NDL on the navy table for a 90 foot dive is 25 minutes.

Once a recreational diver has become comfortable enough in the water to decrease her air consumption, no-decompression dive time is limited by the NDLs rather than the available air. Diving with nitrox allows such a diver to spend more time underwater, without running into the NDLs. When nitrox was first introduced to recreational divers, it was considered a technical specialty. But its evident advantages in increasing available bottom time has made it more and more common in the recreational diving world. Most dive resorts and dive boats now offer nitrox to their patrons.

Nitrogen narcosis

Physiologically inert molecules in the breathing mix have another important effect on divers, when the partial pressure is large enough. For traditional mixes involving nitrogen, this effect is called *nitrogen narcosis* or *rapture of the deep*.

While the physiology of this effect is not completely understood, apparently the inert molecules interfere with the transmission of nerve signals. A familiar example is the use of nitrous oxide[6], which impedes nerve signals enough that it can be used effectively as an anesthetic in dentistry.

Nitrogen narcosis does not have a noticeable intoxicating effect until depths of 100 feet or more, for most divers.[7] Divers can experience impaired judgement, inexplicable elation or paranoia and other symptoms of intoxication. The effect by itself is not dangerous, but can lead to dangerous behavior, like continuing to descend without regard for the risks involved, or even discarding equipment. The effects are transitory, and go away almost immediately if the diver ascends to a lesser depth. It is a crucial responsibility of a diver on a deep dive to watch for signs of this intoxication in his buddy.

The onset of nitrogen narcosis is unpredictable and idiosyncratic to the diver. A diver might have serious effects at a given depth on one dive, but yet be completely unaffected at that depth on another dive.

A diver using a nitrox mix with more oxygen and less nitrogen is perhaps less susceptible to nitrogen narcosis, because there are at a given depth fewer inert gas molecules around to cause the effect. A dive to 100 feet on EANx32 has partial pressure of nitrogen equal to $(100 + 33) \times .68 = 90$ fsw, which

is of course considerably less than the $(100 + 33) \times .79 = 105$ fsw for the corresponding dive on air. The air diver thus appears to be more susceptible to narcosis.[8]

In the case of commercial, military or technical diving to extraordinary depths of 200 feet or more, almost all divers are impaired to some degree. While there is some anecdotal evidence that divers can be acclimated to nitrogen narcosis, it is more likely that such divers learn to deal with the effects rather than that they cease to be intoxicated. For wreck divers and cave divers, who dive to extraordinary depths while dealing with unusually stressful and dangerous conditions, the danger of nitrogen narcosis is considerable.

Consequently, in recent years new dive gas mixes have been introduced, with the intention of further decreasing the likelihood of narcosis. While all inert gases have intoxicating effects, the effect is less with gases having lighter molecules. This has led to dive mixes where some of the nitrogen is replaced by the lighter molecule helium; such dive mixes are called *trimix*. Since helium is lighter, it also off-gasses more quickly. This means that trimix is also less likely to produce the bends in divers.

One disadvantage of such mixes is their considerable expense as compared to air or even nitrox. And technical divers typically use trimix with a relatively low oxygen content, so that they can avoid oxygen toxicity, even at great depths. This leads to such divers using more than one tank: perhaps one for descent, one for depth, and another oxygen-rich tank for decompression and off-gassing. Divers who embark on such plans need to be highly skilled and extremely well trained. Breathing from the wrong tank can be fatal: a rich oxygen mix breathed at depth could lead to oxygen toxicity, while breathing on a lean oxygen mix designed for depth at the deco stop could provide inadequate oxygen for survival. We will not inquire into these matters in detail here.[9]

Exercises

1. A diver makes the following dives in a freshwater lake at sea level. The first dive is to 82 feet for 27 minutes, followed by a 1:55 surface interval. The next dive is to 61 feet for 40 minutes. Draw a dive profile diagram that *takes the adjustment from fresh to salt water into account*, and compute all pressure groups and decompression stops (if necessary). How does this compare to a corresponding series of dives in salt water?

2. What is the equivalent sea-level depth for a dive to 68 feet at 3800 feet of elevation? (Don't bother to adjust for fresh water.) What is the resulting pressure group if a diver makes this dive for 23 minutes? (Assume that the diver has been at 3800 feet for at least 12 hours.)

3. A diver immediately ascends to 4600 feet from sea level. To what pressure group does this translate? Suppose now that the diver goes to 80 feet for 21 minutes. Draw a dive profile diagram, and compute the ending pressure group.

4. What is the limitation on depth imposed by the use of EANx40? (Compute this using both 1.4 and 1.6 ata as a standard.)

5. Compute the equivalent air depth (EAD) for these dives:

   (a) 86 feet, with EANx32.
   (b) 78 feet, with EANx36.
   (c) 65 feet, with EANx40.

   Then suppose that each of these dives is made for 30 minutes. Draw a dive profile for each of these dives, and compute any required decompression and the resulting pressure group.

6. What are the NDLs for the depths 30, 40, 50, 60, 70, 80, 90, 100, and 110 feet, when using EANx32 as a breathing mix? Why weren't you asked for NDLs for depths in excess of 110 feet?

7. A diver in equilibrium at Colorado Springs (altitude 6000 feet) drives to Cottonwood Lake, at 10,000 feet. Suppose she immediately dives to 60 feet. How long could she stay, assuming that she uses the navy tables, and does not want to make a decompression dive?

8. A diver who is unusually frightened about oxygen toxicity decides to dive with a tank mixed with 16% oxygen and 84% nitrogen. Being a Colorado native, he feels he can get adequate oxygen from this mix at all depths. Suppose we breathe from this tank at the surface (at sea level); at what elevation would a breather on the surface get an equivalent partial pressure of oxygen? But what is the NDL for a dive to 80 fsw for this tank? to 60 fsw?

9. A diver claims she suffers symptoms of nitrogen narcosis every time she reaches 90 feet on air. At what depth would she expect the narcotic effect, when diving with EANx32?[10]

10. While in the water, your dive buddy exhibits symptoms that you believe indicate nitrogen narcosis. What action do you take, and what do you expect to occur?

11. While in the water, your dive buddy exhibits symptoms that you believe indicate decompression sickness. What action do you take, and what do you expect to occur?

12. A diver is in equilibrium at an altitude of 5600 feet. He makes the following two dives, using EANx32. The first dive is to 81 feet for 27 minutes, followed by a surface interval of 2:15. The second dive is to 67 feet for 43 minutes. Draw a dive profile diagram, and compute the resulting pressure groups and decompression, taking into account *the altitude, the changed breathing mix, and the fact that she is diving in fresh water.*

Endnotes

1 As usual, when using a table with a round-up rule, additional conservatism is introduced into standard practice. The computations we do will be mathematically accurate, but occasionally more liberal than what the table-user might find.

2 Notice that we are here ignoring the distinction between fresh water and seawater. Technically, we should multiply the resulting value of the calculation by 33/34 to account for this.

3 PADI divers have a rule of thumb for diving after an ascent to altitude, which is based on these same computations, while using the 60 minute tissue and PADI's 26 pressure groups (see the next chapter for details about this). That rule of thumb is to add two pressure groups for each 1000 feet of ascent.

4 Incidentally, it is also incredibly cold!

5 Enriched air nitrox is mixed with pure oxygen and ordinary air; consequently, the non-oxygen portion will actually contain a small mixture of inert gas that is not nitrogen, just as ordinary air does.

6 Laughing gas!

7 Older scuba literature delighted in referring to the "martini rule": the effect of nitrogen narcosis is equivalent to 1 martini consumed for each 50 feet of depth. This "rule" has at best metaphorical value: you may well be impaired at 100 feet, and at 150 feet you are almost undoubtedly impaired.

8 Recent studies suggest that oxygen may also have a narcotic effect at depth, and so this advantage for the nitrox diver is less significant that previously thought.

9 There is yet one more problem: a helium-rich mix is lighter, and so does not provide much thermal protection. This means that some technical drysuit divers use yet another tank for the insulating gas in their drysuits.

10 This diver is unlikely to be correct; nitrogen narcosis does not usually manifest itself in such a predictable way! For the purposes of this problem we will assume that oxygen has no narcotic effect.

Chapter 12

Recreational Diving

In this chapter we will explore the reasons why recreational divers might need dive tables designed for them, and look a bit at some of the further safe diving protocols that good recreational divers follow.

Problems with the Navy Tables

The navy tables are used to this day by many divers.[1] They have an excellent record of helping to prevent the bends in divers who use them carefully. Nonetheless, there are some problems that arise when tables designed for military divers are applied to recreational divers.

First of all, the data upon which the navy tables were based consist of dives by young fit males in the military. However, the community of recreational divers consist of a much broader class of human beings. Many recreational divers have conditions that predispose them to decompression sickness, including age, lack of fitness, obesity, consumption of alcohol and smoking. There was concern within the recreational community that these factors together might make the navy tables too liberal for the typical recreational diver.

Secondly, although the navy tables accommodate repetitive dives, they are really not well-designed for the three or four dives per day that many recreational divers would wish to do. Military divers tend to do one dive per day to achieve a particular objective. Recreational divers, in contrast, tend to do several dives per day.

Thirdly, military divers usually have a particular objective at a fixed depth. This leads to what is called a *flat profile*: this means a dive at a fixed depth. Recreational divers tend to do dives with *multi-level profiles*. Since the typical recreational dive is a tour to see the underwater scenery, recreational divers naturally want to visit several different depths during the course of their dive.

And finally, navy tables include the provision for decompression diving. Within the military context, there is enough infrastructure to make decompression diving safe. However, recreational divers are in many cases not well trained enough (nor good enough with their air consumption) to embark on planned decompression dives.

There consequently developed in the recreational dive community the need for a new set of tables, which would be somewhat more conservative about NDLs than the navy tables, but at the same time accommodate the need for multiple repetitive dives and multi-level profiles. The new tables should avoid all obligatory decompression.

Before discussing the advent of dive tables for recreational divers, a word of caution is in order. *All* dive tables (and dive computers) are based on mathematical models of what is happening in the diver's body, and it is inevitable that these models do not accurately predict outcomes for all dives and all divers. There are certainly cases where divers have followed the latest safe and conservative protocols, but have yet got bent – we can trust mathematical models only so far. Fortunately, these cases are rare indeed, and incidences of decompression sickness can usually be attributed to disregarding safe diving practices.

THE DEVELOPMENT OF THE PADI TABLES

The Professional Association of Diving Instructors (PADI) had originally used a truncated version of the navy tables, even allowing some short decompression stops. In the mid 1980s they commissioned a commercial organization named DSAT[2] to build a set of tables addressing the limitations of the navy tables for recreational divers. DSAT designed what became known as the *Recreational Dive Planner* (or RDP for short).

DSAT did two things to build a table with more conservative NDLs. They expanded to 14 tissue compartments (instead of 6), and used more conservative M-values than the navy had. For example, with the RDP the NDL for a 60 foot dive is 55 minutes, rather than the 60 minutes on the navy table. The following table provides a complete comparison, up through 140 feet[3]:

Depth	Navy NDL	PADI NDL
25	595	205
30	405	205
35	310	205
40	200	140
50	100	80
60	60	55
70	50	40
80	40	30
90	30	25
100	25	20
110	20	16
120	15	13
130	10	10
140	10	8

On the other hand, DSAT adopted a more liberal approach to the calculation of pressure groups for repetitive diving. They believed that for most typical recreational profiles, it was unnecessary to use the 120 minute tissue compartment for tracking residual nitrogen. Instead, they adopted the 60 minute compartment, which means that an RDP repetitive dive is defined as one that occurs less than *six* hours after the last dive. They introduced no less than 26 pressure groups to keep track of residual nitrogen (one for each letter of the alphabet). In their view, the 120 minute compartment would only be relevant in the case of very long shallow dives — quite possible in the military context, but unlikely for the typical recreational diver. This faster off-gassing rate meant that with the RDP divers could get back in the water much more quickly and do the dives they might wish. Of course, operators of dive resorts and dive boats welcomed this more liberal approach to repetitive dives, since it made the two-tank morning dives[4] they use more feasible.

Actually, DSAT eventually added special provisos to call for extra long surface intervals in the unusual case of long and shallow diving. This was essentially an admission that in certain rare cases the 120 minute compartment really was important. These special provisos are called the *XYZ* rules, named after the nitrogen-heavy pressure groups where the rules apply. Since we will not be using the RDP in this book, we will not delve into these rules in detail.

The RDP in its original form was actually not a table at all, but instead a beautifully designed analogue computing device, consisting of turning plastic wheels whose alignments give information about NDLs, pressure groups and surface intervals; it can be compared to a circular slide rule.[5] This PADI called *The Wheel*.[6] They also produced a more traditional table version, similar in appearance to their version of the navy tables. More recently, they have also produced an electronic version called the eRDP.

One considerable advantage of the Wheel is that its depths are given in 5 foot increments; if used to this accuracy, it gave slight but real increases in available no-decompression bottom time. But the major advantage of the wheel was that it incorporated the opportunity to compute multi-level dives. Using a "flat table", a diver must count his entire bottom time as taking place at the deepest depth of the dive. But there is a considerable increase in safe available bottom time if a diver thinks of the dive as coming in two or more separate levels. It is possible to "stair-step" a set of dive tables to do multi-level profiles: merely treat the separate levels as separate dives, with zero surface interval in between; such a procedure gives the diver more available bottom time on a given profile.[7] However, this leads to relatively liberal limits; DSAT built additional conservative factors into the wheel when it is used to compute the limits and pressure-group outcomes for multi-level profiles.

The PADI Recreational Dive Planner (in either the wheel, flat table or electronic form) is now the backbone of PADI training for safe dive profiles. Since PADI is the industry leader in recreational dive training, the RDP is familiar to thousands of recreational divers, and by now has behind it an excellent track record of safety.

In its flat table form, use of the RDP is very similar to (and indeed modeled on) the navy table; for this reason we will not pursue it here. If you are interested in the mathematics of diving (or the beauty of an analogue design), it is well worth it to acquire and play with a Wheel.

Dive computers

By the time the RDP was in place among divers trained by PADI, a much more important development was already underway. This is the advent of the *dive computer*. The basic principle of the dive computer is to extend the lesson of multi-level diving to a much larger scale. A dive computer senses the ambient pressure for the diver, and translates that into the depth. This is read by the computer frequently, often several times a minute. With these data available, the computer can then model nitrogen absorption minute-by-minute. Consequently, instead of modeling the dive as taking place at only one depth (a *flat profile*), or at two or three depths (a typical *multi-level profile*), the computer in essence computes nitrogen on-gassing and off-gassing on many levels over the course of the dive.

The mathematics is not much more complicated than that we've already seen. However, the number of computations is large, and of course only practical with the computational power available in the dive computer. For the most part, the details of the algorithm used to compute nitrogen absorption by a given computer are proprietary, varying as to number of compartments, and as to whether the model is exactly Haldanian, or else incorporates other mathematical approaches to the problem. Independent reviewers of dive computers must consequently evaluate computers by testing them, and placing them on a conservative-to-liberal scale. Obviously, such factors as ease of use and reliability are important too. Many computers allow the down-loading of dive profiles onto personal computers, warn divers as to depth and time restrictions and also rate-of-ascent. Many are even air-integrated, which means that they give the tank pressure information once conveyed by a separate pressure gauge.

In just a few years, the dive computer has moved from the unusual to the almost universal, at least among experienced divers. It is actually quite rare now to see a recreational diver checking his nitrogen absorption using the RDP or the navy tables. However, there are some important problems associated with dive computers that we should touch on here.

The first problem is conceptual and controversial. There are undoubtedly more and more divers who treat their computers as infallible black boxes, and consequently do not think carefully about how risky the profiles they dive might be. Instead, they merely rely on the computer to do their thinking for them. Some people have suggested that this may lead to more aggressive and dangerous profiles by a population of divers who have less and less understanding of DCS. But on the other hand, the average recreational diver paying attention to his computer may well avoid dangerous dives that he might have made by misusing (or ignoring) the dive tables.

The second problem is that decompression sickness is not completely modeled by the on-gassing and off-gassing of nitrogen. There is evidence that suggests that *bounce* profiles (a deep dive followed by a nominally safe but quick ascent) and *saw-tooth* profiles (a dive with many abrupt ups and downs), are rather more likely to result in DCS than a purely mathematical computation of nitrogen might suggest. Dive computers compute (the mathematical model of) the nitrogen absorption very well — but they may not identify otherwise dangerous profiles.

Many dive computers are now fully capable of computing decompression schedules, as well as safe no-decompression profiles (although many recreational computers will lock out a diver for 24 hours if they violate decompression rules).[8] This may induce divers with inadequate skills, training or experience to dive compression profiles whose full danger might not be apparent to the diver. But it has always been the case that under-educated divers have relied more on the resident "expert", whether that be a dive guide or a computer, rather than taking personal responsibility for their own safety in the water.

A third problem is that dive computers can and do fail – their batteries run out, they're flooded or lost, etc. A diver in mid-trip is likely to continue to dive, despite having lost information about his nitrogen load. To avoid this, many divers carry a back-up computer – many modern dive watches double as dive computers. The safe alternative is to attempt to reconstruct his profiles by hand, and use dive tables, or else to stay out of the water for at least 24 hours. Most recreational divers do not follow these protocols.

These issues need further study, but there is no going back. The dive computer is with us forever, with its ever-increasing functionality and flexibility.

Safety stops

One important safety practice that all recreational training agencies emphasize is the *safety stop*, and there is much evidence to suggest that *the safety stop is one of the most important things a diver can do to avoid DCI.* A safety stop is a pause at 10 to 20 feet at the end of the dive for 3 to 5 minutes. This sounds really similar to a decompression stop, and it functions in the same way. The diver waiting at the safety stop is indeed off-gassing nitrogen, which makes it yet safer to finally reach the surface. The safety stop after every dive is now part of the usual practice of most recreational divers.

The technical distinction between a safety stop and a decompression stop is this. On a decompression dive, the mathematical model predicts that a direct ascent entirely to the surface is dangerous; some nitrogen *must* be off-gassed before reaching the surface. A safety stop, on the other hand, is an opportunity is further off-gas nitrogen on a dive that the mathematical model predicts is safe, even with a direct slow ascent to the surface. Realistically, all dives are decompression dives, in the sense that we on-gas as we go deeper, and then off-gas as we move towards the surface at shallower depths. A true decompression dive requires a stop for off-gassing; a safety stop is an optional but recommended off-gassing stop that increases the diver's margin of safety.[9]

For inexperienced divers, the safety stop adds an unexpected additional margin of safety. For when such a diver ascends, there is danger that the diver may lose control and ascend at a rate exceeding safe limits. But when an inexperienced diver is actually planning on a stop at 15 feet, he is then less likely to go all the way to the surface at an over-rapid rate. Pressure is changing most rapidly near the surface, which means that managing buoyancy there is most difficult.

Recently, there has been a lot of discussion in the literature about the value of so-called *deep stops* — safety stops at about half the deepest depth of the dive. The evidence strongly suggests that such stops further decrease the likelihood of DCI. We'll have more to say about deep stops in the next section.

Safe dive profiles

As we have already mentioned, there is evidence that bounce and sawtooth profiles are more dangerous than the amount of nitrogen absorbed would suggest. Traditional scuba educators go further, and suggested that the safest dive profile is a *stair-step* profile: this is a multi-level profile with the deepest level first, followed thereafter by successively shallower levels. Divers who follow such profiles are actually often incorporating a deep stop in their profile, at one of these shallower levels in the stair-step profile.

The rationale behind such profiles is this. On the first deepest level, the diver is absorbing nitrogen into all compartments. The faster tissue compartments (with shorter half-lives) approach saturation more rapidly. Consequently, when the diver ascends to a shallower level, it may well be the case that the nitrogen load in one or more of the faster compartments is actually less than the ambient nitrogen pressure. Thus, at this shallower level, the diver may well be off-gassing in the faster compartments, while still on-gassing in the slower compartments. This process will continue as she goes on to reach the shallower levels of the multi-level dive.

In contrast, consider the *reverse profile.* Suppose the diver does the shallowest level first, followed by deeper and deeper levels. Then she is at each stage experiencing a larger ambient nitrogen pressure than that achieved in any of the compartments, and so she will be on-gassing during the entire dive. It certainly sounds safer to be off-gassing from some compartments during at least some of the dive!

Let's compute a concrete example of this, to illustrate the fact that the reverse profile will on-gas more nitrogen. Consider a dive where the first level is at 100 feet for 15 minutes, followed by 50 feet for 15 minutes.

We compute the nitrogen load as usual for the navy tissue compartments, with $P_a = (33 + 100) \times .79 = 106$ and $P_0 = 33 \times .79 = 26$. After 15 minutes at 100 feet we have the second row in the following table, and after 15 minutes at 50 feet we have the third row:

Tissue	5	10	20	40	80	120
after 100 ft level	95	77	58	44	36	33
after 50 ft level	69	70	61	49	39	35

Notice that at the 50 foot level, the 5 and 10 minute compartments are actually off-gassing, while the slower compartments are still on-gassing!

Now let's repeat this dive, except that we shall do the 50 foot level followed by the 100 foot level. The corresponding table looks like this:

Tissue	5	10	20	40	80	120
after 50 ft level	61	52	42	35	31	29
after 100 ft level	100	86	68	51	40	36

All compartments on the 100 foot level continue to on-gas at the higher ambient pressure there. In every case, the compartment has a larger nitrogen load after the reverse profile than after the stair-step profile. You'll have the chance to compute another example of this in one of the exercises.

Stair-step profiles consequently lead to the opportunity for more bottom time, because reverse profiles may well bring the diver to NDLs sooner. It is usually the case that at recreational dive locations dive sites are selected and mapped by operators to encourage stair-step profiles.

The same reasoning applies to successive repetitive dives during a day. Less nitrogen load (and consequently more available bottom time) results if divers do their deeper dives before their shallower ones. Some off-gassing may occur during a late day shallow dive, just as it does in our example during a later shallower level in a multi-level dive.

It is consequently a sensible choice for divers to make use of stair-step profiles during a given dive, and to do deeper dives before shallow ones. In fact, dive trainers often assert that reverse profiles are more likely to bring on DCS, too. However, recent studies do not support this traditional view.

Exercises

1. A PADI diver and a navy diver have an argument about whose dive tables are better; what are the arguments on either side?

2. Consider a dive on which you dive to 80 feet for 10 minutes, and then ascend to 40 feet for 25 minutes.

   (a) According to standard navy practice, we should consider this as a dive for 35 minutes, *at the deepest depth reached.* In what pressure group is the diver, under this interpretation?
   (b) Now compute the ending pressure group for this dive, if we interpret this as two consecutive dives, with no surface interval between them.
   (c) Explain why your answers are different!

3. Suppose that a diver does two dives: a 50 minute dive to 40 feet, and a 40 minute dive to seventy feet; there is a 1:40 surface interval between.

   (a) What is the resulting pressure group when the diver does the 70 foot dive first?
   (b) What is the resulting pressure group when the diver does the 40 foot dive first?
   (c) Explain why your answers are different!

4. Suppose you do a multi-level dive: 18 minutes at 80 feet, and 32 minutes at 40 feet.

   (a) Compute the nitrogen load in the six navy tissue compartments after these two levels have been completed.
   (b) Now consider the corresponding reverse profile dive: 32 minutes at 40 feet, and then 18 minutes at 80 feet. Compute the nitrogen load in the six navy compartments after these two levels have been completed.

   (c) Explain why your answers are different! Note in particular which compartments were off-gassing and which were on-gassing on the second level in each case.

Endnotes

1 The *National Association of Underwater Instructors* (or *NAUI*) is an important dive certification agency; they use tables based on the navy tables.

2 DSAT stands for Diving Science and Technology; they remain a corporate affiliate of PADI, and have recently developed the technical dive training PADI now offers.

3 The deepest depth on the RDP is 140 feet. PADI recommends that recreational divers observe an absolute depth limit of 130 feet; the 140 foot data is included on the RDP in case of diver error.

4 In the industry, a "two-tank dive" means a boat trip involving two separate dives (with one tank each), separated by a surface interval of an hour or so.

5 Perhaps you've seen one these!

6 You can see the Wheel in the hands of actress Sharon Stone, in an early scene in the movie *Sphere*.

7 In one of the exercises you will "stair-step" the navy tables, as a way to show the considerable increase in bottom time available this way. We do not recommend this use of the navy tables for actual diving however; either use the wheel, or make safe use of a dive computer, as discussed below.

8 And divers have been known to reset locked-out computers by removing the batteries, or to simply resort to another computer for future dives!

9 PADI further muddies these waters by referring to *mandatory safety stops* for some profiles. Although these dive profiles are no-decompression dives, there is enough nitrogen in the body that it would be foolhardy not to take a safety stop for an extra margin of safety.

Appendix A

Units of Measurement

In this Appendix we will describe the basic units used in this book, and how you can change from one unit system to another. In the tables that follow, we will place Imperial units in the left-hand column, and SI units in the right.

Length

1 inch = 1 in = 2.54 cm	1 millimeter = 1 mm = .0394 in
1 foot = 1 ft = 12 in = 30.48 cm	1 centimeter = 1 cm = .394 in
1 mile = 5280 ft = 1609 m	1 meter = 1 m = 100 cm = 1000 mm = 39.37 in
	1 kilometer = 1 km = 1000 m = .62 mi

Area

1 in^2=6.452 cm^2	1 cm^2=.155 in^2
1 ft^2= 144 in^2 = 929.03 cm^2	1 m^2= 104 cm^2=10.76ft^2

Volume

1 in^3= 16.39 cm^3	1 cm^3= 1 cc = .061 in^3
1 quart = 57.75 in^3= 946.3 cm^3 = .946 liters	1 liter = $10^3$$cm^3$=1.0567 quarts
1 gallon = 4 quarts = 231 in^3	
1 ft^3=1728 in^3= 28.3 liters	1 m^3=106cm^3 = 103 liters=35.3 ft^3

Mass

1 ounce = 1 oz = 28.3 g	1 gram = 1 g = .0353 oz
1 pound mass = 1 lb = 16 oz = .454 kg	1 kilogram = 1 kg = 2.2 lb

Time

1 second = 1 s
1 minute = 1 min = 60 seconds
1 hour = 1 hr=60 min

Force

1 pound = 1 lb = 4.448 N	1 newton = 1 N = 1 $\frac{\text{kg m}}{\text{s}^3}$ =.2248 lb

Pressure

1 $\frac{\text{lb}}{\text{in}^2}$ =1 psi = 6894.8 Pa	1 Pascal = 1 Pa=1 $\frac{\text{N}}{\text{m}^2}$
	1 kiloPascal = 1 kPa = 10^3 Pa
1 atmosphere = 1 ata = 14.7 psi = 1.01325 bar	1 bar = 10^5 Pa = 14.5 psi = .9869 ata
1 inHg = .49 psi = 3386.4 Pa	1 mmHg = 133.32 Pa
1 fsw = .445 psi	1 msw = .101 bar
33 fsw = 1 ata= 30 inHg	10 msw = 1 ata = 760 mmHg

Energy

1 foot-pound = 1 ft lb = 1.3558 J	1 joule = 1 J = 1 N m = .7376 ft lb 1 (small) calorie = 1 cal = 4.186 J

Temperature

Fahrenheit degrees F =9/5C + 32
Celsius degrees C =5/9(F – 32)
Freezing Point of Water (at 1 ata) = 32° F= 0° C
Boiling Point of Water (at 1 ata) = 212° F= 100° C
Absolute Zero = –459.7° F= –273.15° C
Degrees Rankine R = F + 459.7
Kelvin K = C + 273.15

Appendix B
Specific Gravity

The specific gravity of a substance can be defined as the ratio of the weights (or masses) of the same volume of the given substance to that of fresh water (at 4° Celsius and 1 ata). In SI units, 1 cubic centimeter of pure water has mass equal to 1 gram, and so specific gravity can be interpreted as the mass in grams of 1 cubic centimeter of the substance. Here are some specific gravities for substances of interest in this book:

Substance	Specific Gravity	Substance	Specific Gravity
Hydrogen	0.000090	Pure Water	1.000
Helium	0.000177	Seawater	1.026
Nitrogen	0.00121	Glycerin	1.26
Air	0.00125	Bone	1.7–2.0
Oxygen	0.001429	Granite	2.64–2.76
Argon	0.001784	Aluminum	2.70
Cedar wood	0.49–.057	Iron	7.87
Oak wood	0.6–0.9	Copper	8.96
Ethyl Alcohol	0.791	Lead	11.35
Butter	0.86	Mercury	13.6

Appendix C
Air

For the most part, we have assumed in this book that air is composed of 21% oxygen and 79% nitrogen. As we discussed in Chapter 4, these percentages are measured in terms of *volume*, or equivalently, in terms of molecule count. This approximation of the truth is close enough for our discussion of the physiology of scuba diving. However, here is a more complete description of the composition of dry air; these figures are quite uniform over the surface of the earth:

Substance	Percentage
Nitrogen	78.084
Oxygen	20.946
Argon	0.934
Carbon Dioxide	.038
Neon	0.0018
Helium	0.0005

It is of course quite important to note that this is the breakdown for *dry* air. In the natural environment there is always some water vapor dissolved into air, but the air in a scuba tank does not contain appreciably much water. This is necessary because of the adverse effects it would have on the regulator and the tank. Indeed, divers often complain about breathing the dry air from a scuba tank over extended periods of time.

When as much water is dissolved into air as possible, we say that we have reached 100% humidity. This amount varies a great deal with temperature. At 30°C, this amounts to almost 30 grams per cubic meter, while at 0°C, it is about 5 grams per cubic meter. If we translate these figures into *volume*, water vapor at most amounts to about 4%.

Appendix D

Rules of Exponents

This appendix is intended to remind you of the basic rules of exponents, and show how these rules turn into corresponding important rules for the logarithm function.

In what follows let's suppose that a is a fixed *positive* number; it might be an integer like 2, or some other real number (like e =2.71728 ...). For positive *integers* n, an exponential expression of the form a^n is just a count of multiplicative factors: $a^3 = a \cdot a \cdot a$. This means that when we multiply two exponential expressions with the same base a, we need only add the exponents:

$$a^n a^m = a^{n+m}.$$

We call this the *addition rule for exponentiation.*

Now consider a expression like $(a^n)^m$. If we count the a factors we have $(a^2)^3 = a^2 \cdot a^2 \cdot a^2 = a \cdot a \cdot a \cdot a \cdot a \cdot a$. That is,

$$(a^n)^m = a^{nm}.$$

We call this the *multiplication rule for exponentiation.*

Now consider the expression a^0. If this use of exponentiation is to follow the addition rule, we must have $a^0 a^1 = a^{0+1} = a^1$. But since $a \neq 0$, we can divide both sides by a and obtain

$$a^0 = 1.$$

A zero exponent on a (non-zero) number gives one.

Next, consider a positive integer n. If the expression a^{-n} is to follow the addition rule, we must have $a^n a^{-n} = a^{n+-n} = a^0 = 1$, and so

$$a^{-n} = \frac{1}{a^n}$$

Negative exponents mean division. More generally,

$$a^{n-m} = a^n/a^m.$$

We call this the *subtraction rule for exponentiation.*

Now mathematicians are actually able to define a^x for any real number x, in such a way that the rules above are still satisfied. So we can and do use all such rules any time we encounter exponents. Any calculator is equipped to handle exponentiations of the form a^x where x can be chosen to be any real number, whether positive, negative or zero (and a is positive).

We can now define *logarithms* as exponents. That is, the logarithm $\ln(A)$ of A is exactly the exponent we need to put on e to obtain A:

$$e^{\ln(A)} = A.$$

In principle, we could define a logarithm function for any given positive base a, and mathematicians do this. You may have even encountered logarithms base 10 or base 2. However, in this book, we only use the logarithm base e, the so-called *natural logarithm.*

With this definition of logarithm, each of the exponentiation rules discussed above is turned inside out! For example, consider the expression $\ln(AB)$. Now,

$$e^{\ln A+\ln B} = e^{\ln A} e^{\ln B} \text{(by the addition rule for exponentiation).}$$

But $e^{\ln A}e^{\ln B} = AB$, and so $\ln A + \ln B$ is a number which when placed as an exponent on e, gives AB. That is,

$$\ln(AB) = \ln A + \ln B.$$

The logarithm of a product is the sum of the logarithms.

Now consider $\ln(A/B)$. Consider the computation

$$e^{\ln A-\ln B} = (e^{\ln A})/(e^{\ln B}) = A/B,$$

where we have used the subtraction rule for exponentiation. Thus $\ln A - \ln B$ is a number which when placed as an exponent on e gives A/B. That is,

$$\ln(A/B) = \ln A - \ln B.$$

The logarithm of a quotient is the difference of the logarithms.

We've seen that $e^0=1$. This translates into the fact that *the logarithm of 1 is zero.*

$$\ln(1) = 0.$$

We can also translate the multiplication rule for exponents to show that

$$\ln(A^k)= k \ln(A).$$

The logarithm of an exponentiation is a product.

A particular case of the previous rule occurs often:

$$\ln(1/A) = \ln (A^{-1}) = - \ln(A).$$

The logarithm of a reciprocal is the negative of the logarithm.

Appendix E

The US Navy Dive Tables

US Navy Dive Table 3

Depth	NDL	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
10	NL	60	120	210	300	797	*									
15	NL	35	70	110	160	225	350	452	*							
20	NL	25	50	75	100	135	180	240	325	390	917	*				
25	595	20	35	55	75	100	125	160	195	245	315	361	540	595		
30	405	15	30	45	60	75	95	120	145	170	205	250	310	344	405	
35	310	5	15	25	40	50	60	80	100	120	140	160	190	220	270	310
40	200	5	15	25	30	40	50	70	80	100	110	130	150	170	200	
50	100		10	15	25	30	40	50	60	70	80	90	100			
60	60		10	15	20	25	30	40	50	55	60					
70	50		5	10	15	20	30	35	40	45	50					
80	40		5	1015	20	25	30	35	40							
90	30		51	0	12	15	20	25	30							
100	25		5	7	10	15	20	22	25							
110	20			5	10	13	15	20								
120	15			5	10	12	15									
130	10			5	8	10										
140	10			5	7	10										
150	5			5												
160	5				5											
170	5				5											
180	5				5											
190	5				5											

US NAVY DIVE TABLE 4

A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	:10
																12:00
B	B	B	B	B	B	B	B	B	B	B	B	B	B	B	:10	3:21
															3:20	12:00
C	C	C	C	C	C	C	C	C	C	C	C	C	C	:10	1:40	4:50
														1:39	4:49	12:00
D	D	D	D	D	D	D	D	D	D	D	D	D	:10	1:10	2:39	5:49
													1:09	2:38	5:48	12:00
E	E	E	E	E	E	E	E	E	E	E	E	:10	:55	1:58	3:25	6:35
												:54	1:57	3:24	6:34	12:00
F	F	F	F	F	F	F	F	F	F	F	:10	:46	1:30	2:29	3:58	7:06
											:45	1:29	2:28	3:57	7:05	12:00
G	G	G	G	G	G	G	G	G	G	:10	:41	:1:16	2:00	2:59	4:26	7:36
										:40	1:15	1:59	2:58	4:25	7:35	12:00
H	H	H	H	H	H	H	H	H	:10	:37	1:07	1:42	2:24	3:21	4:50	8:00
									:36	1:06	1:41	2:23	3:20	4:49	7:59	12:00
I	I	I	I	I	I	I	I	:10	:34	1:00	1:30	2:03	2:45	3:44	5:13	8:22
								:33	:59	:1:29	2:02	2:44	3:43	5:12	8:21	12:00
J	J	J	J	J	J	J	:10	:32	:55	1:20	1:48	2:21	3:05	4:03	5:41	8:51
							:31	:54	1:19	1:47	2:20	3:04	4:02	5:40	8:50	12:00
K	K	K	K	K	K	:10	:29	:50	1:12	1:36	2:04	2:39	3:22	4:20	5:49	8:59
						:28	:49	1:11	1:35	2:03	2:38	3:21	4:19	5:48	8:58	12:00
L	L	L	L	L	:10	:27	:46	1:05	1:26	1:50	2:20	2:54	3:37	4:36	6:03	9:13
					:26	:45	1:04	1:25	1:49	2:19	2:53	3:36	4:35	6:02	9:12	12:00
M	M	M	M	:10	:26	:43	1:00	1:19	1:36	2:06	2:35	3:09	3:53	4:50	6:19	9:29
				:25	:42	:59	1:18	1:35	2:05	2:34	3:08	3:52	4:49	6:18	9:28	12:00
N	N	N	:10	:25	:40	:55	1:12	1:31	1:54	2:19	2:48	3:23	4:05	5:04	6:33	9:44
			:24	:39	:54	1:11	1:30	1:53	2:18	2:47	3:22	4:04	5:03	6:32	9:43	12:00
O	O	:10	:24	:37	:52	1:08	1:25	1:44	2:05	2:30	3:00	3:34	4:18	5:17	6:45	9:55
		:23	:36	:51	1:07	1:24	1:43	2:04	2:29	2:59	3:33	4:17	5:16	6:44	9:54	12:00
Z	:10	:23	:35	:49	1:03	1:19	1:37	1:56	2:18	2:43	3:11	3:46	4:30	5:28	6:57	10:06
	:23	:34	:48	1:02	1:18	1:36	1:55	2:17	2:42	3:10	3:45	4:29	5:27	6:56	10:05	12:00
	Z	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A
10	–	–	–	–	–	–	–	–	–	–	–	797	279	159	88	39
20	–	–	–	–	–	–	917	399	279	208	159	120	88	62	39	18
30	*	*	469	349	279	229	190	159	132	109	88	70	54	39	25	12
40	257	241	213	187	161	138	116	101	87	73	61	49	37	25	17	7
50	169	160	142	124	111	99	87	76	66	56	47	38	29	21	13	6
60	122	117	107	97	88	79	70	61	52	44	36	30	24	17	11	5
70	100	96	87	80	72	64	57	50	43	37	31	26	20	15	9	4
80	84	80	73	68	61	54	48	43	38	32	28	23	18	13	8	4
90	73	70	64	58	53	47	43	38	33	29	24	20	16	11	7	3
100	64	62	57	52	48	43	38	34	30	26	22	18	14	10	7	3
110	57	55	51	47	42	38	34	31	27	24	20	16	13	10	6	3
120	52	50	46	43	39	35	32	28	25	21	18	15	12	9	6	3
130	46	44	40	38	35	31	28	25	22	19	16	13	11	8	6	3
140	42	40	38	35	32	29	26	23	20	18	15	12	10	7	5	2
150	40	38	35	32	30	27	24	22	19	17	14	12	9	7	5	2
160	37	36	33	31	28	26	23	20	18	16	13	11	9	6	4	2
170	35	34	31	29	26	24	22	19	17	15	13	10	8	6	4	2
180	32	31	29	27	25	22	20	18	16	14	12	10	8	6	4	2
190	31	30	28	26	24	21	19	17	15	13	11	10	8	6	4	2

US NAVY DIVE TABLE 5

Depth	Time	50	40	30	20	10	Letter
40	200					0	N
40	210					2	N
40	230					7	N
40	250					11	O
40	270					15	O
40	300					19	Z
40	360					23	–
40	480					41	–
40	720					69	–
50	100					0	L
50	110					3	L
50	120					5	M
50	140					10	M
50	160					21	N
50	180					29	O
50	200					35	O
50	220					40	Z
50	240					47	Z
60	60					0	J
60	70					2	K
60	80					7	L
60	100					14	M
60	120					26	N
60	140					39	O
60	160					48	Z
60	180					56	Z
60	200				1	69	Z
60	240				2	79	–
60	360				20	119	–
60	480				44	148	–
60	720				78	187	–
70	50					0	J
70	60					8	K
70	70					14	L
70	80					18	M
70	90					23	N
70	100					33	N
70	110				2	41	O
70	120				4	47	O
70	130				6	52	O
70	140				8	56	Z
70	150				9	61	Z
70	160				13	72	Z
70	170				19	79	Z

US Navy Dive Table 5, continued

Depth	Time	50	40	30	20	10	Letter
80	40					0	I
80	50					10	K
80	60					17	L
80	70					23	M
80	80				2	31	N
80	90				7	39	N
80	100				11	46	O
80	110				13	53	O
80	120				17	56	Z
80	130				19	63	Z
80	140				26	69	Z
80	150				32	77	Z
80	180				35	85	–
80	240			6	52	120	–
80	360			29	90	160	–
80	480			59	107	187	–
80	720		17	108	142	187	–
90	30					0	H
90	40					7	J
90	50					18	L
90	60					25	M
90	70				7	30	N
90	80				13	40	N
90	90				18	48	O
90	100				21	54	Z
90	110				24	61	Z
90	120				32	69	Z
90	130			5	36	74	Z
100	25					0	H
100	30					3	I
100	40					15	K
100	50				2	24	L
100	60				9	28	N
100	70				17	39	O
100	80				23	48	O
100	90			3	23	57	Z
100	100			7	23	66	Z
100	110			10	34	72	Z
100	120			12	41	78	Z
100	180		1	29	53	118	–
100	240	1	4	42	84	142	–
100	360	2	42	73	111	187	–
100	480	21	61	91	142	187	–
100	720	55	106	122	142	187	–

US Navy Dive Table 5, continued

Depth	Time	50	40	30	20	10	Letter
110	20					0	G
110	25					3	H
110	30					7	J
110	40				2	21	L
110	50				8	26	M
110	60				18	36	N
110	70			1	23	48	O
110	80			7	23	57	Z
110	90			12	30	64	Z
110	100			15	37	72	Z
120	15					0	F
120	20					2	H
120	25					6	I
120	30					14	J
120	40				5	25	L
120	50				15	31	N
120	60			2	22	45	O
120	70			9	23	55	O
120	80			15	27	63	Z
120	90			19	37	74	Z
120	100			23	45	80	Z
120	120		10	19	47	98	–
120	180	5	27	37	76	137	–
120	240	23	35	60	97	179	–
130	10					0	E
130	15					1	F
130	20					4	H
130	25					10	J
130	30				3	18	M
130	40				10	25	N
130	50			3	21	37	O
130	60			9	23	52	Z
130	70			16	24	61	Z
130	80		3	19	35	72	Z
130	90		8	19	45	80	Z

Suggestions for Further Reading

There are many directions to go if you wish to read further about the topics discussed in this book. Here are my comments on a few books that I have found particularly instructive or useful.

Bennett, Peter and David Elliott. *The Physiology and Medicine of Diving.* London: Saunders, 1993.

This is a treasure-trove of information; Peter Bennett was for many years the head of the Divers Alert Network (DAN). The article by Hempleman (pp. 342–375) on the history of decompression was particularly useful for this book.

Bookspan, Jolie. *Diving Physiology in Plain English.* Kensington, Maryland: Undersea and Hyperbaric Medical Society, 1995.

This book is appropriately named, and a delight to read and consult.

Chowdhury, Bernie. *The Last Dive: A Father and Son's Fatal Descent into the Ocean's Depths.* New York: HarperCollins, 2000.

There is a large and growing literature of popular books on technical diving on wrecks and in caves. This is my favorite example.

Cousteau, Jacques Y. (with James Dugan). *The Living Sea.* New York: Harper & Row, 1963.

Any bibliography on scuba diving has to include a book by Captain Cousteau. This is a great choice, as is *The Silent World*, of course.

Epcott, Tim. *Neutral Buoyancy: Adventures in a Liquid World.* New York: Atlantic Monthly Press, 2001.

This is an excellent popular account of scuba diving, including a lot of interesting history.

Jablonski, Jarrod. *Doing it Right: The Fundamentals of Better Diving.* High Springs, Florida: Global Underwater Explorers, 2001.

This describes the one correct way to dive, according to the author and his group. Despite that, this book contains a lot of useful information.

Joiner, James T. (ed.). *NOAA Diving Manual: Diving for Science and Technology.* Flagstaff, Arizona: Best Publishing, 2001.

This is a tremendous resource, worth looking at if you're serious about diving. It includes a careful and example-driven description of the use of the Navy Dive Tables.

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This includes a good layman's account of decompression theory; needless to say, by now the technology of dive computers has gone beyond what is included in this book.

Lippmann, John. *Deeper into Diving.* Victoria, Australia: J. L. Publications, 1996.

This book includes some great information about the Navy and PADI dive tables, and lots more besides.

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This is a recent, fascinating account of Cousteau's career.

PADI. *PADI Divemaster Manual.* Santa Ana, California: PADI, 1991.
PADI. *The PADI Open Water Diver Manual.* Santa Ana, California: PADI, 1994.
PADI. *The Encyclopedia of Recreational Diving.* Santa Ana, California: PADI, 1996.

I was trained in the PADI system, and originally learned much of my diving theory from PADI publications. The *Encylopedia* is filled with valuable information. These books are all revised regularly, as the PADI system of education evolves – these citations are to the editions I actually learned from.

Phillips, John L. *The Bends: Compressed Air in the History of Science, Diving and Engineering.* New Haven: Yale University, 1998.

This is a valuable popular account of the bends, with lots of nice historical detail.

Weinke, B. R. *Basic Decompression Theory and Application.* Flagstaff, Arizona: Best Publishing, 1991.
Weinke, B. R. *Basic Diving Physics and Applications.* Flagstaff, Arizona: Best Publishing, 1994.

Bruce Weinke is a dive instructor and scientist, who has done a lot of work in decompression theory. His books include a wealth of information, but they are not written for the layman.

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