Bayesian and optimal gain methods for estimating DCS model parameters

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Introduction

- Probabilistic DCS model fitting.
- Maximum Likelihood Estimation (MLE) maximizes $P(X|\beta)$ – the likelihood.
- Bayesian estimation (BE) finds $P(\beta|X)$ – the posterior.
- What role can/do MLE and BE* play in DCS modeling?
- Confidence intervals vs. credible intervals.
- The gain problem, noise, confidence intervals in model-predicted DCS probabilities.
- Present an optimal (exact) gain technique for MLE or BE.

*Eftedal, Tjelmeland & Brubakk Av. Space & Env. Med. 78(2): 94-99, 2007.

Bayes and frequentist views

- Log likelihood (frequentist) estimation – reject any prior knowledge and examine the likelihood that the model (parameters) fits the data.
- Bayes method:

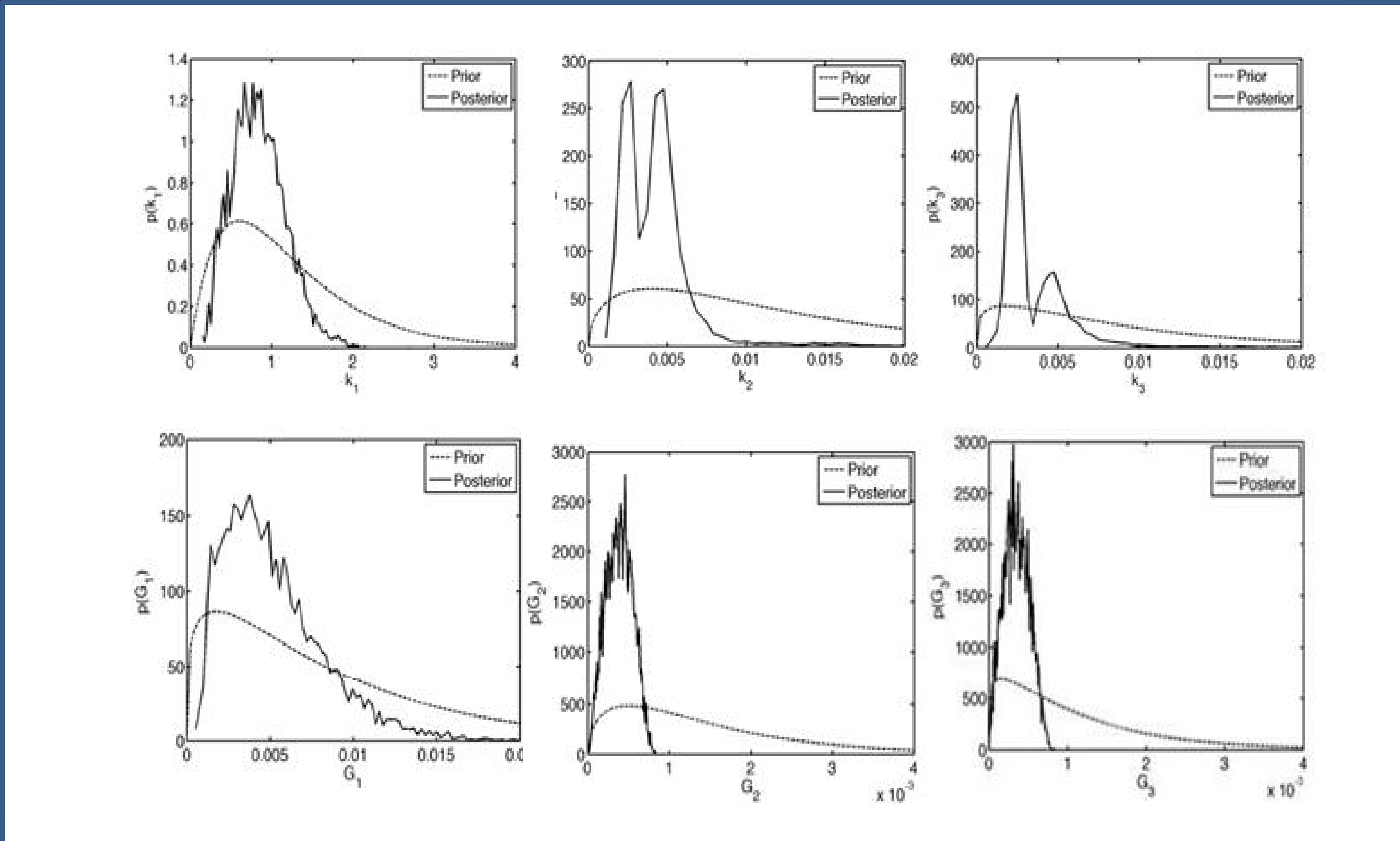
$$\mathrm{P}(\beta \mid X) = \frac{\mathrm{P}(X \mid \beta)\mathrm{P}(\beta)}{\mathrm{P}(X)} \propto \mathrm{P}(X \mid \beta)\mathrm{P}(\beta)$$

- Of what use is BE?
- MLE gives the "optimal" β but not the distribution of β.
- BE gives the distribution of β but and the most probable parameter values.

Calculating priors

- Posterior distributions sampled using the Markov Chain Monte Carlo method.
- Metropolis-Hastings algorithm generates candidate values from a "proposal" distribution.
- M-H algorithm accepts/rejects candidate parameter values based on posterior probability
- Accepted values *are* samples from the desired posterior.

Bayes distributions on parameters



Note bimodal k_1 and k_2 – we will see this again.

Parameter credible bounds

What is the difference between a confidence Interval (MLE) and a credible interval (BE)?

Parameter	MLE parameter estimate	MLE 95% confidence interval	BE 95% credible lower bound	BE 95% credible upper bound
k_1	0.6289	±0.9968	0.303	1.5457
k_2	0.004473	±0.02904	0.0017	0.0101
k_3	0.002005	±0.007556	0.0015	0.0092
g_1	0.00343	±0.003105	0.0012	0.0127
g_2	0.000488	±0.0001663	0.0000949	0.0006913
g_3	0.000237	±0.0001576	0.0000646	0.0006638

Bayesian estimation role in DCS modeling

- BE gives model parameter distributions and most probable parameter values.
- MLE gives the "optimal" solution but does not give parameter distributions. The local vs. global optimum problem is not addressed with MLE.

1. Use BE to find parameter distributions.
2. Use the most probable parameter values from (1) in the model or as initial parameter values in MLE ("double dipping").
3. Many initial starts no longer required for MLE parameters to address the local/global optimum issue.

Optimal (exact) gain solution

$$P = 1 - e^{-\sum g \int r(\beta)\, dt} = 1 - e^{-\vec{g}\cdot\vec{R}}$$

$$\text{let} \int_{t_1}^{t_2} r(\beta)\, dt \equiv \overset{t_1 t_2}{R}$$

$$\text{then } P_{t_1 \to t_2} = 1 - e^{-\vec{g}\cdot\overset{12}{\vec{R}}} = 1 - \overset{12}{\xi}$$

The log likelihood becomes

$$LL = \sum_{d=1}^{D} \ln(1-\xi)^{\delta} \ln(\xi)^{1-\delta}$$

Optimal gain – C coupled equations

$$\sum_{s=1}^{S}\left[\frac{\overset{12}{R}_{c,s}}{\overset{12}{\xi}{}_{s}^{-1}-1}\right]+\delta\sum_{n=1}^{N}\left[\frac{\overset{12}{R}_{c,n}}{\overset{12}{\xi}{}_{n}^{-1}-1}\right]=$$

$$\sum_{s=1}^{S}\overset{01}{R}_{c,s}+\sum_{n=1}^{N}\left[\overset{03}{R}_{c,n}-\delta\,\overset{13}{R}_{c,n}\right]+\sum_{z=1}^{Z}\overset{03}{R}_{c,z}$$

$$\overset{12}{\xi}{}_{n}^{-1}=\mathrm{e}^{\sum_{c} g_c \int_{t_1}^{t_2} r_c\,dt}$$

S: full DCS producing profiles

N: marginal DCS producing profiles

Z: profiles resulting on no DCS

c: tissue compartment index

How do we use the optimal gain - MLE, BE, P_{DCS}?

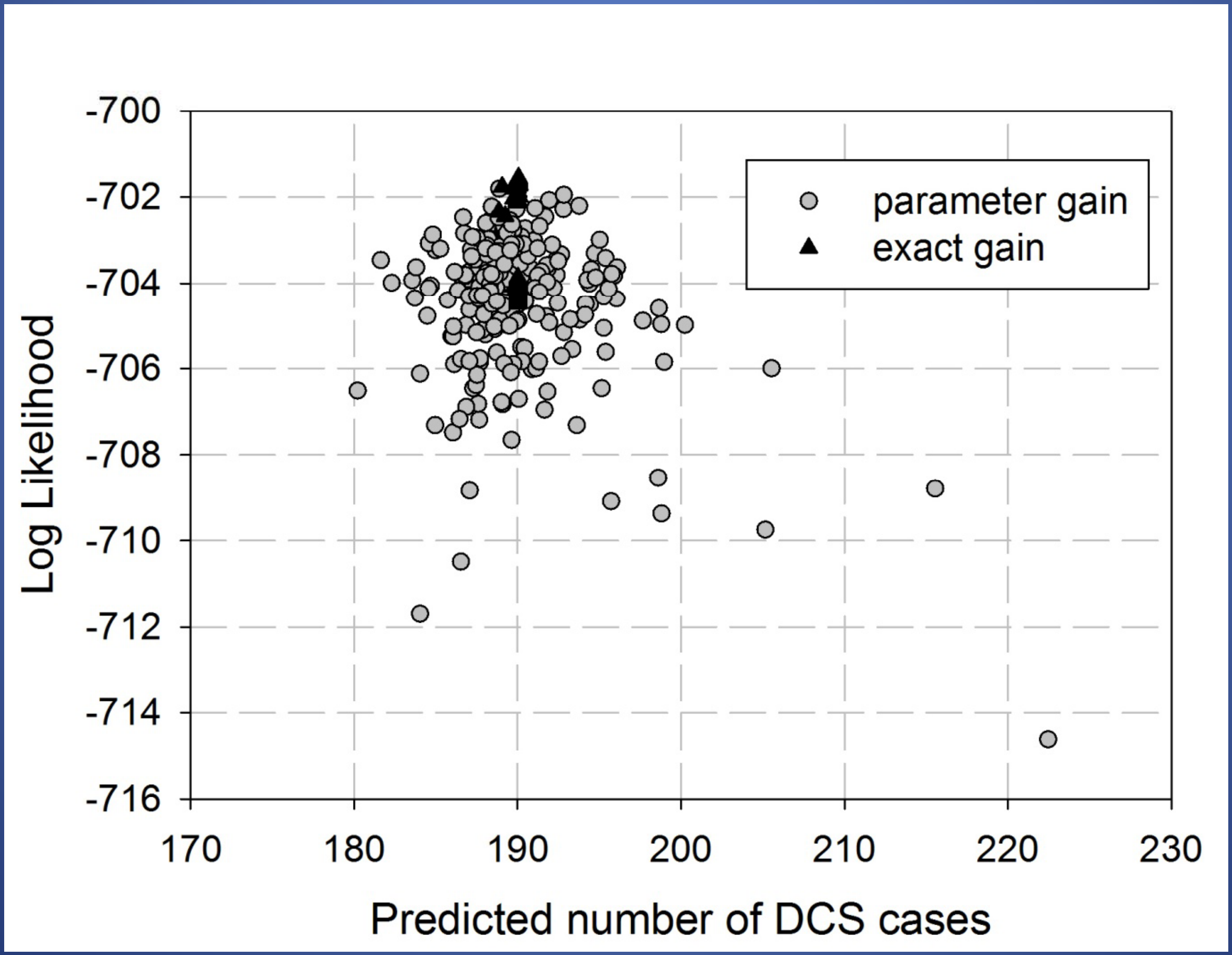
Maximum likelihood optimization with and without exact gain.

- Big292 dive data set
- EE1(nt) model with 3 “tissues”
- Incidence only, no failure times
- MLE estimation of 256 solutions (3 DOF & 6 DOF)

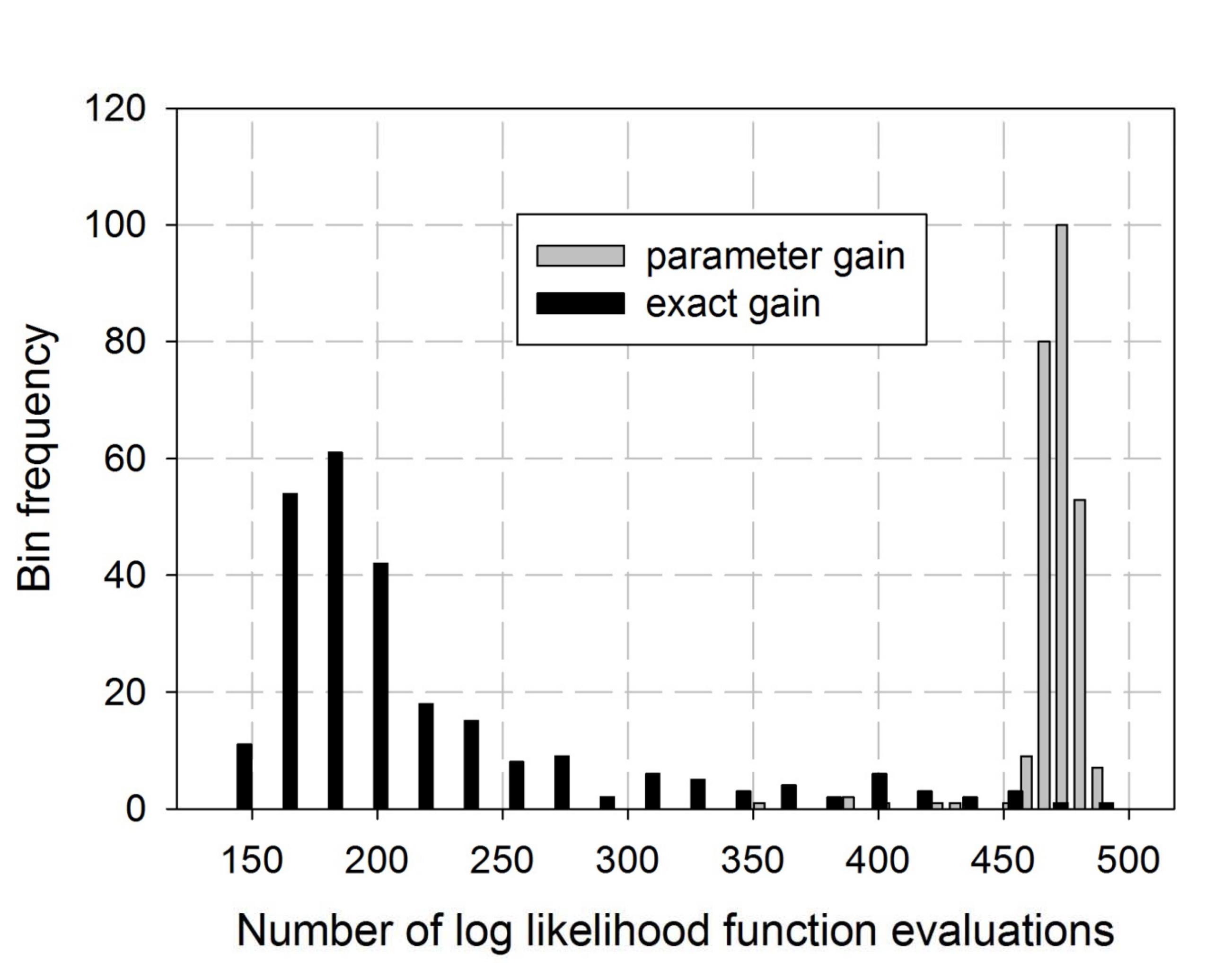
	EE1 model maximum log likelihood parameters					
	τ_1 (min)	τ_2 (min)	τ_3 (min)	g_1	g_2	g_3
exact gain (3 DOF)	0.4075	151.46	827.82	0.019550	0.000585	0.000181
95% C.I.	±0.5271	±75.13	±488.2	-	-	-
param gain (6DOF)	0.9435	152.52	796.07	0.004611	0.000561	0.000193
95% C.I.	±1.066	±74.35	±462.4	±0.00635	±0.000248	±0.00019
	EE1 model results and performance					
	PDCS	LL	time (ms)	evals.	cond.	
exact gain (3 DOF)	190.06	-701.55	76,206	282	9.39E+05	
95% C.I.	±0.1496					
param gain (6DOF)	188.88	-701.81	94,544	475	8.06E+13	
95% C.I.	±25.91					

Maximum LL solutions

190 full DCS in the Big 292 database

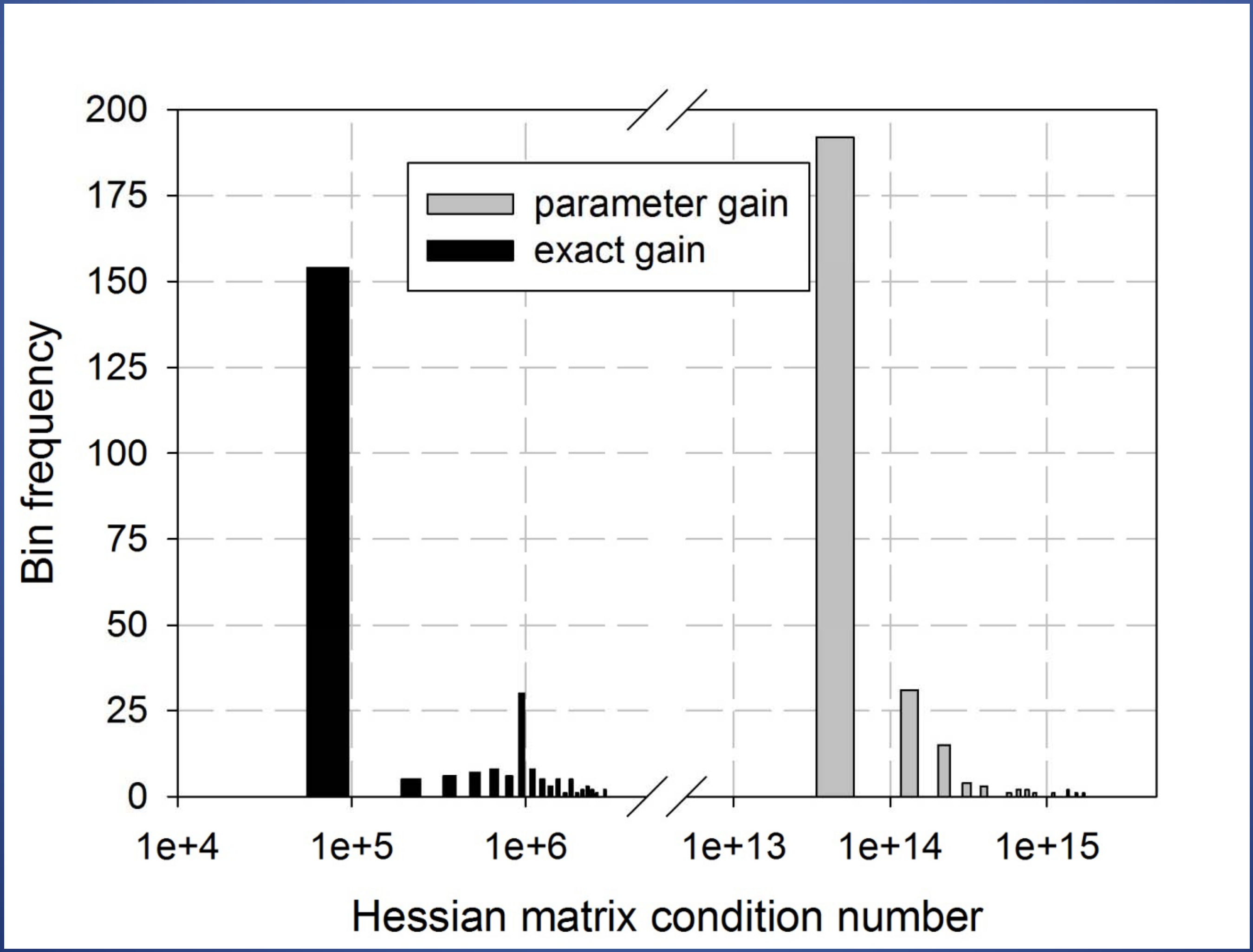


Function evaluations required

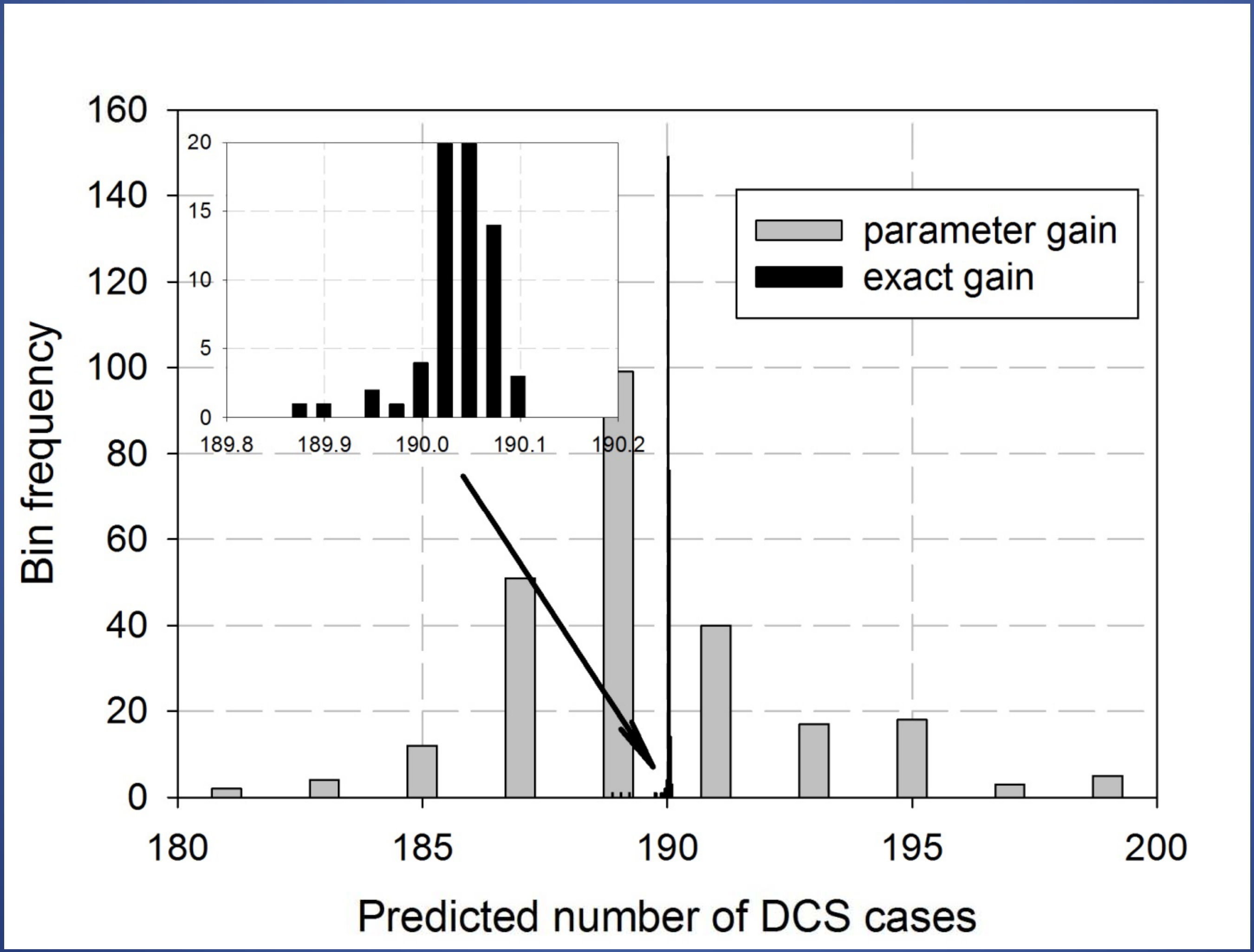


Hessian matrix condition number

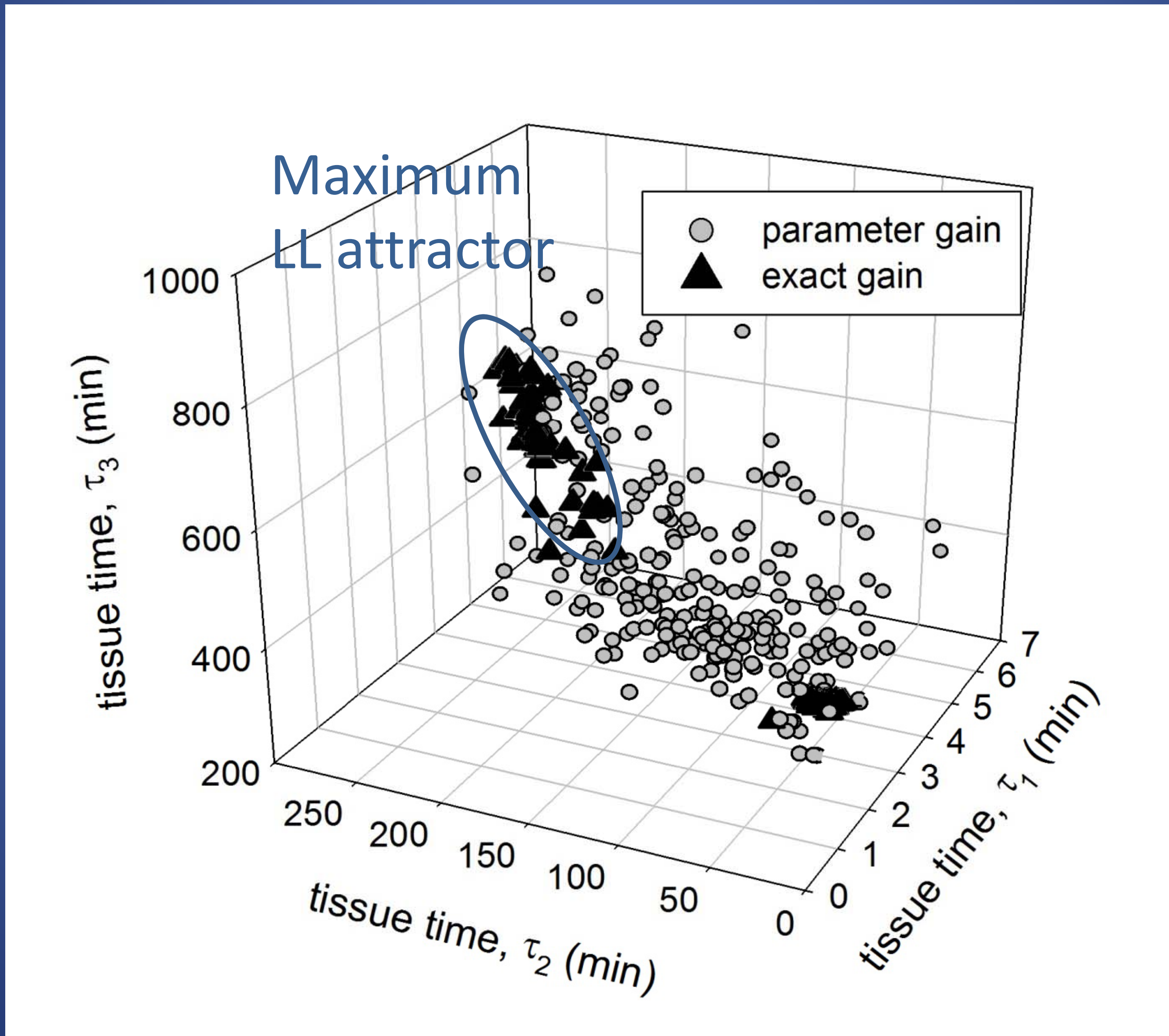
Nelder-Mead optimization method, identical random initial parameter distributions



Predicted DCS case histograms



Solution attractors



Conclusions

- MLE and BE are complementary for probabilistic DCS modeling.
- Only BE gives credible intervals on model parameters.
- Much of the scatter in the Poisson survival model, as applied to DCS modeling, is due to the ill-conditioned and highly covariate gain parameter.
- Eliminating the gain from the fitted parameters vastly improves the quality of the fit.
- The optimal gain method shown here is general and applies to any Poisson survival model using a multiplicative constant parameter.

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