Statistics

Pocket Guide

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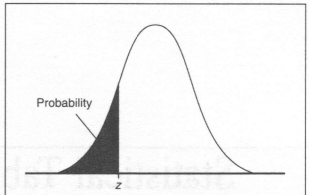
Contents

Preface

This guide is intended to provide a handy reference to all the key formulas, rules, and tables in *Statistics: Concepts and Applications* by Amir D. Aczel. There are two main sections. First, commonly used statistical tables that are also included in Appendix B in the text. Providing these in the form you see here is intended to ease your use of these while studying or doing homework assignments.

Second, in Part Two are all the key formulas and properties extracted from the text. These will not substitute for thoroughly understanding the text, however, we hope they will serve as a handy reference while studying and for future reference in coursework that may require statistics.

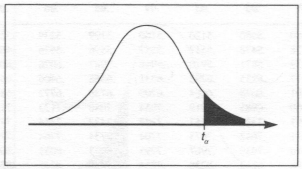


The table gives the probability that the standard normal random variable will fall at or below *z*.

The Standard Normal Probability Distribution **TABLE B.2**

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.01258	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
−3.5	.0002									
−4.0	.00003									
−4.5	.000003									
−5.0	.0000003									

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998									
4.0	.99997									
4.5	.999997									
5.0	.9999997									
6.0	.999999999									



Critical Values of the t Distribution

TABLE B.3

Degrees of Freedom	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

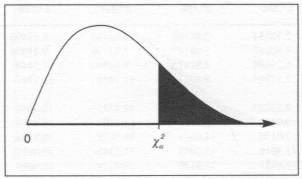


TABLE B.5

Critical Values of the Chi-Square Distribution

Degrees of Freedom	$X^2.995$	$X^2.990$	$X^2.975$	$X^2.950$	$X^2.900$
1	0.0000393	0.0001571	0.0009821	0.0039321	0.0157908
2.	0.0100251	0.0201007	0.0506356	0.102587	0.210720
3	0.0717212	0.114832	0.215795	0.351846	0.584375
4	0.206990	0.297110	0.484419	0.710721	1.063623
5	0.411740	0.554300	0.831211	1.145476	1.61031
6	0.675727	0.872085	1.237347	1.63539	2.20413
7	0.989265	1.239043	1.68987	2.16735	2.83311
8	1.344419	1.646482	2.17973	2.73264	3.48954
9	1.734926	2.087912	2.70039	3.32511	4.16816
10	2.15585	2.55821	3.24697	3.94030	4.86518
11	2.60321	3.05347	3.81575	4.57481	5.57779
12	3.07382	3.57056	4.40379	5.22603	6.30380
13	3.56503	4.10691	5.00874	5.89186	7.04150
14	4.07468	4.66043	5.62872	6.57063	7.78953
15	4.60094	5.22935	6.26214	7.26094	8.54675
16	5.14224	5.81221	6.90766	7.96164	9.31223
17	5.69724	6.40776	7.56418	8.67176	10.0852
18	6.26481	7.01491	8.23075	9.39046	10.8649
19	6.84398	7.63273	8.90655	10.1170	11.6509
20	7.43386	8.26040	9.59083	10.8508	12.4426
21	8.03366	8.89720	10.28293	11.5913	13.2396
22	8.64272	9.54249	10.9823	12.3380	14.0415
23	9.26042	10.19567	11.6885	13.0905	14.8479
24	9.88623	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100	67.3276	70.0648	74.2219	77.9295	82.3581

Degrees of Freedom	$X^2.100$	$X^2.050$	$X^2.025$	$X^2.010$	$X^2.005$
1	2.70554	3.84146	5.02389	6.63490	7.87944
2	4.60517	5.99147	7.37776	9.21034	10.5966
3	6.25139	7.81473	9.34840	11.3449	12.8381
4	7.77944	9.48773	11.1433	13.2767	14.8602
5	9.23635	11.0705	12.8325	15.0863	16.7496
6	10.6446	12.5916	14.4494	16.8119	18.5476
7	12.0170	14.0671	16.0128	18.4753	20.2777
8	13.3616	15.5073	17.5346	20.0902	21.9550
9	14.6837	16.9190	19.0228	21.6660	23.5893
10	15.9871	18.3070	20.4831	23.2093	25.1882
11	17.2750	19.6751	21.9200	24.7250	26.7569
12	18.5494	21.0261	23.3367	26.2170	28.2995
13	19.8119	22.3621	24.7356	27.6883	29.8194
14	21.0642	23.6848	26.1190	29.1413	31.3193
15	22.3072	24.9958	27.4884	30.5779	32.8013
16	23.5418	26.2962	28.8454	31.9999	34.2672
17	24.7690	27.5871	30.1910	33.4087	35.7185
18	25.9894	28.8693	31.5264	34.8053	37.1564
19	27.2036	30.1435	32.8523	36.1908	38.5822
20	28.4120	31.4104	34.1696	37.5662	39.9968
21	29.6151	32.6705	35.4789	38.9321	41.4010
22	30.8133	33.9244	36.7807	40.2894	42.7956
23	32.0069	35.1725	38.0757	41.6384	44.1813
24	33.1963	36.4151	39.3641	42.9798	45.5585
25	34.3816	37.6525	40.6465	44.3141	46.9278
26	35.5631	38.8852	41.9232	45.6417	48.2899
27	36.7412	40.1133	43.1944	46.9630	49.6449
28	37.9159	41.3372	44.4607	48.2782	50.9933
29	39.0875	42.5569	45.7222	49.5879	52.3356
30	40.2560	43.7729	46.9792	50.8922	53.6720
40	51.8050	55.7585	59.3417	63.6907	66.7659
50	63.1671	67.5048	71.4202	76.1539	79.4900
60	74.3970	79.0819	83.2976	88.3794	91.9517
70	85.5271	90.5312	95.0231	100.425	104.215
80	96.5782	101.879	106.629	112.329	116.321
90	107.565	113.145	118.136	124.116	128.299
100	118.498	124.342	129.561	135.807	140.169

Source: C. M. Thompson, "Tables of the Percentage Points of the X^2-Distribution," *Biometrika* 32 (1941), pp. 188–89. Reproduced by permission of the *Biometrika* Trustees.

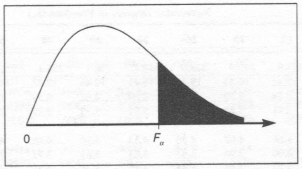


TABLE B.4 Critical Values of the F Distribution for $\alpha = 0.05$

Denominator Degrees of Freedom (k_2)	Numerator Degrees of Freedom (k_1)								
	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

Denominator Degrees of Freedom (k_2)	Numerator Degrees of Freedom (k_1)									
	10	12	15	20	24	30	40	60	120	∞
1	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

TABLE 1 The Binomial Distribution

		p								
n	x	.01	.02	.03	.04	.05	.06	.07	.08	.09
2	0	.9801	.9604	.9409	.9216	.9025	.8836	.8649	.8464	.8281
	1	.0198	.0392	.0582	.0768	.0950	.1128	.1302	.1472	.1638
	2	.0001	.0004	.0009	.0016	.0025	.0036	.0049	.0064	.0081
3	0	.9703	.9412	.9127	.8847	.8574	.8306	.8044	.7787	.7536
	1	.0294	.0576	.0847	.1106	.1354	.1590	.1816	.2031	.2236
	2	.0003	.0012	.0026	.0046	.0071	.0102	.0137	.0177	.0221
	3	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0005	.0007
4	0	.9606	.9224	.8853	.8493	.8145	.7807	.7481	.7164	.6857
	1	.0388	.0753	.1095	.1416	.1715	.1993	.2252	.2492	.2713
	2	.0006	.0023	.0051	.0088	.0135	.0191	.0254	.0325	.0402
	3	.0000	.0000	.0001	.0002	.0005	.0008	.0013	.0019	.0027
	4	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
5	0	.9510	.9039	.8587	.8154	.7738	.7339	.6957	.6591	.6240
	1	.0480	.0922	.1328	.1699	.2036	.2342	.2618	.2866	.3086
	2	.0010	.0038	.0082	.0142	.0214	.0299	.0394	.0498	.0610
	3	.0000	.0001	.0003	.0006	.0011	.0019	.0030	.0043	.0060
	4	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003
	5	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6	0	.9415	.8858	.8330	.7828	.7351	.6899	.6470	.6064	.5679
	1	.0571	.1085	.1546	.1957	.2321	.2642	.2922	.3164	.3370
	2	.0014	.0055	.0120	.0204	.0305	.0422	.0550	.0688	.0833
	3	.0000	.0002	.0005	.0011	.0021	.0036	.0055	.0080	.0110
	4	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005	.0008
	5	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
7	0	.9321	.8681	.8080	.7514	.6983	.6485	.6017	.5578	.5168
	1	.0659	.1240	.1749	.2192	.2573	.2897	.3170	.3396	.3578
	2	.0020	.0076	.0162	.0274	.0406	.0555	.0716	.0886	.1061
	3	.0000	.0003	.0008	.0019	.0036	.0059	.0090	.0128	.0175
	4	.0000	.0000	.0000	.0001	.0002	.0004	.0007	.0011	.0017
	5	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001
	6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	0	.9227	.8508	.7837	.7214	.6634	.6096	.5596	.5132	.4703
	1	.0746	.1389	.1939	.2405	.2793	.3113	.3370	.3570	.3721
	2	.0026	.0099	.0210	.0351	.0515	.0695	.0888	.1087	.1288
	3	.0001	.0004	.0013	.0029	.0054	.0089	.0134	.0189	.0255
	4	.0000	.0000	.0001	.0002	.0004	.0007	.0013	.0021	.0031
	5	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002
	6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

		p								
n	x	.01	.02	.03	.04	.05	.06	.07	.08	.09
9	0	.9135	.8337	.7602	.6925	.6302	.5730	.5204	.4722	.4279
	1	.0830	.1531	.2116	.2597	.2985	.3292	.3525	.3695	.3809
	2	.0034	.0125	.0262	.0433	.0629	.0840	.1061	.1285	.1507
	3	.0001	.0006	.0019	.0042	.0077	.0125	.0186	.0261	.0348
	4	.0000	.0000	.0001	.0003	.0006	.0012	.0021	.0034	.0052
	5	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005
	6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
10	0	.9044	.8171	.7374	.6648	.5987	.5386	.4840	.4344	.3894
	1	.0914	.1667	.2281	.2770	.3151	.3438	.3643	.3777	.3851
	2	.0042	.0153	.0317	.0519	.0746	.0988	.1234	.1478	.1714
	3	.0001	.0008	.0026	.0058	.0105	.0168	.0248	.0343	.0452
	4	.0000	.0000	.0001	.0004	.0010	.0019	.0033	.0052	.0078
	5	.0000	.0000	.0000	.0000	.0001	.0001	.0003	.0005	.0009
	6	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
12	0	.8864	.7847	.6938	.6127	.5404	.4759	.4186	.3677	.3225
	1	.1074	.1922	.2575	.3064	.3413	.3645	.3781	.3837	.3827
	2	.0060	.0216	.0438	.0702	.0988	.1280	.1565	.1835	.2082
	3	.0002	.0015	.0045	.0098	.0173	.0272	.0393	.0532	.0686
	4	.0000	.0001	.0003	.0009	.0021	.0039	.0067	.0104	.0153
	5	.0000	.0000	.0000	.0001	.0002	.0004	.0008	.0014	.0024
	6	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0003
	7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	11	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
15	0	.8601	.7386	.6333	.5421	.4633	.3953	.3367	.2863	.2430
	1	.1303	.2261	.2938	.3388	.3658	.3785	.3801	.3734	.3605
	2	.0092	.0323	.0636	.0988	.1348	.1691	.2003	.2273	.2496
	3	.0004	.0029	.0085	.0178	.0307	.0468	.0653	.0857	.1070
	4	.0000	.0002	.0008	.0022	.0049	.0090	.0148	.0223	.0317
	5	.0000	.0000	.0001	.0002	.0006	.0013	.0024	.0043	.0069
	6	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0006	.0011
	7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001
	8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	11	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

		p								
n	x	.01	.02	.03	.04	.05	.06	.07	.08	.09
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
18	0	.8345	.6951	.5780	.4796	.3972	.3283	.2708	.2229	.1831
	1	.1517	.2554	.3217	.3597	.3763	.3772	.3669	.3489	.3260
	2	.0130	.0443	.0846	.1274	.1683	.2047	.2348	.2579	.2741
	3	.0007	.0048	.0140	.0283	.0473	.0697	.0942	.1196	.1446
	4	.0000	.0004	.0016	.0044	.0093	.0167	.0266	.0390	.0536
	5	.0000	.0000	.0001	.0005	.0014	.0030	.0056	.0095	.0148
	6	.0000	.0000	.0000	.0000	.0002	.0004	.0009	.0018	.0032
	7	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0005
	8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	11	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
20	0	.8179	.6676	.5438	.4420	.3585	.2901	.2342	.1887	.1516
	1	.1652	.2725	.3364	.3683	.3774	.3703	.3526	.3282	.3000
	2	.0159	.0528	.0988	.1458	.1887	.2246	.2521	.2711	.2818
	3	.0010	.0065	.0183	.0364	.0596	.0860	.1139	.1414	.1672
	4	.0000	.0006	.0024	.0065	.0133	.0233	.0364	.0523	.0703
	5	.0000	.0000	.0002	.0009	.0022	.0048	.0088	.0145	.0222
	6	.0000	.0000	.0000	.0001	.0003	.0008	.0017	.0032	.0055
	7	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0005	.0011
	8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	11	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

		p								
n	x	.10	.15	.20	.25	.30	.35	.40	.45	.50
2	0	.8100	.7225	.6400	.5625	.4900	.4225	.3600	.3025	.2500
	1	.1800	.2550	.3200	.3750	.4200	.4550	.4800	.4950	.5000
	2	.0100	.0225	.0400	.0625	.0900	.1225	.1600	.2025	.2500
3	0	.7290	.6141	.5120	.4219	.3430	.2746	.2160	.1664	.1250
	1	.2430	.3251	.3840	.4219	.4410	.4436	.4320	.4084	.3750
	2	.0270	.0574	.0960	.1406	.1890	.2389	.2880	.3341	.3750
	3	.0010	.0034	.0080	.0156	.0270	.0429	.0640	.0911	.1250
4	0	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915	.0625
	1	.2916	.3685	.4096	.4219	.4116	.3845	.3456	.2995	.2500
	2	.0486	.0975	.1536	.2109	.2646	.3105	.3456	.3675	.3750
	3	.0036	.0115	.0256	.0469	.0756	.1115	.1536	.2005	.2500
	4	.0001	.0005	.0016	.0039	.0081	.0150	.0256	.0410	.0625
5	0	.5905	.4437	.3277	.2373	.1681	.1160	.0778	.0503	.0312
	1	.3280	.3915	.4096	.3955	.3602	.3124	.2592	.2059	.1562
	2	.0729	.1382	.2048	.2637	.3087	.3364	.3456	.3369	.3125
	3	.0081	.0244	.0512	.0879	.1323	.1811	.2304	.2757	.3125
	4	.0004	.0022	.0064	.0146	.0284	.0488	.0768	.1128	.1562
	5	.0000	.0001	.0003	.0010	.0024	.0053	.0102	.0185	.0312
6	0	.5314	.3771	.2621	.1780	.1176	.0754	.0467	.0277	.0156
	1	.3543	.3993	.3932	.3560	.3025	.2437	.1866	.1359	.0938
	2	.0984	.1762	.2458	.2966	.3241	.3280	.3110	.2780	.2344
	3	.0146	.0415	.0819	.1318	.1852	.2355	.2765	.3032	.3125
	4	.0012	.0055	.0154	.0330	.0595	.0951	.1382	.1861	.2344
	5	.0001	.0004	.0015	.0044	.0102	.0205	.0369	.0609	.0938
	6	.0000	.0000	.0001	.0002	.0007	.0018	.0041	.0083	.0156
7	0	.4783	.3206	.2097	.1335	.0824	.0490	.0280	.0152	.0078
	1	.3720	.3960	.3670	.3115	.2471	.1848	.1306	.0872	.0547
	2	.1240	.2097	.2753	.3115	.3177	.2985	.2613	.2140	.1641
	3	.0230	.0617	.1147	.1730	.2269	.2679	.2903	.2918	.2734
	4	.0026	.0109	.0287	.0577	.0972	.1442	.1935	.2388	.2734
	5	.0002	.0012	.0043	.0115	.0250	.0466	.0774	.1172	.1641
	6	.0000	.0001	.0004	.0013	.0036	.0084	.0172	.0320	.0547
	7	.0000	.0000	.0000	.0001	.0002	.0006	.0016	.0037	.0078
8	0	.4305	.2725	.1678	.1001	.0576	.0319	.0168	.0084	.0039
	1	.3826	.3847	.3355	.2670	.1977	.1373	.0896	.0548	.0312
	2	.1488	.2376	.2936	.3115	.2965	.2587	.2090	.1569	.1094
	3	.0331	.0839	.1468	.2076	.2541	.2786	.2787	.2568	.2188
	4	.0046	.0185	.0459	.0865	.1361	.1875	.2322	.2627	.2734
	5	.0004	.0026	.0092	.0231	.0467	.0808	.1239	.1719	.2188
	6	.0000	.0002	.0011	.0038	.0100	.0217	.0413	.0703	.1094
	7	.0000	.0000	.0001	.0004	.0012	.0033	.0079	.0164	.0312
	8	.0000	.0000	.0000	.0000	.0001	.0002	.0007	.0017	.0039

		p								
n	x	.10	.15	.20	.25	.30	.35	.40	.45	.50
9	0	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046	.0020
	1	.3874	.3679	.3020	.2253	.1556	.1004	.0605	.0339	.0176
	2	.1722	.2597	.3020	.3003	.2668	.2162	.1612	.1110	.0703
	3	.0446	.1069	.1762	.2336	.2668	.2716	.2508	.2119	.1641
	4	.0074	.0283	.0661	.1168	.1715	.2194	.2508	.2600	.2461
	5	.0008	.0050	.0165	.0389	.0735	.1181	.1672	.2128	.2461
	6	.0001	.0006	.0028	.0087	.0210	.0424	.0743	.1160	.1641
	7	.0000	.0000	.0003	.0012	.0039	.0098	.0212	.0407	.0703
	8	.0000	.0000	.0000	.0001	.0004	.0013	.0035	.0083	.0176
	9	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0008	.0020
10	0	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025	.0010
	1	.3874	.3474	.2684	.1877	.1211	.0725	.0403	.0207	.0098
	2	.1937	.2759	.3020	.2816	.2335	.1757	.1209	.0763	.0439
	3	.0574	.1298	.2013	.2503	.2668	.2522	.2150	.1665	.1172
	4	.0112	.0401	.0881	.1460	.2001	.2377	.2508	.2384	.2051
	5	.0015	.0085	.0264	.0584	.1029	.1536	.2007	.2340	.2461
	6	.0001	.0012	.0055	.0162	.0368	.0689	.1115	.1596	.2051
	7	.0000	.0001	.0008	.0031	.0090	.0212	.0425	.0746	.1172
	8	.0000	.0000	.0001	.0004	.0014	.0043	.0106	.0229	.0439
	9	.0000	.0000	.0000	.0000	.0001	.0005	.0016	.0042	.0098
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010
12	0	.2824	.1422	.0687	.0317	.0138	.0057	.0022	.0008	.0002
	1	.3766	.3012	.2062	.1267	.0712	.0368	.0174	.0075	.0029
	2	.2301	.2924	.2835	.2323	.1678	.1088	.0639	.0339	.0161
	3	.0853	.1720	.2362	.2581	.2397	.1954	.1419	.0923	.0537
	4	.0213	.0683	.1329	.1936	.2311	.2367	.2128	.1700	.1208
	5	.0038	.0193	.0532	.1032	.1585	.2039	.2270	.2225	.1934
	6	.0005	.0040	.0155	.0401	.0792	.1281	.1766	.2124	.2256
	7	.0000	.0006	.0033	.0115	.0291	.0591	.1009	.1489	.1934
	8	.0000	.0001	.0005	.0024	.0078	.0199	.0420	.0762	.1208
	9	.0000	.0000	.0001	.0004	.0015	.0048	.0125	.0277	.0537
	10	.0000	.0000	.0000	.0000	.0002	.0008	.0025	.0068	.0161
	11	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010	.0029
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
15	0	.2059	.0874	.0352	.0134	.0047	.0016	.0005	.0001	.0000
	1	.3432	.2312	.1319	.0668	.0305	.0126	.0047	.0016	.0005
	2	.2669	.2856	.2309	.1559	.0916	.0476	.0219	.0090	.0032
	3	.1285	.2184	.2501	.2252	.1700	.1110	.0634	.0318	.0139
	4	.0428	.1156	.1876	.2252	.2186	.1792	.1268	.0780	.0417
	5	.0105	.0449	.1032	.1651	.2061	.2123	.1859	.1404	.0916
	6	.0019	.0132	.0430	.0917	.1472	.1906	.2066	.1914	.1527
	7	.0003	.0030	.0138	.0393	.0811	.1319	.1771	.2013	.1964
	8	.0000	.0005	.0035	.0131	.0348	.0710	.1181	.1647	.1964
	9	.0000	.0001	.0007	.0034	.0116	.0298	.0612	.1048	.1527
	10	.0000	.0000	.0001	.0007	.0030	.0096	.0245	.0515	.0916
	11	.0000	.0000	.0000	.0001	.0006	.0024	.0074	.0191	.0417
	12	.0000	.0000	.0000	.0000	.0001	.0004	.0016	.0052	.0139

		p								
n	x	.10	.15	.20	.25	.30	.35	.40	.45	.50
	13	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010	.0032
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
18	0	.1501	.0536	.0180	.0056	.0016	.0004	.0001	.0000	.0000
	1	.3002	.1704	.0811	.0338	.0126	.0042	.0012	.0003	.0001
	2	.2835	.2556	.1723	.0958	.0458	.0190	.0069	.0022	.0006
	3	.1680	.2406	.2297	.1704	.1046	.0547	.0246	.0095	.0031
	4	.0700	.1592	.2153	.2130	.1681	.1104	.0614	.0291	.0117
	5	.0218	.0787	.1507	.1988	.2017	.1664	.1146	.0666	.0327
	6	.0052	.0301	.0816	.1436	.1873	.1941	.1655	.1181	.0708
	7	.0010	.0091	.0350	.0820	.1376	.1792	.1892	.1657	.1214
	8	.0002	.0022	.0120	.0376	.0811	.1327	.1734	.1864	.1669
	9	.0000	.0004	.0033	.0139	.0386	.0794	.1284	.1694	.1855
	10	.0000	.0001	.0008	.0042	.0149	.0385	.0771	.1248	.1669
	11	.0000	.0000	.0001	.0010	.0046	.0151	.0374	.0742	.1214
	12	.0000	.0000	.0000	.0002	.0012	.0047	.0145	.0354	.0708
	13	.0000	.0000	.0000	.0000	.0002	.0012	.0045	.0134	.0327
	14	.0000	.0000	.0000	.0000	.0000	.0002	.0011	.0039	.0117
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0009	.0031
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0006
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
20	0	.1216	.0388	.0115	.0032	.0008	.0002	.0000	.0000	.0000
	1	.2702	.1368	.0576	.0211	.0068	.0020	.0005	.0001	.0000
	2	.2852	.2293	.1369	.0669	.0278	.0100	.0031	.0008	.0002
	3	.1901	.2428	.2054	.1339	.0716	.0323	.0123	.0040	.0011
	4	.0898	.1821	.2182	.1897	.1304	.0738	.0350	.0139	.0046
	5	.0319	.1028	.1746	.2023	.1789	.1272	.0746	.0365	.0148
	6	.0089	.0454	.1091	.1686	.1916	.1712	.1244	.0746	.0370
	7	.0020	.0160	.0545	.1124	.1643	.1844	.1659	.1221	.0739
	8	.0004	.0046	.0222	.0609	.1144	.1614	.1797	.1623	.1201
	9	.0001	.0011	.0074	.0271	.0654	.1158	.1597	.1771	.1602
	10	.0000	.0002	.0020	.0099	.0308	.0686	.1171	.1593	.1762
	11	.0000	.0000	.0005	.0030	.0120	.0336	.0710	.1185	.1602
	12	.0000	.0000	.0001	.0008	.0039	.0136	.0355	.0727	.1201
	13	.0000	.0000	.0000	.0002	.0010	.0045	.0146	.0366	.0739
	14	.0000	.0000	.0000	.0000	.0002	.0012	.0049	.0150	.0370
	15	.0000	.0000	.0000	.0000	.0000	.0003	.0013	.0049	.0148
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0013	.0046
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0011
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Selected Random Numbers

1559	9068	9290	8303	8508	8954	1051
5550	6245	7313	0117	7652	5069	6354
4735	6214	8037	1385	1882	0828	2957
5333	1313	3063	1134	8676	6241	9960
8495	2956	1121	8484	2920	7934	0670
1947	3353	1197	7363	9003	9313	3434
4785	6325	1868	5020	9100	0823	7379
9972	9163	5833	0100	5758	3696	6496
0472	4629	2007	4464	3312	8728	1193
4727	6994	1175	5622	2341	8562	5192
3658	3226	5981	9025	1080	1437	5721
6906	9758	0244	0259	4609	1269	5957
3793	6916	0132	8873	8987	4975	4814
3376	5966	1614	4025	0721	1537	6695
6126	0224	7169	3596	1593	5097	7286
0466	7566	1320	8777	8470	5448	9575
9908	9832	8185	8835	0384	3699	1272
7594	3636	1224	6808	1184	3404	6752
5715	9301	5847	3524	0077	6674	8061
7932	4739	4567	6797	4540	8488	3639
6311	2025	5250	6099	6718	7539	9681
0476	1624	3470	1600	0675	3261	7749
5317	3903	6098	9438	3482	5505	5167
7474	8876	1918	9829	2061	6664	0391
7460	6800	1987	2758	0737	6880	1500
1002	1494	9972	3877	6104	4006	0477
5449	6891	9047	6297	1075	7762	8091
9453	0809	7151	9982	0411	1120	6129
0471	2725	7588	6573	0546	0110	6132
5469	2668	1996	2249	3857	6637	8010
2782	9603	1877	4159	9809	2570	4544
3129	7217	5020	3788	0853	9465	2186
7092	9885	3714	8557	7804	9524	6228
9566	0501	8352	1062	0634	2401	0379
5863	7000	1714	9276	7218	6922	1032
5881	9151	2321	3147	6755	2510	5759
6416	9939	9569	0439	1705	4680	9881
9568	3012	6316	9065	0710	2158	1639

Control Chart Constants

For Estimating Sigma			For X Chart		For X Chart (Standard Given)	For R Chart		For R Chart (Standard Given)		For s Chart (Standard Given)			
n	c_4	d_2	A_2	A_3	A	D_3	D_4	D_1	D_2	B_3	B_4	B_5	B_6
2	0.7979	1.128	1.880	2.659	2.121	0	3.267	0	3.686	0	3.267	0	2.606
3	0.8862	1.693	1.023	1.954	1.732	0	2.575	0	4.358	0	2.568	0	2.276
4	0.9213	2.059	0.729	1.628	1.500	0	2.282	0	4.698	0	2.266	0	2.088
5	0.9400	2.326	0.577	1.427	1.342	0	2.115	0	4.918	0	2.089	0	1.964
6	0.9515	2.534	0.483	1.287	1.225	0	2.004	0	5.078	0.030	1.970	0.029	1.874
7	0.9594	2.704	0.419	1.182	1.134	0.076	1.924	0.205	5.203	0.118	1.882	0.113	1.806
8	0.9650	2.847	0.373	1.099	1.061	0.136	1.864	0.387	5.307	0.185	1.815	0.179	1.751
9	0.9693	2.970	0.337	1.032	1.000	0.184	1.816	0.546	5.394	0.239	1.761	0.232	1.707
10	0.9727	3.078	0.308	0.975	0.949	0.223	1.777	0.687	5.469	0.284	1.716	0.276	1.669
15	0.9823	3.472	0.223	0.789	0.775	0.348	1.652	1.207	5.737	0.428	1.572	0.421	1.544
20	0.9869	3.735	0.180	0.680	0.671	0.414	1.586	1.548	5.922	0.510	1.490	0.504	1.470
25	0.9896	3.931	0.153	0.606	0.600	0.459	1.541	1.804	6.058	0.565	1.435	0.559	1.420

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PART

2 Formulas and Definitions

CHAPTER

1 Information Everywhere

The mean of a sample is defined as follows:

$$\bar{x} = \frac{\text{Sum of } x}{n} = (x_1 + x_2 \ldots + x_n)/n$$

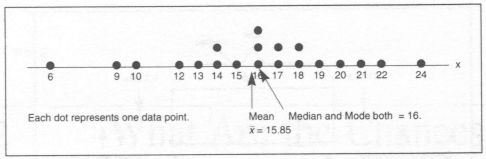
The mean of a sample:

$$\mu = \frac{\text{Sum of } x}{N}$$

The mean of the observations in Example 1.4 is found as

$$\bar{x} = (x_1 + x_2 + \cdots + x_{20})/20 = (9 + 6 + 12 + 10 + 13 + 15 + 16 + 14 + 14 + 16 + 17 + 16 + 24 + 21 + 22 + 18 + 19 + 18 + 20 + 17)/20 = 317/20 = 15.85$$

Mean, Median, and Mode **FIGURE 1.1**



A sample variance may be found by using the following formula:

$$s^2 = \frac{\text{Sum } (x_i - \bar{x})^2}{n - 1}$$

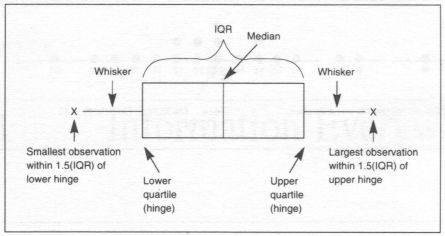
A sample standard deviation may be found by using the following equation:

$$s = \sqrt{s^2} = \sqrt{\frac{\text{Sum } (x_i - \bar{x})^2}{n - 1}}$$

Shortcut formula for the sample variance:

$$s^2 = \frac{\text{Sum } (x_i^2) - [\text{Sum } (x_i)]^2/n}{n - 1}$$

FIGURE 1.2 The Box Plot



CHAPTER

2 What Are the Chances?

PROBABILITY RULES

For any event, the probability $P(\mathrm{A})$ satisfies:

$$0 \le P(\mathrm{A}) \le 1$$

Probability of the complement:

$$P(\bar{\mathrm{A}}) = 1 - P(\mathrm{A})$$

The rule of unions:

$$P(\mathrm{A} \cup \mathrm{B}) = P(\mathrm{A}) + P(\mathrm{B}) - P(\mathrm{A} \cap \mathrm{B})$$

The **conditional probability** of event A given the occurrence of event B is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

assuming $P(B) \neq 0$.

Variations of the conditional probability formula:

$$P(A \cap B) = P(A|B)P(B)$$

and

$$P(A \cap B) = P(B|A)P(A)$$

INDEPENDENCE

Events A and B are said to be independent of each other if and only if the following three conditions hold.

Conditions for the **independence of two events** A and B:

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

and, most useful:

$$P(A \cap B) = P(A)P(B)$$

Bayes' theorem:

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})}$$

Extended Bayes' theorem:

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{\text{Sum}[P(A|B_i)P(B_i)]}$$

CHAPTER

3 Chance Quantities

DEFINITIONS

A **continuous random variable** is a random variable that can take on any value in an interval of numbers.

The probabilities associated with a continuous random variable X are determined by the **probability density function** of the random variable. The function, denoted $f(x)$, has the following properties.

1. $f(x) \geq 0$ for all x.
2. The probability that X will be between two numbers a and b is equal to the area under $f(x)$ between a and b.
3. The total area under the entire curve of $f(x)$ is equal to 1.00.

The **cumulative distribution function** of a continuous random variable:

$F(x) = P(X \leq x) =$ Area under $f(x)$ between the *smallest* possible value of X (often $-\infty$) and the point x.

FORMULAS

The **expected value** of a discrete random variable X is equal to the sum of all values of the random variable, each value multiplied by its probability:

$$\mu = E(X) = \text{Sum}[xP(x)]$$

The **variance of a discrete random variable** X is given by the following formula:

$$\sigma^2 = V(X) = E[(X - \mu)^2] = \text{Sum}\,(x - \mu)^2 P(x)$$

Computational formula for the variance of a random variable:

$$\sigma^2 = V(X) = E(X^2) - [E(X)]^2$$

Formula for the **standard deviation of a random variable:**

$$\sigma = SD(X) = \sqrt{V(X)}$$

Formula for the **binomial probability distribution**:

$$P(x) = \frac{n!}{x!(n - x)!} p^x q^{n-x}$$

where

p = the probability of success in a single trial
$q = 1 - p$
n = the number of trials
x = the number of successes
$n!$ = n-factorial, which is $n(n - 1)(n - 2) \ldots 1$

Mean of a binomial distribution:

$$\mu = E(X) = np$$

Variance of a binomial distribution:

$$\sigma^2 = V(X) = npq$$

Standard deviation of a binomial distribution:

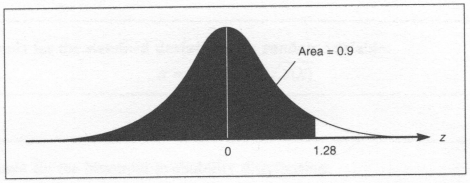
$$\sigma = SD(X) = \sqrt{npq}$$

CHAPTER

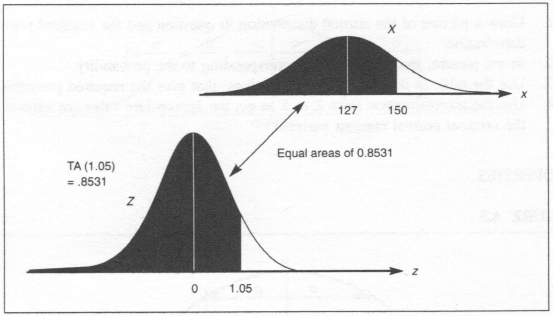
4 The Bell-Shaped Curve

TRANSFORMATIONS

FIGURE 4.1 Finding such that $P(Z \leq z) = 0.9$



Probability Computation for Example 4.2 **FIGURE 4.2**



Transformation formulas of X to Z, where a and b are numbers:

$$P(X < a) = P\left(Z < \frac{a-\mu}{\sigma}\right)$$

$$P(X > b) = P\left(Z > \frac{b-\mu}{\sigma}\right)$$

$$P(a < X < b) = P\left(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma}\right)$$

The normal transformation:

$$Z = \frac{X-\mu}{\sigma}$$

The inverse normal transformation:

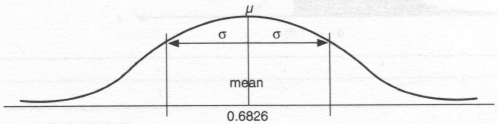
$$x = \mu + z\sigma$$

Summary of the procedure of obtaining values of a normal random variable, given a probability:

1. Draw a picture of the normal distribution in question and the standard normal distribution.
2. In the picture, shade in the area corresponding to the probability.
3. Use the table to find the z value (or values) that give the required probability.
4. Use the transformation from Z to X to get the appropriate value (or values) of the original normal random variable.

PROPERTIES

FIGURE 4.3



1. The probability that a normal random variable will be within a distance of *1 standard deviation* from its mean (on either side) is 0.6826, or *approximately 0.68.*
2. The probability that a normal random variable will be within *2 standard deviations* of its mean is .09544, or *approximately 0.95.*
3. The probability that a normal random variable will be within *3 standard deviations* of its mean is 0.9974.

FIGURE 4.4

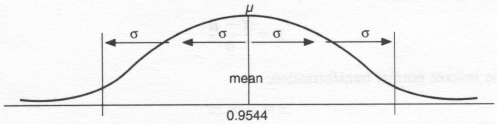
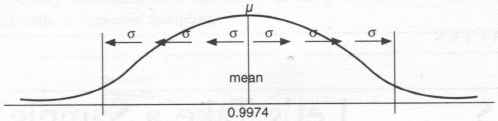


FIGURE 4.5



CHAPTER

5 Let's Take a Sample

CENTRAL LIMIT THEOREM

The *central limit theorem*: When sampling from a population with mean μ and finite standard deviation σ, the sampling distribution of the sample mean, $\bar{X}$, will tend to a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$ as the sample size n becomes large. For a large enough n,

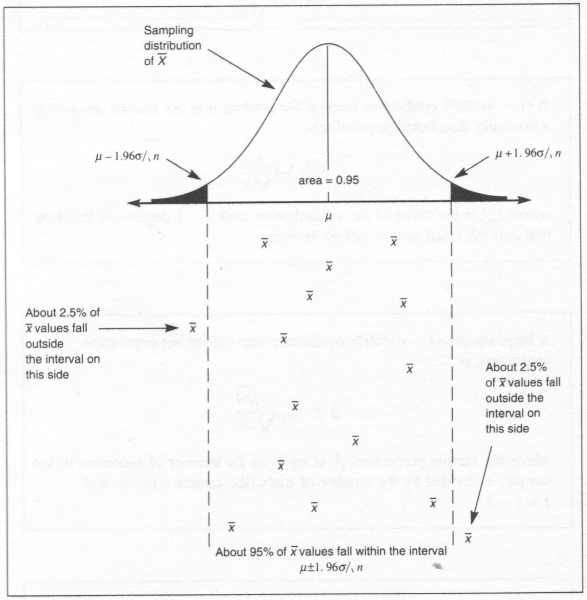
$$\bar{X} \text{ is } N(\mu, \sigma/\sqrt{n})$$

DEGREES OF FREEDOM

The number of **degrees of freedom** is equal to the total number of measurements (these are not always raw data points), less the total number of restrictions on the measurements. A *restriction* is a quantity computed from the measurements.

CONFIDENCE INTERVALS

The Probability Distribution of $\bar{X}$ and Some Resulting Values of the Statistic in Repeated Samplings **FIGURE 5.1**



A 95 percent confidence interval for μ when σ is known and sampling is done from a normal population, or a large sample is used:

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

A $(1 - \alpha)100\%$ confidence interval for μ when σ is known and sampling is done from a normal population, or with a large sample:

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

A $(1 - \alpha)100\%$ confidence interval for μ when σ is not known (assuming a normally distributed population):

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$$

where $t_{\alpha/2}$ is the value of the t distribution with $n - 1$ degrees of freedom that cuts off a tail area of $\alpha/2$ to its right.

A large-sample $(1 - \alpha)100\%$ confidence interval for the population proportion, p:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

where the sample proportion, $\hat{p}$, is equal to the number of successes in the sample, x, divided by the number of trials (the sample size), n, and $\hat{q} = 1 - \hat{p}$.

Minimum required sample size in estimating the population mean, μ:

$$n = \frac{z^2_{\alpha/2}\sigma^2}{D^2}$$

Minimum required sample size in estimating the population proportion, p:

$$n = \frac{z^2_{\alpha/2}\, pq}{D^2}$$

CHAPTER

6 Trial by Probability

CONDITIONS

FIGURE 6.1 The State of Nature, the Decision, and the Two Possible Errors

Decision	State of Nature: H_0	State of Nature: H_1
H_0	Correct decision	Type II error β
H_1	Type I error α	Correct decision

$\alpha = P(\text{Reject } H_0 \text{ when } H_0 \text{ is true})$
$\beta = P(\text{Not reject } H_0 \text{ when } H_0 \text{ is false})$

TESTING

The level of significance of a statistical hypothesis test is α, the probability of committing a Type I error.

The **rejection region** of a statistical hypothesis test is the range of numbers that will lead us to reject the null hypothesis in case the test statistic falls within this range.

The **nonrejection region** is the range of values (also determined by the critical points) that will lead us *not* to reject the null hypothesis if the test statistic should fall within this region.

TYPES OF TESTS

The elements of a two-tailed test for the population mean, μ:

The null hypothesis:	H_0: $\mu = \mu_0$
The alternative hypothesis:	H_1: $\mu \neq \mu_0$
The significance level of the test:	α (often, $\alpha = 0.05$ or 0.01)
The test statistic:	$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ (assuming σ is known; otherwise, with large n, use $s/\sqrt{n}$ in the denominator)
The critical points:	These depend on α. They are the bounds $\pm z_{\alpha/2}$ that capture between them an area of $1 - \alpha$. (When $\alpha = 0.05$, the critical points are ± 1.96; when $\alpha = 0.01$, the critical points are ± 2.576. For other values of α, the critical points may be obtained from the standard normal table.)
The decision rule:	Reject the null hypothesis if either $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$, where $z_{\alpha/2}$ and $-z_{\alpha/2}$ are the two critical points.

The elements of a right-tailed test for the population mean, μ:

The null hypothesis:	H_0: $\mu \leq \mu_0$
The alternative hypothesis:	H_1: $\mu > \mu_0$
The significance level of the test:	α (often, $\alpha = 0.05$ or 0.01)
The test statistic:	$z = \dfrac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$ (assuming σ is known; otherwise, for large n, use $s/\sqrt{n}$ in the denominator)
The critical point:	This depends on α. It is the bound z_α that captures an area of α to its right. (When $\alpha = 0.05$, the critical point is 1.645; when $\alpha = 0.01$, the critical point is 2.326. For other values of α, the critical point may be obtained from the standard normal table.)
The decision rule:	Reject the null hypothesis if $z > z_\alpha$ where z_α is the critical point.

SIGNIFICANCE

Critical Points of Z for Selected Levels of Significance

	Level of Significance α:		
	0.10	**0.05**	**0.01**
One-Tailed Test	+ or 1.28 −	+ or 1.645 −	+ or 2.326 −
Two-Tailed Test	+ and 1.645 −	+ and 1.96 −	+ and 2.576 −

Note: + or − means we use the positive value in the table for a right-tailed test and the negative value for a left-hand-tailed test. In a two-tailed test, we use both the positive and the negative values.

- When the p-value is *smaller than 0.01,* the result is called *very significant.*
- When the p-value is *between 0.01 and 0.05,* the result is called *significant.*

- When the p-value is *between 0.05 and 0.10,* the result is considered by some as *marginally significant* (and by others as not significant).
- When the p-value is *greater than 0.10,* the result is considered by most as *not significant.*

The further away in the tail of the distribution our test statistic falls, the smaller is the p-value and, hence, the more convinced we are that the null hypothesis is false and should be rejected.

In a two-tailed test, we find the p-value by *doubling* the area in the tail of the distribution beyond the value of the test statistic.

The test statistic in tests for μ when σ is unknown:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

When the population is normally distributed and the null hypothesis is true, the test statistic has a t distribution with $n - 1$ degrees of freedom.

Large-sample test statistic for the population proportion, p:

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}}$$

where $q_0 = 1 - p_0$.

SUMMARY

In a right-tailed test, the p-value is the area to the right of the test statistic if the test statistic is positive.

In a left-hand-tailed test, the p-value is the area to the left of the test statistic if the test statistic is negative.

In a two-tailed test, the p-value is twice the area to the right of a positive test statistic or to the left of a negative test statistic.

For a quick interpretation of reported p-values you may want to use the following rule.

For a given level of significance, α:
Reject the null hypothesis if and only if $\alpha \geq p$-value.

POWER
The **power** of a statistical hypothesis test is the probability of rejecting the null hypothesis when the null hypothesis is false.

The factors of **power** are

1. The power depends on the distance between the value of the parameter under the null hypothesis and the *true value* of the parameter in question. *The greater this distance, the greater the power.*
2. The power depends on the population standard deviation. *The smaller the population standard deviation, the greater the power.*
3. The power depends on the sample size used. *The larger the sample, the greater the power.*
4. The power depends on the level of significance of the test. *The smaller the level of significance, α, the smaller the power.*

CHAPTER

7 Making Comparisons

1. Test statistic for the paired-observations t test:

$$t = \frac{\bar{D} - \mu_{D_0}}{s_D/\sqrt{n}}$$

where $\bar{D}$ is the sample average difference between each pair of observations, s_D is the sample standard deviation of these differences, and the sample size, n, is the number of pairs of observations. The symbol μ_{D_0} is the population mean difference under the null hypothesis. When the null hypothesis is true and the population mean difference is μ_{D_0}, the statistic has a t distribution with $n - 1$ degrees of freedom.

A $(1 - \alpha)100\%$ confidence interval for the mean difference μ_D:

$$\bar{D} \pm t_{\alpha/2}\frac{s_D}{\sqrt{n}}$$

where $t_{\alpha/2}$ is the value of the t distribution with $n - 1$ degrees of freedom that cuts off an area of $\alpha/2$ to its right. When the sample size, n, is large, we may use $z_{\alpha/2}$ instead.

2.

A test statistic for the difference between two population means:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

The term $(\mu_1 - \mu_2)_0$ is the difference between μ_1 and μ_2 under the null hypothesis. It is equal to zero in situations I and II, and to the prespecified value D in situation III. The term in the denominator is the standard deviation of the difference between the two sample means. (It relies on the assumption that the two samples are independent.) If the population variances, σ_1^2 and σ_2^2 are unknown, they are replaced by the sample variances, s_1^2 and s_2^2. For large samples, the standard normal distribution may be assumed.

A $(1 - \alpha)100\%$ confidence interval for the difference between two population means, $\mu_1 - \mu_2$, using independent random sampling:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

If σ_1^2 and σ_2^2 are unknown, they are replaced by s_1^2 and s_2^2. When sample sizes are small, an approximate t distribution applies, as discussed earlier.

A **pooled estimate** of the common population variance, based on a sample variance s_1^2 from a sample of size n_1 and a sample variance s_2^2 from a sample of size n_2, is given by:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

The estimate of the standard error of $(\bar{X}_1 - \bar{X}_2)$ is given by:

$$\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

3.

Test statistic for the difference between two population means, assuming equal population variances:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $(\mu_1 - \mu_2)_0$ is the difference between the two population means under the null hypothesis (zero or some number, D). The degrees of freedom of the test statistic t are $n_1 + n_2 - 2$ (the degrees of freedom associated with s_p^2, the pooled estimate of the population variance).

A $(1 - \alpha)100\%$ confidence interval for $(\mu_1 - \mu_2)$, assuming equal population variances:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

4.

Test statistic for the difference between two population proportions where the null-hypothesized difference is zero:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\hat{p}_1 = x_1/n_1$ is the sample proportion in sample 1 and $\hat{p}_2 = x_2/n_2$ is the sample proportion in sample 2. The symbol $\hat{p}$ stands for the *combined sample proportion in both samples,* considered as a single sample. That is,

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

A large-sample $(1 - \alpha)100\%$ confidence interval for the difference between two population proportions:

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$$

The hypothesis test of analysis of variance:

$$H_0\text{: } \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_r$$

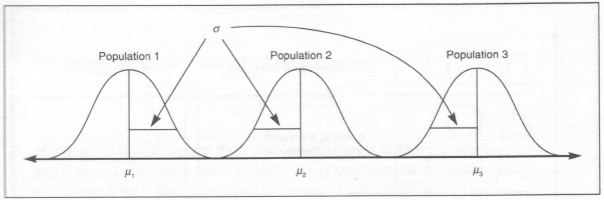
$$H_1\text{: Not all } \mu_i\ (i = 1, \ldots, r) \text{ are equal}$$

ANOVA ASSUMPTIONS

The required assumptions of ANOVA:

1. We assume *independent random sampling* from each of the r populations.
2. We assume that the r populations under study are *normally distributed,* with means μ_i that may or may not be equal, but with *equal variances,* σ^2.

Three Normally Distributed Populations with Different Means but Equal Variance **FIGURE 7.1**



ANOVA test statistic = $F_{(r-1,n-r)}$

The Distribution of the ANOVA Test Statistic for $r = 4$ Populations and a Total Sample Size $n = 54$ **FIGURE 7.2**

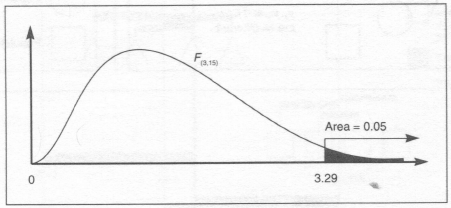
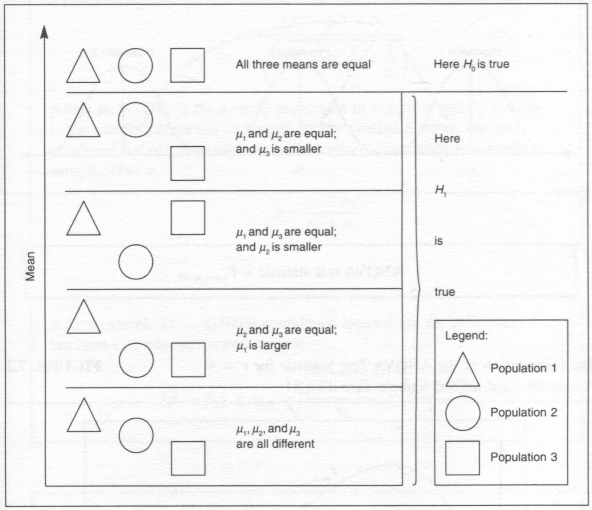


FIGURE 7.3 The Possible Relationships among the Relative Magnitudes of the Three Population Means μ_1, μ_2, μ_3



The mean of sample i ($i = 1, \ldots, r$):

$$\overline{X}_i = \frac{\text{sum } s_{ij}}{n_i}$$

The **grand mean,** the mean of all the data points:

$$\overline{X} = \frac{\text{sum } x_{ij}}{n}$$

where x_{ij} is the particular data point in position j within the sample from population i. The subscript i denotes the population, or treatment, and runs from 1 to r. The subscript j denotes the data point within the sample from population i; thus, j runs from 1 to n.

FIGURE 7.4 The Deviations of the Triangles, Squares, and Circles from Their Sample Means and the Deviation of the Sample Means from the Grand Mean

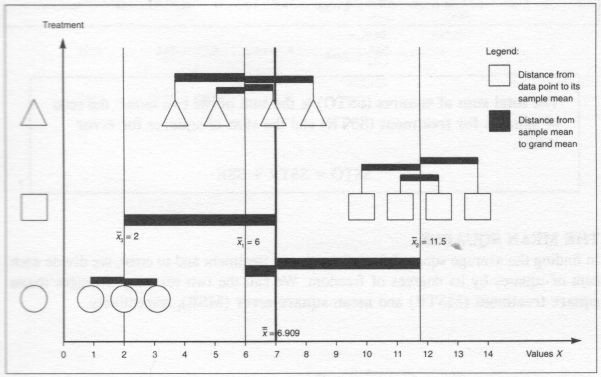
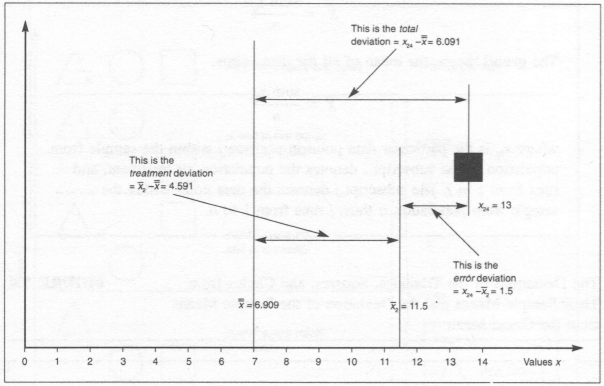


FIGURE 7.5 The *Total Deviation* as the Sum of the Treatment Deviation and the Error Deviation for a Particular Data Point



The **total sum of squares (SSTO)** is the sum of the two terms: the **sum of squares for treatment (SSTR)** and the **sum of squares for error (SSE):**

$$\text{SSTO} = \text{SSTR} + \text{SSE}$$

THE MEAN SQUARES

In finding the average squared deviations due to treatment and to error, we divide each sum of squares by its degrees of freedom. We call the two resulting averages **mean square treatment (MSTR)** and **mean square error (MSE),** respectively.

$$\text{MSTR} = \frac{\text{SSTR}}{r-1}$$

$$\text{MSE} = \frac{\text{SSE}}{n-r}$$

Under the assumptions of ANOVA, the ratio MSTR/MSE possesses an F distribution with $r - 1$ degrees of freedom for the numerator and $n - r$ degrees of freedom for the denominator when the null hypothesis is true. Hence, the test statistic in analysis of variance is:

$$F_{(r-1,n-r)} = \frac{\text{MSTR}}{\text{MSE}}$$

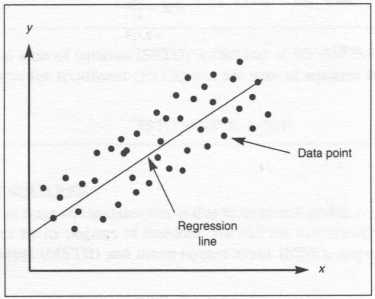
TABLE 8.4 ANOVA Table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Ratio	p-Value
Treatment	SSTR = 159.9	$r - 1 = 2$	$\text{MSTR} = \frac{\text{SSTR}}{r-1} = 79.95$	$F = \frac{\text{MSTR}}{\text{MSE}} = 37.62$	0.000
Error	SSE = 17.0	$n - r = 8$	$\text{MSE} = \frac{\text{SSE}}{n-r} = 2.125$		
Total	SSTO = 176.9	$n - 1 = 10$			

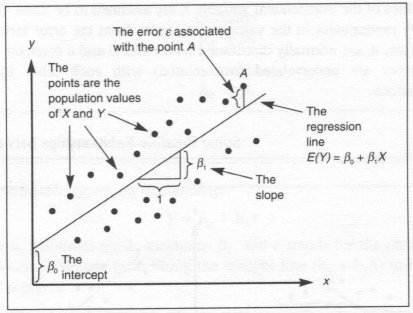
CHAPTER

8 Making Extended Comparisons

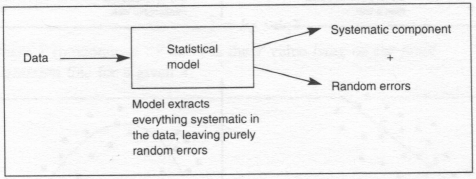
FIGURE 8.1 Simple Linear Regression



The Population Regression Line **FIGURE 8.2**



A Statistical Model **FIGURE 8.3**



The population simple linear regression model:

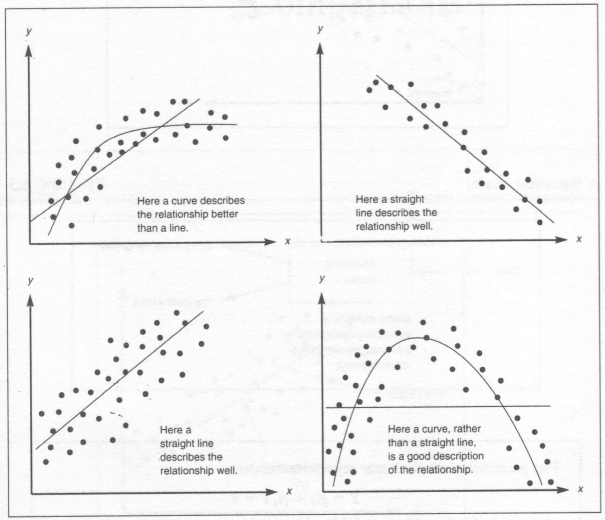
$$Y = \beta_0 + \beta_1 X + \epsilon$$

where Y is the dependent variable, the variable we wish to explain or predict; X is the independent variable, also called the *predictor* variable; and ϵ is the error term, the only random component in the model and thus the only source of randomness in Y.

Model assumptions:

1. The relationship between X and Y is a straight-line relationship.
2. The values of the independent variable X are assumed to be fixed (not random); the only randomness in the values of Y comes from the error term, ϵ.
3. The errors, ϵ, are normally distributed with mean 0 and a constant variance σ^2. The errors are uncorrelated (not related) with each other in successive observations.

FIGURE 8.4 Some Possible Relationships between X and Y



The least-squares estimators:

estimates
$b_0 \rightarrow \beta_0$

estimates
$b_0 \rightarrow \beta_1$

The estimated regression relationship:

$$Y = b_0 + b_1X + e$$

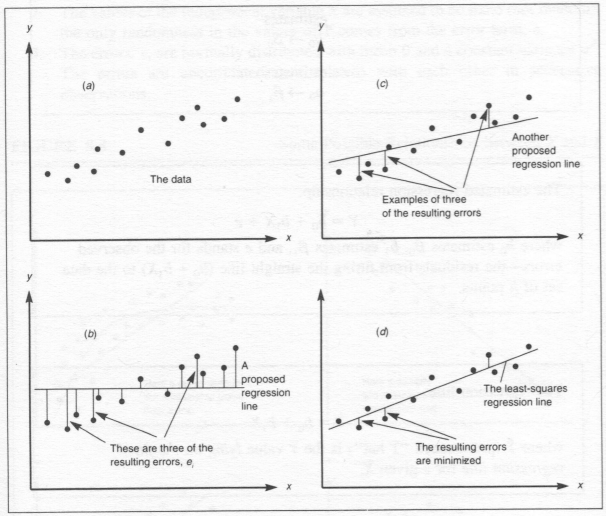
where b_0 estimates β_0, b_1 estimates β_1, and e stands for the observed errors—the residuals from fitting the straight line $(b_0 + b_1X)$ to the data set of n points.

The regression line:

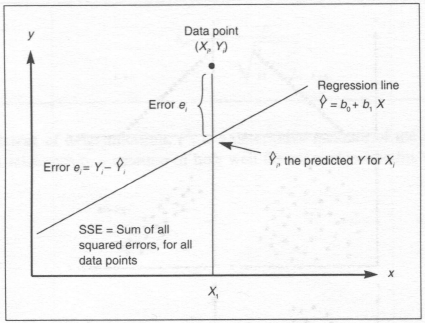
$$\hat{Y} = b_0 + b_1X$$

where $\hat{Y}$ (pronounced "Y hat") is the Y value *lying on the fitted regression line* for a given X.

FIGURE 8.5 Proposed Straight Lines to Describe the Data



The Regression Errors Leading to SSE **FIGURE 8.6**



CORRELATION

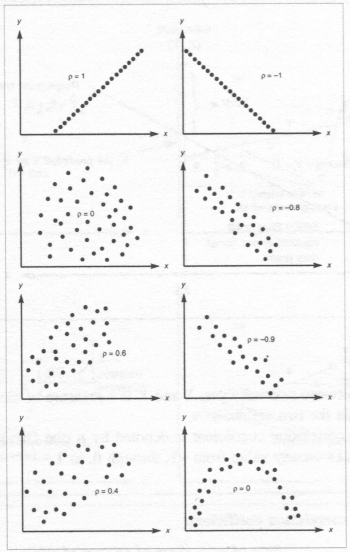
The **correlation** between two variables, X and Y, is a measure of the *degree of linear association* between the two variables.

The population correlation coefficient is denoted by ρ (the Greek letter rho). The coefficient ρ can take on any value from −1, through 0, to 1.

The **sample correlation coefficient:**

$$r = \frac{\text{Sum of } xy - (\text{Sum of } x)(\text{Sum of } y)/n}{\sqrt{(\text{Sum of } x^2 - (\text{Sum of } x)^2/n)(\textit{Sum of } y^2 - (\text{Sum of } y)^2/n)}}$$

FIGURE 8.7 Several Possible Correlations between Two Variables



TESTS FOR CORRELATION

The most common test is a test of whether or not two random variables, X and Y, are correlated. The hypothesis test is

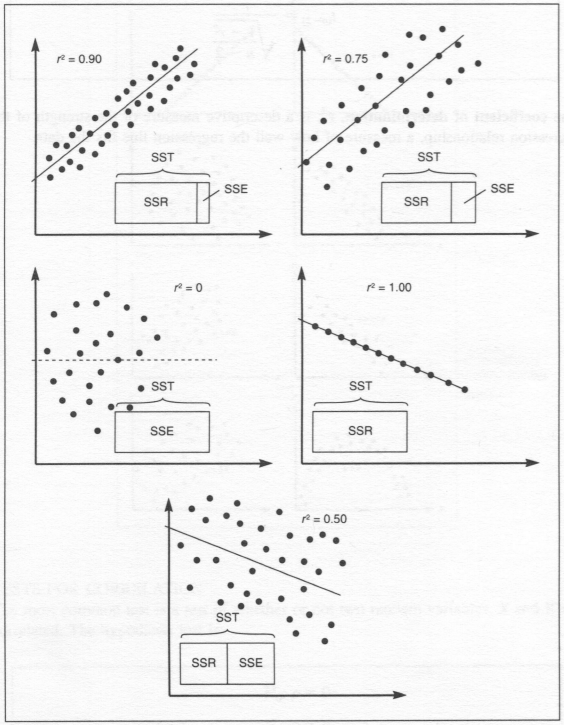
$$H_0: \rho = 0$$
$$H_1: \rho \neq 0$$

The test statistic for this particular test is

$$t_{(n-2)} = \frac{r}{\sqrt{\dfrac{1 - r^2}{n - 2}}}$$

The **coefficient of determination, r^2,** is a descriptive measure of the strength of the regression relationship, a measure of how well the regression line fits the data.

FIGURE 8.8 The Value of the Coefficient of Determination in Different Regressions



CHAPTER

10 The Quest for Quality

Process in, and out of, Statistical Control **FIGURE 10.1**

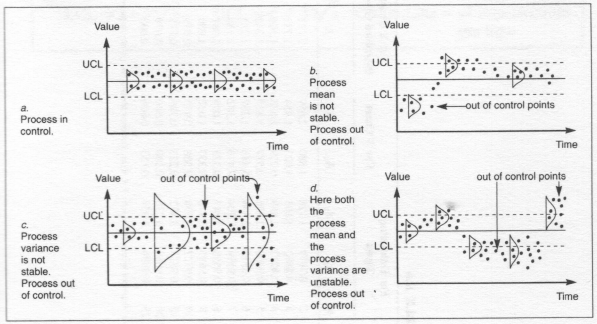


TABLE B.6

	For Estimating Sigma		For X Chart		For X Chart (Standard Given)	For R Chart		For R Chart (Standard Given)		For s Chart (Standard Given)		
n	c_4	d_2	A_2	A_3	A	D_3	D_4	D_1	D_2	B_3	B_4	B_5	B_6
2	0.7979	1.128	1.880	2.659	2.121	0	3.267	0	3.686	0	3.267	0	2.606
3	0.8862	1.693	1.023	1.954	1.732	0	2.575	0	4.358	0	2.568	0	2.276
4	0.9213	2.059	0.729	1.628	1.500	0	2.282	0	4.698	0	2.266	0	2.088
5	0.9400	2.326	0.577	1.427	1.342	0	2.115	0	4.918	0	2.089	0	1.964
6	0.9515	2.534	0.483	1.287	1.225	0	2.004	0	5.078	0.030	1.970	0.029	1.874
7	0.9594	2.704	0.419	1.182	1.134	0.076	1.924	0.205	5.203	0.118	1.882	0.113	1.806
8	0.9650	2.847	0.373	1.099	1.061	0.136	1.864	0.387	5.307	0.185	1.815	0.179	1.751
9	0.9693	2.970	0.337	1.032	1.000	0.184	1.816	0.546	5.394	0.239	1.761	0.232	1.707
10	0.9727	3.078	0.308	0.975	0.949	0.223	1.777	0.687	5.469	0.284	1.716	0.276	1.669
15	0.9823	3.472	0.223	0.789	0.775	0.348	1.652	1.207	5.737	0.428	1.572	0.421	1.544
20	0.9869	3.735	0.180	0.680	0.671	0.414	1.586	1.548	5.922	0.510	1.490	0.504	1.470
25	0.9896	3.931	0.153	0.606	0.600	0.459	1.541	1.804	6.058	0.565	1.435	0.559	1.420

Source: Reprinted by permission from T. P. Ryan, *Statistical Methods for Quality Improvement* (New York: John Wiley and Sons, 1989).

FORMULAS

Upper and lower control limits for the process mean:

$$\text{LCL} = \overline{X} - A_2\overline{R} \qquad \text{UCL} = \overline{X} + A_2\overline{R} \qquad \text{Center Line} = \overline{X} = \frac{\text{Sum of sample means}}{\text{number of samples}}$$

Upper and lower control limits for the process range:

$$\text{LCL} = D_3\overline{R} \qquad \text{UCL} = D_4\overline{R} \qquad \text{Center Line} = \overline{R} = \frac{\text{Sum of group ranges}}{\text{number of groups}}$$

$$\text{LCL} = B_3\overline{s} \qquad \text{UCL } B_4\overline{s} \qquad \text{Center Line} = \frac{\text{Sum of group standard deviations}}{\text{number of groups}}$$

$$\text{LCL} = \overline{p} = 3\sqrt{\overline{p}(1-\overline{p})/n} \quad \text{UCL} = \overline{p} - 3\sqrt{\overline{p}(1-\overline{p})/n}$$

Center Line = $\overline{p}$ = proportion of defectives in combined, overall sample.

Upper and lower control limits for the number of defects:

$$\text{LCL} = \overline{c} - 3\sqrt{\overline{c}} \qquad \text{UCL} = \overline{c} + 3\sqrt{\overline{c}} \qquad \text{Center Line} = \text{average number of imperfections per item}$$

Statistics

Pocket Guide

Amir D. Aczel