

CLASSICAL THEORY OF DECOMPRESSION AND THE DESIGN OF SCUBA DIVING TABLES

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Abstract

We summarise the classical (gas diffusion) theory of decompression, which is an interesting application of elementary differential equations. We show that the derivation of recreational scuba diving tables from this theory is an ill-defined problem in optimisation.

Keywords: compartment models; Dalton's Law; diffusion; Fick's Law; Lagrange multipliers; optimisation; ordinary differential equations.

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1. Introduction

All scuba divers are trained to follow procedures that limit the time spent at depth, in order to minimise the risk of decompression sickness. These procedures, encoded either on waterproof tables or in the algorithm of a diving computer, are based on a mathematical theory that is relatively simple in structure. Classical decompression theory, first developed by J.S. Haldane [4, 6], describes the diffusion of nitrogen in the diver's body by a simple independent-compartment model, consisting of a small system of ordinary differential equations governed by some simple constraints.

As millions of young people have now been certified as scuba divers [9, p. 32],

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educators may find it useful to give decompression theory as a motivating example in calculus classes. We shall see that much of the analysis requires little more than careful consideration of the solutions of relatively straightforward differential systems.

In addition to explaining classical decompression theory, we aim to show how the design of optimal diving procedures is a rather ill-defined process which is worthy of further study. A day of recreational diving often involves successive dives, to different depths, separated by a rest period at the surface (known as the surface interval). The design of recreational diving tables can be formulated as an optimisation problem. Existing tables are often protected by commercial secrecy, and we are not aware of any detailed discussion of table design in the open literature.

The remainder of the article is set out as follows. In §2 we consider some of the basic theory behind the constraints that limit the duration of a dive. This sets the parameters of the problem and enables the subsequent calculations to be considered in the following sections. We investigate two relatively simple cases; in §3 we examine the limits to the duration of a single no-decompression dive. This analysis is extended in §4 to considering the optimal plan for two dives separated by a prescribed surface interval. We close with a short discussion.

2. The basic theory

The definitive modern text on diving physiology and medicine is that by Bennett & Elliott [5]; decompression theory is covered in a chapter by Tikvisis & Gerth [10] which should be consulted for technical references. Accessible, popular introductions to diving physiology are given by Bookspan [3], Martin [8] and Lippmann [7]. What follows is a simplified summary.

Scuba equipment is designed to deliver the breathing air to the diver at a pressure equal to that of the ambient water. The classical explanation for decompression sickness [2] is that, at this increased pressure, nitrogen in the breathing air diffuses into the diver's body. When the diver returns to the surface, the ambient pressure is

reduced and the body now contains an excess of nitrogen. Nitrogen can diffuse from the body tissues back into the lungs where it is exhaled. However if the quantity of dissolved nitrogen in a tissue exceeds a certain critical value, nitrogen will come out of solution and form bubbles of nitrogen gas, either in the blood or in other tissues. The presence of these bubbles leads to decompression sickness and other ills. For a popular demonstration, take two bottles of carbonated soft drink: opening the first bottle rapidly leads to the formation of large bubbles, while opening the second bottle slowly does not.

2.1. The diffusion of nitrogen

The ambient water pressure p increases linearly with depth so at a distance d beneath the surface it is $p = p_0 + \lambda d$ atmospheres, where p_0 is the pressure at the water surface. Typically $p_0 = 1$ atmosphere at sea level and λ is approximately 0.1 atmospheres per metre in sea water; in other words at a depth of 10 metres the water pressure is roughly twice the sea-level surface pressure p_0 and increases by one atmosphere for every further 10 metres descended. Since air is composed of approximately 21% oxygen and 79% nitrogen, the nitrogen component of the diver's breathing air exerts a partial pressure of $0.79p$ atmospheres according to Dalton's Law. More generally we may consider dives in lakes at higher altitudes (that is a reduced surface pressure p_0) or using a breathing gas which has a different fraction μ of nitrogen. Naturally the partial pressure of nitrogen in the breathing gas is then simply μp .

Nitrogen diffuses from the breathing gas in the lung air space into the diver's blood, and then in turn, into other bodily tissues. The diffusion from air into blood occurs so quickly that it can be treated as a virtually instantaneous process and the blood then effectively also carries a nitrogen partial pressure of μp atmospheres. This diffuses slowly into other bodily tissues and in the classical theory it is assumed there are a finite number of these, say m , each connected independently to the blood. If we label these tissues i with $i = 1, \dots, m$, and suppose that the nitrogen saturation (or tension) in tissue i is x_i atmospheres, then diffusion is assumed to satisfy Fick's law. This asserts

that the rate of diffusion of nitrogen from the blood into the tissue is proportional to the difference in their respective nitrogen concentrations. Mathematically, this means that the time derivative of nitrogen tension is simply

$$\dot{x}_i = -k_i(x_i - \mu p) \quad (1)$$

where the value of the diffusion constant k_i depends on the characteristics of tissue i . Thus the complete state of the body can be summarised in the form of a vector $\mathbf{x} = (x_1, \dots, x_m)$ of tissue nitrogen tensions governed by independent diffusion equations.

The constants k_i in equation (1) are usually quoted in the literature in terms of ‘half-times’ τ_i defined to be the time elapsed for an initial saturation $x_i = x_i(0)$ to be reduced by a factor two when the surrounding ambient pressure $p = 0$. It is easy to show that $\tau_i = \ln(2)/k_i$; for human tissues these half-times typically lie in the range from about 10 minutes to as long as 6 hours. The original model formulated by Haldane [4, 6] consisted of five tissues (or compartments) with halftimes between 5 and 75 minutes; a more up-to-date example with eight compartments is the DSAT model with parameters as listed in Table 1.

compartment i	1	2	3	4	5	6	7	8
halftime τ_i (minutes)	5	10	20	30	40	60	80	120
saturation $M_{0,i}$ (atm)	3.035	2.533	2.049	1.830	1.707	1.576	1.507	1.438

TABLE 1: Typical halftime and surfacing M -values for the eight compartments used in the DSAT model.

Classical decompression theory is founded on the assumption that each tissue i can tolerate a maximum nitrogen tension of $M_i(p)$ before nitrogen bubbles begin to form. This critical tension, commonly referred to as the M -value, is a function of the ambient pressure p and increases with p . In the original Haldane model [4] it was assumed that, for a diver breathing compressed air, the critical nitrogen tension is simply twice the partial pressure of nitrogen in the breathing air at ambient pressure; this then implies that $M_i(p) = 1.58p$ for all compartments i . In more sophisticated models, the critical nitrogen tension is taken to be a linear function of p with coefficients that depend on

the properties of the particular tissue under investigation. In this case

$$M_i(p) = M_{0,i} + (p - 1)M'_i \quad (2)$$

where $M_{0,i}$ is the maximum nitrogen saturation that can be tolerated at a pressure of one atmosphere; this is often known as the “surfacing M-value” when the water surface is at sea level. Table 1 includes representative surfacing M -values extracted from Bookspan [3, pp. 16, 23].

2.2. Dive planning constraints

Before descending it is normal practice for the diving party to agree on the duration and purpose of the impending dive. This ‘dive plan’ specifies the intended depth $d(t)$ as a function of time t during a dive and generally is one of two basic kinds. The first, a *no-decompression* dive, is one which, in theory, can be aborted at any time without requiring special decompression procedures. For this to be possible, it has to be ensured that the nitrogen saturation in each of the diver’s tissue compartments never exceeds the maximum nitrogen saturation that can be tolerated at sea level: this demands that

$$x_i(t) \leq M_i(p_0) \quad \text{for all } t. \quad (3)$$

In contrast, on a *decompression* dive, the diver may not be able to ascend immediately to the surface at any time: obligatory decompression stops are required during the ascent. This more complicated type of dive plan satisfies only the minimal requirement for avoiding decompression sickness that the nitrogen saturation in each tissue is less than the maximum saturation appropriate to the present depth. This then imposes the weaker requirement that

$$x_i(t) \leq M_i(p(t)) = M_i(p_0 + \lambda\mu d(t)) \quad \text{for all } t \quad (4)$$

and, as we have already remarked that $M_i(p)$ is frequently taken to be a linear function of p , the constraint (4) becomes linear in depth so

$$x_i(t) \leq (M_{0,i} + (p_0 - 1)M'_i) + \lambda\mu M'_i d(t) \quad \text{for all } t. \quad (5)$$

It is worth noting that the Haldane model was developed and used for planning staged decompression dives in military and commercial circumstances, rather than no-decompression dives which are more common for recreational purposes.

3. No-decompression limits

3.1. Theory

A simple optimization problem is to determine the “no-decompression limit” for a given depth D . Put simply, this is the maximum permissible duration T of a no-decompression dive to a constant depth D , starting and ending at the surface. To a first approximation it may be assumed that ascent and descent occurs quickly, so that the dive profile can be idealised as a simple square form, $d(t) = D$ if $0 < t < T$ and $d(t) = 0$ otherwise. For the purposes of the analysis it is assumed that the diver is fresh, that is, they have not had recent exposure to changes in pressure; their initial state $\mathbf{x}(0)$ is in equilibrium with air at sea level. Then $x_i(0) = 0.79$ atmospheres for all compartments i although for planning multiple dives during a day the no-decompression limit for a diver is critically dependent on their particular recent diving history.

Thus we seek to maximise T subject to the constraints

$$x_i(t) \leq M_i(p_0) \quad \text{for all } t$$

where

$$\dot{x}_i = -k_i(x_i - \mu p(t))$$

with initial condition $x_i(0) = 0.79$ unless otherwise stated. Writing $P = p_0 + \lambda D$ for

the ambient pressure at depth D , and $Q = \mu P = \mu p_0 + \mu \lambda D$ for the nitrogen partial pressure at this depth, we have

$$\dot{x}_i = -k_i x_i + k_i Q.$$

This first order DE has solution

$$x_i = Q + A_i e^{-k_i t}$$

where the constant A_i is determined by the initial value $x_i(0)$ yielding

$$x_i(t) = Q + (x_i(0) - Q)e^{-k_i t}. \quad (6)$$

Since $x_i(t)$ is monotonically increasing, the maximum permitted nitrogen tension $M_i(p_0)$ is reached at time

$$T_i = -\frac{1}{k_i} \ln \left[\frac{Q - M_i(p_0)}{Q - x_i(0)} \right] \quad (7)$$

provided $Q \geq M_i(p_0)$; otherwise this limit is never reached.

Thus, let $\text{NDL}(D, \mathbf{x})$ be the no-decompression time limit for a square dive to depth D for a diver with initial state vector \mathbf{x} . Then we have shown that

$$\text{NDL}(D, \mathbf{x}) = -\min_i \frac{1}{k_i} \ln \left[\frac{Q - M_i(p_0)}{Q - x_i(0)} \right] \quad (8)$$

where the minimum is taken over those i such that $M_i(p_0) \leq Q$, where $Q = \mu p_0 + \mu \lambda D$.

In particular, for a dive at sea level ($p_0 = 1$) on air ($\mu = 0.79$) by a fresh diver ($x_i(0) = 0.79$) we have $M_i(p_0) = M_{0,i}$, $Q = \mu(1 + \lambda D)$ and $Q - x_i(0) = \mu \lambda D$ in (8).

We define the ‘*controlling tissue*’ for a no-decompression dive to be the tissue index i which achieves the minimum in (8).

3.2. Calculations

The no-decompression limits depend crucially on the behaviour of the surfacing M -values. In the Haldane model, $M_{0,i} = 1.58$ for all i , so that for a fresh diver, the expression

$$\ln \left[\frac{Q - M_i(p_0)}{Q - x_i(0)} \right]$$

is equal for all i . Hence the minimum in (8) is achieved by taking k_i as large as possible, and the no-decompression limit for a fresh diver (at sea level using compressed air) is always dictated by the compartment with the shortest halftime. The resulting no-decompression limits are shown in Table 2.

Depth (m)	NDL (minutes)		Controlling tissue i
	Haldane	DSAT	
10	∞	275.5	7
12	12.9	152.8	6
14	9.0	102.0	5
16	7.1	74.6	5
18	5.8	56.9	4
20	5.0	46.0	3
22	4.4	37.2	3
24	3.9	31.5	3
26	3.5	27.2	2
28	3.2	22.4	2
30	2.9	19.2	2

TABLE 2: Comparison of theoretical no-decompression limits (NDL) under the Haldane and DSAT models and calculated using (8). The rightmost column identifies the tissue compartment i in the DSAT model which fixes the NDL. The calculations assume that the diver is fresh, breathes compressed air, and the surface is at sea-level.

For the DSAT model (given in Table 1), the surfacing M -values decrease with i , and range from $M_{0,1} = 3.03 > 1.58$ for the fastest compartment to $M_{0,8} = 1.43 < 1.58$ for the slowest. These lead to no-decompression limits (NDLs) which are much longer than the Haldane predictions and, perhaps surprisingly, it is not always the fastest compartment which fixes the upper limit on the duration of the dive. The sample results in Table 2 suggest that at relatively shallow depths the slow compartments determine the length of the dive and the expected importance of the faster compartments

only becomes apparent for deeper excursions.

It is easy to show that the NDL is a decreasing function of depth and much of its attraction lies in the conservative assumptions applied for its calculation. We have mentioned already that when finding the NDL the diver is supposed to reach his maximum depth D immediately, and stay at that depth for the entire duration of the dive. In practice of course the diver cannot descend and ascend instantaneously but it can be shown that the set of all no-decompression dives is ‘monotone’ in the sense that if $d_1(t)$ is a no-decompression dive and $d_2(t) \leq d_1(t)$ for all t , then $d_2(t)$ is also a no-decompression dive. Hence any dive with a maximum depth of D and maximum duration $t \leq \text{NDL}(D)$ is guaranteed to be a no-decompression one and the diver can proceed safe in the knowledge that decompression will not be required.

It may be of interest to determine how the predicted NDL is affected should the descent and/or ascent be specified. Divers frequently change from one depth to another at a steady speed and it turns out that the governing equation for $x_i(t)$ can still be solved explicitly if the depth is taken to be a linear function of time. Further elaboration can be introduced by assuming that the diver is not fresh and has dived previously on the same day and we consider this now.

4. Optimal planning for two no-decompression dives

4.1. Double dives

Recreational divers normally plan two dives for the day separated by a period at the surface. Suppose the first dive has a square profile to a depth d_1 metres for time t_1 minutes, followed by a surface interval of s minutes, and then a second dive which is also a square profile to depth d_2 for time t_2 minutes. This simple profile is illustrated in Figure 1.

In all that follows we shall assume that the depths d_1, d_2 and the surface interval s are all fixed, and the task is to optimise (some function of) the dive durations t_1, t_2 subject to the no-decompression constraint.

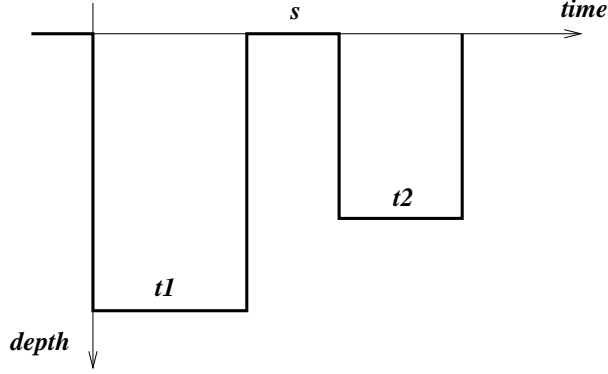


FIGURE 1: A typical double dive plan profile

Let us assume that the diver begins fresh; then by equation (6), on surfacing from the first dive, the diver has tissue states

$$x_i(t_1) = \mu p_0 + \mu \lambda d_1 (1 - e^{-k_i t_1}).$$

While back on the surface, tissues desaturate according to

$$\dot{x}_i = -k_i x_i + k_i \mu p_0$$

so that at the end of the surface interval of s minutes and immediately before the second dive,

$$\begin{aligned} x_i(t_1 + s) &= x_i(t_1) e^{-k_i s} + \mu p_0 (1 - e^{-k_i s}) \\ &= [\mu p_0 + \mu \lambda d_1 (1 - e^{-k_i t_1})] e^{-k_i s} + \mu p_0 (1 - e^{-k_i s}) \\ &= \mu p_0 + \mu \lambda d_1 (1 - e^{-k_i t_1}) e^{-k_i s}. \end{aligned} \tag{9}$$

During the second dive, tissues again take in nitrogen, according to

$$\dot{x}_i + k_i x_i = k_i \mu (p_0 + \lambda d_2)$$

with general solution

$$x_i = C_i e^{-k_i t} + \mu(p_0 + \lambda d_2)$$

so that on surfacing again

$$\begin{aligned} x_i(t_1 + s + t_2) &= x_i(t_1 + s) e^{-k_i t_2} + \mu(p_0 + \lambda d_2)(1 - e^{-k_i t_2}) \\ &= [\mu p_0 + \mu \lambda d_1 (1 - e^{-k_i t_1}) e^{-k_i s}] e^{-k_i t_2} + \mu(p_0 + \lambda d_2)(1 - e^{-k_i t_2}) \\ &= \mu p_0 + \mu \lambda d_1 (1 - e^{-k_i t_1}) e^{-k_i (s+t_2)} + \mu \lambda d_2 (1 - e^{-k_i t_2}). \end{aligned}$$

Thus the final tissue saturation is

$$x_i(t_1 + t_2 + s) = \mu p_0 + \mu \lambda d_1 (1 - e^{-k_i t_1}) e^{-k_i (s+t_2)} + \mu \lambda d_2 (1 - e^{-k_i t_2}). \quad (10)$$

If we define

$$Q_0 = \mu p_0, \quad Q_1 = \mu \lambda d_1 \quad \text{and} \quad Q_2 = \mu \lambda d_2$$

then these results can be rewritten as

$$\begin{aligned} x_i(t_1) &= Q_0 + Q_1 - Q_1 e^{-k_i t_1} \\ x_i(t_1 + t_2 + s) &= Q_0 + Q_2 + (Q_1 e^{-k_i s} - Q_2) e^{-k_i t_2} - Q_1 e^{-k_i (t_1 + t_2 + s)}. \end{aligned}$$

To satisfy the no-decompression requirement (3) for all $0 \leq t \leq t_1 + t_2 + s$, it suffices (by monotonicity) to simply apply this constraint at the ends of each of the two dives, i.e. to demand that

$$x_i(t_1) \leq M_{0,i} \quad \text{and} \quad x_i(t_1 + t_2 + s) \leq M_{0,i}.$$

The first inequality just forces $t_1 \leq \text{NDL}(d_1)$, the no-decompression limit for the depth d_1 . For a fixed $t_1 \leq \text{NDL}(d_1)$, the second constraint is then satisfied in the interval $t_2 \in [0, T_2]$ where $T_2 = g(t_1) = \text{NDL}(d_2, \mathbf{x}(t_1 + s))$ is the no-decompression limit for a

diver with initial state (9). Hence the set of solutions (t_1, t_2) that together satisfy our two constraints is the subgraph

$$S = \{(t_1, t_2) : 0 \leq t_2 \leq g(t_1), 0 \leq t_1 \leq T_1\}$$

where $T_1 = \text{NDL}(d_1)$. The boundary is piecewise differentiable, with knots occurring where the controlling tissue changes (i.e. where the minimum in (8) with $\mathbf{x} = \mathbf{x}(t_1 + s)$ is achieved by two different tissues i).

Figure 2 shows an example solution set for the choices of $d_1 = 40$ metres, $d_2 = 12$ metres and $s = 15$ minutes. As might have been expected, the boundary is approximately linear with a negative slope steeper than unity (left panel). Closer inspection (right panel) reveals that the boundary is convex and slightly nonlinear.

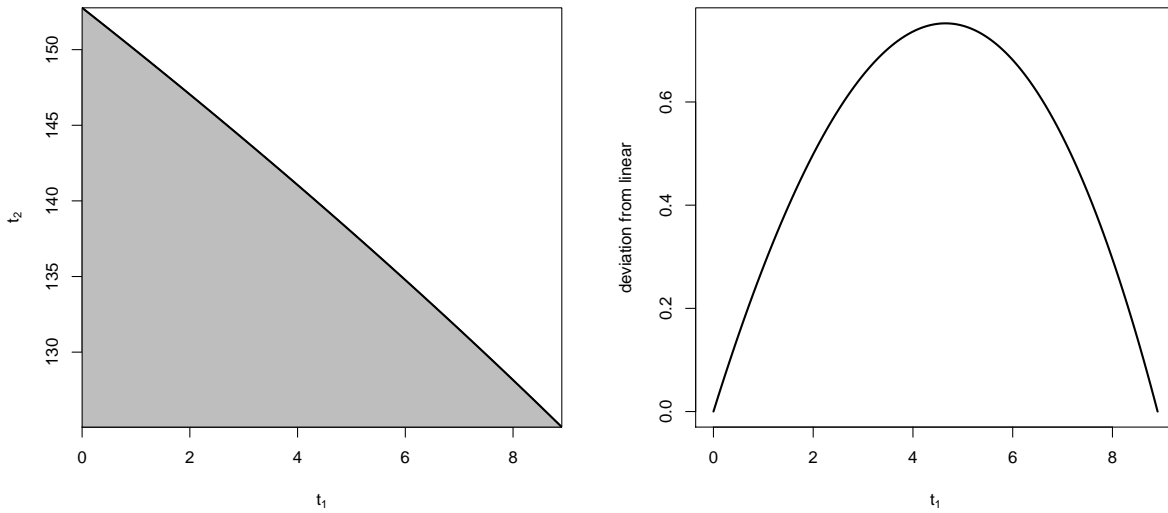


FIGURE 2: Example of solution set for double dive. *Left:* Maximum permissible duration of the second dive, t_2 , as a function of the duration of the first dive t_1 . *Right:* Deviation of the boundary from a straight line. Here the parameters are $d_1 = 40$ metres, $d_2 = 12$ metres, $s = 15$ minutes and the times are computed using the DSAT model.

4.2. The optimal double dive

We now consider how one might attempt to design the most desirable double dive. Of course before much progress can be made some appropriate objective function must be defined that measures the attractiveness of any particular dive plan. In the absence of any other function, an obvious start point would be to consider the total time $t_1 + t_2$ spent underwater. In that case the problem reduces to maximising $T = t_1 + t_2$ subject to

$$Q_0 + Q_1 - Q_1 e^{-k_i t_1} \leq M_{0,i} \quad (11)$$

$$Q_0 + Q_2 + (Q_1 e^{-k_i s} - Q_2) e^{-k_i t_2} - Q_1 e^{-k_i (t_1 + t_2 + s)} \leq M_{0,i} \quad (12)$$

for all i . However it can easily be shown that the solution is trivial: if $d_1 > d_2$ then the solution is just $t_1 = 0$ and $t_2 = \text{NDL}(d_2)$. This result is intuitively obvious when it is realised that the deeper a diver goes so the build-up in nitrogen concentration progressively increases. Thus to spend the maximum total time the diver simply elects to go to the shallower depth for as long as the no-decompression limits allow, and does not dive the deeper depth at all.

This result implies that maximising the total dive time is not a sensible measure of a good dive plan. Most divers will attest that deep dives are in some sense more exhilarating and fulfilling than shallow ones; so as a refined objective function, let us look to the integral of depth over time

$$\Phi = t_1 d_1 + t_2 d_2$$

subject to the constraints (11) and (12).

To calculate the maximum value of the objective function Φ we use the technique of Lagrange multipliers. We therefore introduce constants A_1, \dots, A_m and B_1, \dots, B_m

and define

$$Y = d_1 t_1 + d_2 t_2 + \sum_{i=1}^m A_i a_i(t_1) + \sum_{i=1}^m B_i b_i(t_1, t_2) \quad (13)$$

where

$$\begin{aligned} a_i(t_1) &= M_{0,i} - Q_0 - Q_1 + Q_1 e^{-k_i t_1} \\ b_i(t_1, t_2) &= M_{0,i} - Q_0 - Q_1 e^{-k_i t_1} (1 - e^{-k_i t_2}) - Q_2 (1 - e^{-k_i t_2}). \end{aligned}$$

are the constraint functions corresponding to (11) and (12) respectively.

The optimal solution may occur either at

- (a) a boundary point of the time domain, or at
- (b) a generic stationary point of Y , or at
- (c) a point where two of the constraint functions are equal to zero.

Case (a) We must consider the boundary solutions occurring when either $t_1 = 0$ or $t_2 = 0$ corresponding to only a single dive. The possible boundary points are then

- (a1) $(t_1, t_2) = (0, \text{NDL}(d_2))$ and
- (a2) $(t_1, t_2) = (\text{NDL}(d_1), 0)$

but case (a2) can be shown to be suboptimal as follows. For a fixed value of t_1 , the value of $d_1 t_1 + d_2 t_2$ is clearly greatest when t_2 is maximised subject to the constraints. This is to say that t_2 should equal $\text{NDL}(d_2, \mathbf{x}(t_1 + s))$ with $\mathbf{x}(t_1 + s)$ given by (9). If $t_1 = \text{NDL}(d_1)$ then, for any nonzero surface interval s , the tissue saturations $\mathbf{x}(t_1 + s)$ at the start of the second dive are clearly sub-critical, so that $\text{NDL}(d_2, \mathbf{x}(t_1 + s)) > 0$, and sub-case (a2) is not optimal, and can be safely excluded from further consideration.

Case (b) Stationary points of Y are found by considering the partial derivatives

$$\begin{aligned}\frac{\partial Y}{\partial t_1} &= d_1 - Q_1 \sum_i A_i k_i e^{-k_i t_1} - Q_1 \sum_i B_i k_i e^{-k_i(s+t_1+t_2)} \\ &= d_1 - Q_1 \sum_i A_i k_i e^{-k_i t_1} - \sum_i B_i k_i Q_{1,i} e^{-k_i(t_1+t_2)}\end{aligned}\quad (14)$$

$$\begin{aligned}\frac{\partial Y}{\partial t_2} &= d_2 + \sum_i B_i k_i \left[Q_1 e^{-k_i(s+t_2)} (1 - e^{-k_i t_1}) - Q_2 e^{-k_i t_2} \right] \\ &= d_2 + \sum_i B_i k_i e^{-k_i t_2} \left[Q_{1,i} (1 - e^{-k_i t_1}) - Q_2 \right]\end{aligned}\quad (15)$$

where we have written $Q_{1,i} = Q_1 e^{-k_i s}$ since the surface interval duration s is taken to be fixed.

Generic stationary points of (13) are determined by solving

$$\frac{\partial Y}{\partial t_1} = \frac{\partial Y}{\partial t_2} = 0$$

subject to the additional requirement that one of the constraint functions has value zero (while the other constraint functions are all positive). There are essentially two sub-cases:

(b1) $a_i(t_1) = 0$ for some i whereupon the corresponding Lagrange multiplier A_i is nonzero while all other constraint functions are positive and their Lagrange multipliers zero.

(b2) $b_i(t_1, t_2) = 0$ for some i . Now B_i is nonzero with all the other constraint functions positive with corresponding Lagrange multipliers zero.

Sub-case (b1) implies that t_1 is equal to the no-decompression limit $\text{NDL}(d_1)$ and that t_2 is strictly less than the resulting no-decompression limit $\text{NDL}(d_2, \mathbf{x}(t_1 + s))$ for the second dive. However, this combination is suboptimal and can be excluded by the same argument used to dismiss (a2).

Thus we need only consider sub-case (b2) which implies that $t_1 < \text{NDL}(d_1)$ and $t_2 = \text{NDL}(d_2, \mathbf{x}(t_1 + s))$. All the Lagrange multipliers A_i can safely be set to zero and

a solution sought in which the partial derivatives (14) and (15) are zero together with

$$\begin{aligned}\frac{\partial Y}{\partial B_I} &= 0 \\ B_i &= 0 \quad \text{for } i \neq I\end{aligned}$$

if they exist within the constraint set. Effectively I is the controlling tissue: the final tissue saturation in tissue I is then equal to the surfacing limit $M_{0,I}$.

There are m possible candidates for the identity of the controlling tissue I . Putting $\partial Y/\partial t_1 = 0$ gives

$$\begin{aligned}0 &= d_1 + \sum_i B_i [-k_i Q_1 e^{-k_i(s+t_1+t_2)}] \\ &= d_1 - Q_1 k_I B_I e^{-k_I(s+t_1+t_2)}\end{aligned}$$

implying

$$k_I B_I Q_1 = d_1 e^{k_I(s+t_1+t_2)}. \quad (16)$$

Moreover $\partial Y/\partial t_2 = 0$ forces

$$\begin{aligned}0 &= d_2 + \sum_i B_i [k_i Q_{1,i} (1 - e^{-k_i t_1}) e^{-k_i t_2} - k_i Q_2 e^{-k_i t_2}] \\ &= d_2 + B_I [k_I Q_{1,I} (1 - e^{-k_I t_1}) e^{-k_I t_2} - k_I Q_2 e^{-k_I t_2}] \\ &= d_2 + k_I B_I [Q_{1,I} (1 - e^{-k_I t_1}) e^{-k_I t_2} - Q_2 e^{-k_I t_2}]\end{aligned}$$

and then substituting (16) gives

$$\begin{aligned}0 &= d_2 + d_1 e^{k_I(t_1+t_2)} \left[1 - e^{-k_I t_1} - \frac{Q_2}{Q_{1,I}} \right] e^{-k_I t_2} \\ &= d_2 + d_1 e^{k_I t_1} \left[1 - e^{-k_I t_1} - \frac{Q_2}{Q_{1,I}} \right] \\ &= d_2 - d_1 + d_1 \frac{Q_{1,I} - Q_2}{Q_{1,I}} e^{k_I t_1}\end{aligned}$$

so that

$$e^{k_I t_1} = \frac{Q_{1,I}(d_1 - d_2)}{d_1(Q_{1,I} - Q_2)}. \quad (17)$$

For $\partial Y/\partial B_I = 0$ then

$$M_{0,I} - Q_0 - Q_{1,I}(1 - e^{-k_I t_1})e^{-k_I t_2} - Q_2(1 - e^{-k_I t_2}) = 0$$

yielding

$$e^{k_I t_2} = \frac{Q_{1,I}(1 - e^{-k_I t_1}) - Q_2}{M_{0,I} - Q_0 - Q_2}.$$

The numerator on the right hand side of the last expression is, using (17),

$$\begin{aligned} Q_{1,I} - Q_2 - Q_{1,I}e^{-k_I t_1} &= Q_{1,I} - Q_2 - Q_{1,I} \frac{d_1(Q_{1,I} - Q_2)}{Q_{1,I}(d_1 - d_2)} \\ &= (Q_{1,I} - Q_2) \left[1 - \frac{d_1}{d_1 - d_2} \right] \\ &= -\frac{(Q_{1,I} - Q_2)d_2}{(d_1 - d_2)} \end{aligned}$$

so that

$$e^{k_I t_2} = -\frac{(Q_{1,I} - Q_2)d_2}{(d_1 - d_2)} \frac{1}{M_{0,I} - Q_0 - Q_2}. \quad (18)$$

Equations (17) and (18) together determine at most m candidates for optimal solutions (t_1, t_2) at generic stationary points.

Case (c) If two constraint functions are to be simultaneously zero then one of three possibilities occur: either

- (c1) $a_i(t_1) = a_j(t_1) = 0$ for some $i \neq j$; or
- (c2) $b_i(t_1, t_2) = b_j(t_1, t_2) = 0$ for some $i \neq j$; or
- (c3) $a_i(t_1) = 0$ for some i and $b_j(t_1, t_2) = 0$ for some j ,

while all other constraint functions are assumed to take nonzero values. The equality $a_i(t_1) = 0$ implies that $t_1 = \text{NDL}(d_1)$; similarly $b_j(t_1, t_2) = 0$ leads to $t_2 = \text{NDL}(d_2, \mathbf{x}(t_1 + s))$. Of the three possibilities the first (c1) can be excluded because

it implies $t_2 < \text{NDL}(d_2, \mathbf{x}(t_1 + s))$. Thus we only need to examine the other two; we remark that (c3) requires that $t_1 = \text{NDL}(d_1)$ and $t_2 = \text{NDL}(d_2, \mathbf{x}(t_1 + s))$ which is the double dive conducted to the no-decompression limits on each occasion.

Subcase (c2) is equivalent to identifying those cases when the no-decompression limit for the second dive is controlled by two tissues; that is, when the minimum in (8) is attained by two tissues i and j , for initial state (9). First consider the single tissue i . Observe that $b_i(t_1, t_2) = 0$ if and only if

$$Q_{1,i}(1 - e^{-k_i t_1})\alpha + Q_2(1 - \alpha) = E_i$$

where $\alpha = e^{-k_i t_2}$ and $E_i = M_{0,i} - Q_0$. A solution with $\alpha \in (0, 1)$ occurs whenever either

$$Q_{1,i}(1 - e^{-k_i t_1}) < E_i < Q_2$$

or

$$Q_{1,i}(1 - e^{-k_i t_1}) > E_i > Q_2.$$

If the set

$$D_i = \{t_1 > 0 : b_i(t_1, t_2) = 0 \text{ for some } t_2 > 0\}$$

then solutions with $\alpha \in (0, 1)$ are feasible for all $t_1 > 0$ if $Q_{1,i} \leq E_i \leq Q_2$. In contrast, if $Q_{1,i} \leq E_i$ and $E_i > Q_2$ then no solutions occur whatever the value of t_1 . Lastly, $D_i = (0, c_i)$ if $Q_{1,i} > E_i$ and $E_i \leq Q_2$; and $D_i = (c_i, \infty)$ if $Q_{1,i} > E_i > Q_2$, where

$$c_i = -\frac{1}{k_i} \log \left[1 - \frac{E_i}{Q_{1,i}} \right].$$

We remark that for any $t_1 \in D_i$ the solution of $b_i(t_1, t_2) = 0$ in t_2 exists and equals

$$f_i(t_1) = \frac{1}{k_i} \log \frac{M_{0,i} - Q_0 - Q_2}{Q_{1,i}(1 - e^{-k_i t_2}) - Q_2}.$$

Thus, for two tissues i and j , a solution of

$$b_i(t_1, t_2) = b_j(t_1, t_2) = 0$$

exists precisely when the function $g_{ij}(t_1) = f_i(t_1) - f_j(t_1)$ has a root t_1^* in the intersection $D_i \cap D_j$. If this happens then putting $t_2^* = f_i(t_1^*) = f_j(t_1^*)$ yields the required solution (t_1^*, t_2^*) .

The implication is that to compute subcase (c2) we examine each pair of tissues i and j in turn. It has to be determined whether the intersection $I_{ij} = D_i \cap D_j$ is nonempty; if this is so we have to find whether a root of g_{ij} exists in I_{ij} . In this eventuality the solution (t_1^*, t_2^*) has to be computed and checks made to ensure that the solution satisfies the remaining constraints, $a_k(t_1^*) \geq 0$ for all k and $b_k(t_1^*, t_2^*) \geq 0$ for all $k \neq i, j$.

4.3. A numerical example

We illustrate the calculation for the optimum double dive taking the parameters used in constructing Figure 2, i.e. successive dives to $d_1 = 40$ and $d_2 = 12$ metres separated by a surface interval of $s = 15$ minutes. We again use the DSAT model parameters summarised in Table 1.

Candidates for the optimum double dive were calculated following the procedure described above. The possibilities are listed in Table 3 which reveals that the best plan is achieved by (b2), the stationary point, with $t_1 = 7.33$ minutes and $t_2 = 130.4$ minutes. For comparison if the first dive is conducted up to its no-decompression limit of $t_1 = 8.94$ minutes (case (c3)), the second dive is then restricted to $t_2 = 125$ minutes. We remark that for case (c2) there is actually a crossing between tissues 5 and 8 at $(t_1, t_2) = (3.19, 188.97)$ but this is an infeasible solution since this value of t_2 exceeds the NDL for the second dive.

Figure 3 shows the maximum value of $\Phi = d_1 t_1 + d_2 t_2$ taken over the possible range of t_2 for each fixed t_1 . The optimum clearly occurs at the stationary point of this graph,

case	t_1	t_2	$t_1d_1 + t_2d_2$
(a1)	0	152.8	1833.2
(b2)	7.32	130.4	1858.2
(c3)	8.94	125.0	1856.7

TABLE 3: Candidates for the optimum double dive using the parameters of Figure 2.

where $t_1 \approx 7.32$ minutes. We noted earlier that, to a first approximation, the graph of t_2 against t_1 in Figure 2 is virtually linear with a slope of approximately -3.1 . Since $d_2/d_1 = 40/1.2 = 3.33$, the graph of $d_1t_1 + d_2t_2$ against t_1 is also essentially linear with an almost negligible slope. Figures that gives rise to the stationary point in Figure 3.

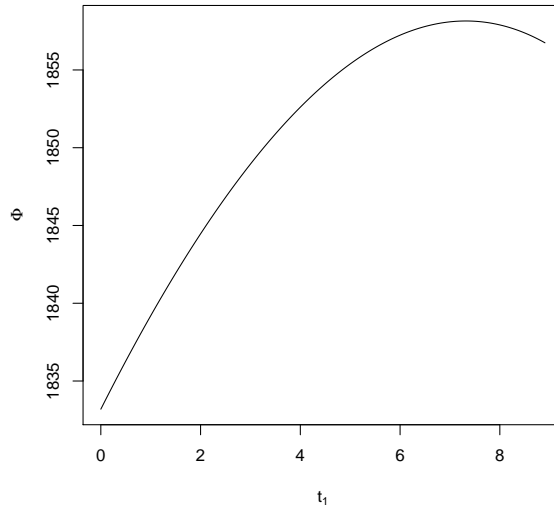


FIGURE 3: Maximised value of $d_1t_1 + d_2t_2$ (maximised over t_2 for fixed t_1) plotted against the duration of first dive t_1 . Same parameters as used in Figure 2.

Continuing with the same example, Figure 4 shows the effect of varying the surface interval s . For small values of s (less than about 3 minutes) the optimum is achieved when $t_1 = 0$; informally this arises because the nitrogen build-up is great during the first (deep) dive and the recovery period on the surface very short. Then the reduction in the second dive duration due to the after-effects of the first is so severe that the optimal double dive would forego the first dive altogether. When s is large, that is greater than about 20 minutes, the optimum is achieved when the first dive is extended

to its full NDL $t_1 = \text{NDL}(d_1)$. For intermediate values of s the optimum typically occurs at a stationary point.

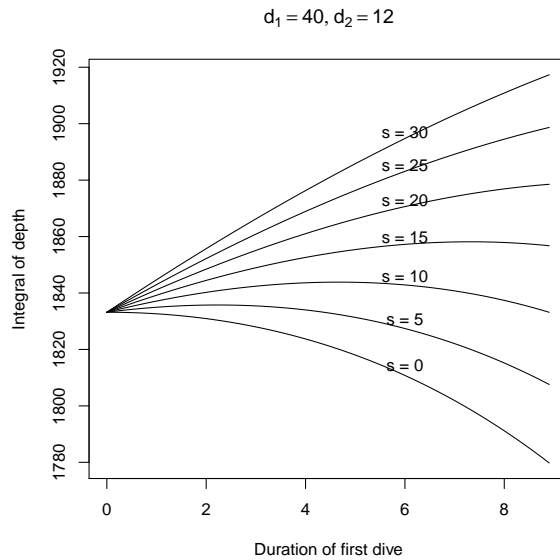


FIGURE 4: Family of curves analogous to previous Figure but allowing for various surface intervals s .

5. Discussion

In this article we have shown how elementary decompression theory can be developed using simple differential equation models. It is seen how optimal dive plans can be deduced, although this issue is not as simple as might have been envisaged at the outset for the reason that the profile of the dive pattern is sensitively dependent on the definition of a good dive. In principle what we have developed is sufficient to formulate a complete set of no-decompression tables for both single and combination dives. Software for performing the calculations in the paper is available [1].

We have deliberately steered clear of discussing decompression dives. There is no technical reason why such dives cannot be handled using exactly the same technology as used here subject to the complication that stops at specified depths would need

to be incorporated into the model. Of course there comes a point where the intrinsic attractiveness of analytic formulae is overtaken by the sheer number of free parameters and a numerical solution is then more efficient. Nevertheless, we would argue that the modelling described here is easily extended in many directions and thus ideal for further investigation. Of particular interest might be an examination of the effect on the NDL of imposing specified descent and ascent time-histories and the construction of the best dive plan comprising of more than two individual dives. The modelling of decompression is a topic that contains a richness of possibilities although there is nothing quite as exciting as putting the theory into practice on a sunny day.

References

- [1] BADDELEY, A. The `scuba` package for the R system. URL: cran.r-project.org/web/packages/scuba.
- [2] BERT, P. (1878). *La pression barométrique*. Masson, Paris.
- [3] BOOKSPAN, J. (1995). *Diving physiology in plain English*. Undersea and Hyperbaric Medicine Society, Kensington, Maryland, USA. ISBN 0-930406-13-3.
- [4] BOYCOTT, A., DAMANT, G. AND HALDANE, J. (1908). The prevention of compressed air illness. *Journal of Hygiene (Cambridge)* **8**, 342–443.
- [5] BRUBAKK, A. AND NEUMAN, T., Eds. (2003). *Bennett and Elliott's Physiology and Medicine of Diving* fifth ed. Saunders. ISBN 0-7020-2571-2.
- [6] HALDANE, J. (1922). *Respiration*. Yale University Press.
- [7] LIPPMANN, J. (1990). *Deeper into diving*. J.L. Publications, Carnegie, Victoria, Australia. ISBN 0-9590306-3-8.
- [8] MARTIN, L. (1997). *Scuba diving explained*. Diversification series. Best Publishing Company, Flagstaff, Arizona, USA. ISBN 0-941332-56-X.

- [9] STRAUSS, M. AND AKSENOV, I. (2004). *Diving Science*. Human Kinetics, Champaign, Illinois, USA.

- [10] TIKVISIS, P. AND GERTH, W. (2003). Decompression theory. In *Bennett and Elliott's Physiology and Medicine of Diving*. ed. A. Brubakk and T. Neuman. Fifth ed. Saunders ch. 10.1, pp. 419–454. ISBN 0-7020-2571-2.

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Module 767

The Mathematics of Scuba Diving

D. R. Westbrook



**Applications of Beginning Calculus to
Physiology**

INTERMODULAR DESCRIPTION SHEET:	UMAP Unit 767
TITLE:	The Mathematics of Scuba Diving
AUTHOR:	D.R. Westbrook Dept. of Mathematics and Statistics University of Calgary Calgary, Alberta, Canada T2N 1N4 westbroo@acs.ucalgary.ca
MATHEMATICAL FIELD:	Beginning calculus
APPLICATION FIELD:	Physiology
TARGET AUDIENCE:	Students in beginning calculus
ABSTRACT:	Exponential solutions of differential equations are used to construct decompression schedules for dives of various durations to various depths.
PREREQUISITES:	A knowledge of differential and integral calculus related to exponential functions.
RELATED UNITS:	Unit 676: <i>Compartment Models in Biology</i> , by Ron Barnes. <i>The UMAP Journal</i> 8 (2): 133–160. Reprinted in <i>UMAP Modules: Tools for Teaching 1987</i> , edited by Paul J. Campbell, 207–234. Arlington, MA: COMAP, 1988.

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The Mathematics of Scuba Diving

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Paul J. Campbell
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Editor
Executive Director, COMAP

1. Introduction

Are you a scuba diver? Can you use the diving tables? Do you know the mathematical basis for the diving tables? Could you construct your own diving tables? The purpose of this module is to describe the physiological basis for the diving tables and the mathematics used for the calculations.

2. A Brief History of Diving

Diving is an ancient pastime. Diving for profit—the collectinn of sponges, shells, and pearls—and diving for food have been with us for some time, and probably so has diving for pleasure. Divers were used for military purposes by the Greeks and are still of strategic importance today.

Ancient diving was essentially free (or breath-hold) diving, although Alexander the Great was reported to have used a primitive diving bell around 330 B.C. A diving bell is essentially a weighted inverted receptacle that retains its air (or other gases) as it is lowered into the water, giving a source of oxygen at depth to which the diver may return as needed or even be connected by a flexible tube. The air in the bell deteriorates in quality as the dive progresses, and various methods have been devised to replenish it.

In 1691, Sir Edmund Halley (of comet fame) built and patented what may have been the first practical diving bell, with a volume of approximately 60 cubic feet. The air was replenished from barrels, and the fouled air was vented out by means of a valve. (A 6-foot-high cylinder of diameter $3\frac{1}{2}$ ft has volume $\simeq 56$ ft³.) Nearly 100 years passed before a successful forcing pump was developed to enable a supply of fresh air to be pumped to the bell from the surface. This technique later developed into personal diving suits supplied from the surface and then to self-contained underwater breathing apparatus (SCUBA).

As dives became deeper and longer, it became apparent that there were various physiological risks involved. One such risk is decompression sickness, or the “bends,” which was associated with a rapid return to the surface after a long or deep dive.

In addition to diving, the nineteenth century saw the introduction of “caissons,” large chambers equipped with an air lock and kept under high pressure, which enabled tunnellers and bridge builders to work underground or underwater without the chamber flooding. It soon became clear that special procedures were needed so that the workers, who may have been working in a high-pressure environment for several hours, did not suffer injuries or even death when they returned to normal atmospheric pressure. The need for a careful decompression sequence became obvious. In 1854, physicians B. Pol and T.J.J. Wattelle stated in a report, “The danger does not lie in entering a shaft containing compressed air; nor in remaining there a longer or shorter time; decompression alone is dangerous” [Hills 1977].

The decompression routines of this time were usually linear (i.e., a reduction in pressure at a fixed constant rate in atmospheres per minute) and were generally devised by experience that involved much pain and some deaths on the part of the experimental subjects. Of the approximately 600 men who worked on the St. Louis bridge, 119 suffered serious neurological decompression sickness, and 14 died. The name “the bends” apparently originated from the gait of these bridge workers, caused by pains in their joints. This resembled the “Grecian bend” of fashionable ladies of the time, who walked voluntarily in this manner.

In the early twentieth century, military needs led various navies to become interested in decompression sickness, and more careful research was begun. The most influential of this research was performed by the physiologist J.S. Haldane for the Royal Navy in 1906. Haldane’s diving tables (1908) were remarkably effective in almost eliminating decompression sickness as a diving hazard and were used for some time. As more experience was gained, it became clear that Haldane’s tables were somewhat conservative for short dives, so adjustments were made. Then, as longer deeper dives were undertaken, it was found that the tables were not conservative enough for such dives, and more refinements were made. Many further refinements have taken place in more recent times, but the tables are still essentially based on adaptations of Haldane’s original ideas.

In the following sections, we examine these basic ideas and the mathematics behind them. To construct adequate universal tables is arithmetically intensive, but we will use the ideas in simplified form to construct our own tables.

**The tables that we construct are not to be used in any dive!
Use the tables that your scuba instructors give you.**

3. Haldane’s Model

When Haldane began his experiments, it had been established that the major cause of decompression sickness was the release of bubbles of nitrogen, an inert gas in the air, into various tissues and into the arterial bloodstream. While a diver is underwater, she is breathing air under high pressure and, as a result, more nitrogen is forced into her blood. When she ascends, the air that she is breathing returns to a lower pressure, and the nitrogen dissolved in her blood forms bubbles. (Because oxygen in the air that is dissolved in the blood is metabolized, it does not cause a problem.) The effect can be seen when the lid of a pop bottle is unscrewed. The gas in the fluid is under pressure that is suddenly reduced when the lid is unscrewed, and bubbles rapidly form.

Initially it was thought that there would be a critical drop in pressure above which sickness would occur; but Haldane’s experiments, which were performed on goats, led to a different conclusion. (Haldane had found that the sensitivity of goats to decompression sickness was acceptably close to that of

humans.) He found that no matter what the original pressure is, decompression sickness does not occur if the pressure is reduced by less than some fixed fraction. That is, there is a value M for which a pressure P_1 can be reduced to $P_2 = MP_1$ without the occurrence of “the bends.” Haldane suggested a value M just slightly less than $1/2$. We will use $1/2.15 \approx .465$ in our calculations.

The subjects of these experiments were exposed to the higher pressure for long periods, so the dissolved gases were brought to saturation levels. In dives, this might not be the case. In addition, for long dives at an absolute pressure of more than twice atmospheric pressure, the subject could not be brought to atmospheric pressure without one or several intermediate stops. (An absolute pressure of two atmospheres occurs at a depth of about 10 m \approx 33 ft of water.)

To determine an appropriate set of stops, a model of how gases are dissolved in and released from body tissues is needed. First, it is known that the pressure of inert gas in the pulmonary circuit is almost instantaneously equalized with that in the lungs, which is the ambient external pressure. Thus, blood entering the arterial system has gas pressure equal to the ambient pressure. A model must now be made of the distribution of the gas to the various tissues in the body.

The simple model that we use in this Module is based on the following assumptions:

- The blood flows through a tissue at a constant volume rate ν ml/sec.
- If the gas pressure in the blood and tissue is p , then the concentration of the gas in the blood is s_1p g/ml and in the tissue is s_2p g/ml, where s_1, s_2 are constants with different values of s_2 for different tissues.

The model is a simple compartment model (see Barnes [1987]). Gas enters the pulmonary circuit from the lungs at pressure p_e , the ambient external pressure. We assume that the gas pressure in the blood as it enters a tissue compartment is p_e . The pressure in the tissue and the blood is quickly equalized to the local pressure p , and the blood leaves the compartment at pressure p .

A balance of mass for the gas must hold:

$$\begin{aligned} \text{The rate of increase of mass in the compartment} &= \\ \text{Rate at which mass flows in} &- \text{Rate at which mass flows out.} \end{aligned}$$

The mass of gas in the compartment at any time is $V_1s_1p + V_2s_2p$, where V_1 and V_2 are measured in ml and represent the respective volumes of blood and tissue in the compartment. The rate of increase of mass is then

$$\frac{d}{dt}[(V_1s_1 + V_2s_2)p] \quad \text{g/sec.}$$

Gas enters the compartment at a rate νs_1p_e g/sec and leaves at a rate of νs_1p g/sec. The balance of mass gives

$$[V_1s_1 + V_2s_2] \frac{dp}{dt} = \nu s_1(p_e - p) \quad \text{or} \quad \frac{dp}{dt} = k(p_e - p),$$

where $k = \nu s_1 / (V_1 s_1 + V_2 s_2)$ is a constant for the tissue. A simple diagram for this model is presented in **Figure 1**.

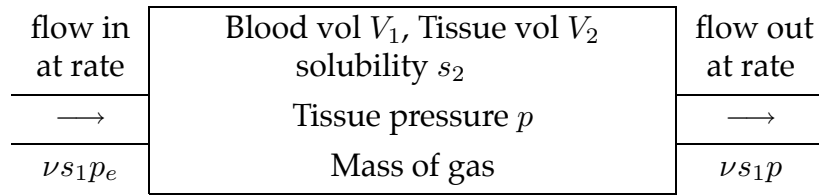


Figure 1. Diagram for the compartment model.

In Haldane's time, this model was thought to be appropriate for both compression ($p_e \geq p$) and decompression ($p \geq p_e$). It was known already that various tissues in the body required different values of s_2 , V_1 , V_2 , and ν , and the same blood does not flow through all tissues. In devising his tables, Haldane considered five different values for the constant k in the differential equation. His calculations were based on solutions of the differential equation and on the experimental result that the external absolute pressure could be reduced by the factor M at any time without an attack of the bends occurring.

In the work that follows, we assume for simplicity that air is all nitrogen. It can be shown that this in fact makes no significant difference to the results (see **Exercise 6**).

4. Solution of the Differential Equation

The differential equation

$$\frac{dp}{dt} = k(p_e - p), \quad (1)$$

where k and p_e are known constants, can be solved to find the pressure p at any time t , provided that the pressure p is known at one instant of time, usually taken to be $t = 0$ (we measure the elapsed time from the instant at which the pressure is known), i.e., $p(0) = p_0$, a known constant. If you know enough integral calculus, you can find the solution of the equation, as shown below, by the method of separation of variables. If you do not know integral calculus, the solution can be verified directly by substitution in (1).

To separate variables, we write (1) as

$$\frac{1}{p_e - p} \frac{dp}{dt} = k$$

and integrate (antidifferentiate) both sides with respect to t . This gives

$$\int \frac{1}{p_e - p} \frac{dp}{dt} dt = \int \frac{1}{p_e - p} dp = \int k dt.$$

Performing the integrations, we get

$$-\ln |p_e - p| = kt + c$$

where c is an arbitrary constant. Taking exponentials of both sides gives

$$|p_e - p| = e^{-(kt+c)} = e^{-kt} e^{-c} = Ae^{-kt},$$

where A is an arbitrary constant, $A = e^{-c}$. Since we also require $p(0) = p_0$, it follows that $|p_e - p_0| = A$, and we obtain the solution

$$p = p_e - (p_e - p_0)e^{-kt}. \tag{2}$$

Graphs of solutions for the case $p_0 = 1$ atm, $p_e = 3$ atm with (a) $k = 0.2$ min⁻¹, (b) $k = 0.1$ min⁻¹ are given in **Figure 2**. The curves represent the pressure p in the tissues of a diver at time t min after descending from the surface ($p = 1$ atm) to a depth of about 66 ft ($p = 3$ atm). Similarly, graphs for the case $p_0 = 3$ atm, $p_e = 1$ atm with (a) $k = 0.2$ min⁻¹, (b) $k = 0.1$ min⁻¹ are given in **Figure 3**. Here the curves represent the pressure t minutes after ascending to the surface from a point where the tissue pressure is 3 atm.

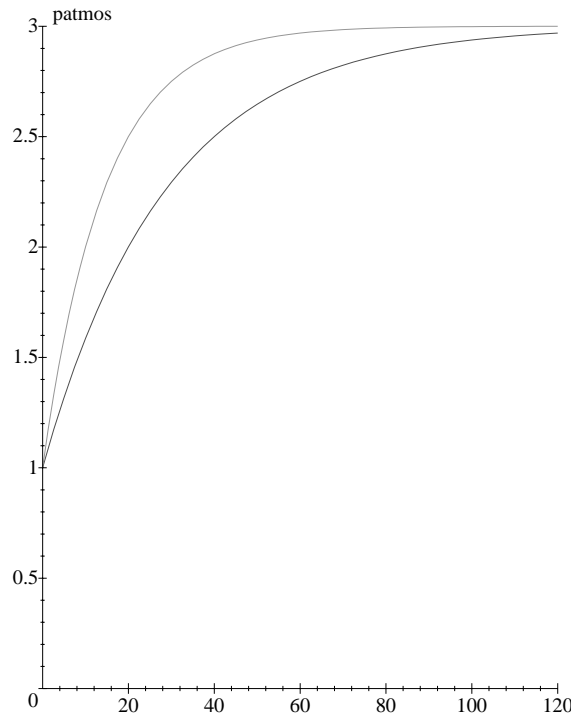


Figure 2. Solutions for the case $p_0 = 1$ atm, $p_e = 3$ atm. The lower curve is for $k = 0.2$ min⁻¹ and the upper curve is for $k = 0.1$ min⁻¹. The curves give the pressure p in the tissues of a diver at time t min after descending from the surface ($p = 1$ atm) to a depth of about 66 ft ($p = 3$ atm).

The role of the constant k , which is measured in min⁻¹ if t is measured in min, is indicated in **Figures 2** and **3**. When p_0 and p_e are held constant, it takes

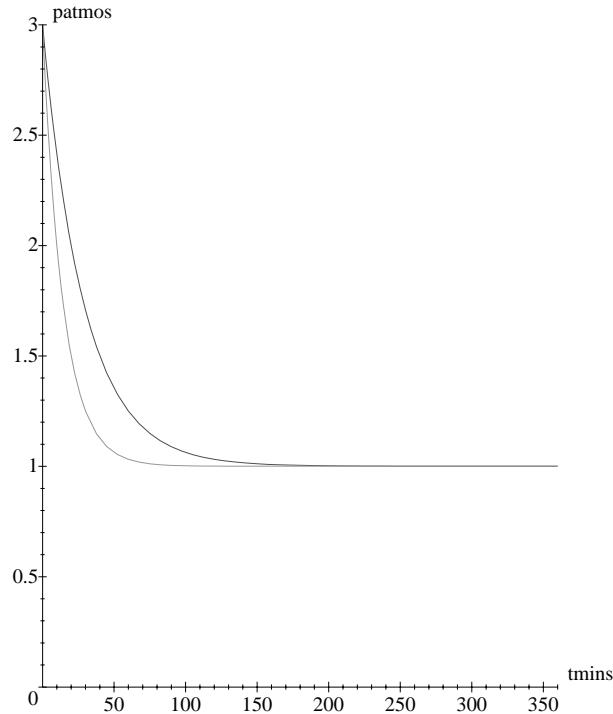


Figure 3. Solutions for the case $p_0 = 3$ atm, $p_e = 1$ atm. The lower curve is for $k = 0.2 \text{ min}^{-1}$ and the upper curve is for $k = 0.1 \text{ min}^{-1}$. The curves give the pressure t min after ascending to the surface from a point where the tissue pressure is 3 atm.

twice as long to attain a given pressure when $k = 0.1$ as it does when $k = 0.2$. We also see that for any positive k , p approaches the constant external pressure p_e as t becomes large ($t \rightarrow \infty$) no matter what the value of p_0 . In other words, the pressure equalizes over time, as expected.

5. The Half-Time

Because solutions of the exponential nature of (2) all have the same asymptote $p = p_e$ for all positive values of k , they are often characterized by their half-time, or half-life as it is called in the case of radioactive decay.

The half-time is the time required for the difference between p and the external pressure p_e to drop to exactly one half of its original value, that is, the time at which $(p - p_e) = (p_0 - p_e)/2$.

From (2), we see that if T is the half-time, then

$$p - p_e = (p_0 - p_e)e^{-kT} = \frac{1}{2}(p_0 - p_e)$$

and hence

$$e^{-kT} = \frac{1}{2} \Rightarrow e^{kT} = 2 \Rightarrow kT = \ln 2. \quad (3)$$

From this equation we see that $k = \ln 2/T$ no matter what the values of p_0 and p_e are, and that the half-time T for a tissue completely determines the value of k in (2). This makes the half-time extremely useful in characterizing the various tissues in the body.

The relationships between bottom times and decompression programmes differ for different half-times. The human body contains many different tissues, as Haldane knew, and a safe decompression programme must make sure that the bends do not occur in any of them. Haldane did not have exact values for half-times, so to compile his tables he used five different values (5, 10, 20, 40, and 75 min) in the belief that this would cover any reasonable spectrum of half-times. His tables were successful over the wide range of dives undertaken at that time and for some considerable time thereafter.

Noting that

$$e^{-kT} = \frac{1}{2}, \quad e^{-kt} = e^{-kT(\frac{t}{T})} = (e^{-kT})^{(t/T)} = \left(\frac{1}{2}\right)^{t/T},$$

we rewrite (2) as

$$p = p_e + (p_0 - p_e) \left(\frac{1}{2}\right)^{t/T}. \tag{4}$$

6. Scuba and No-Stop Dives

Most recreational divers usually dive to a given depth, remain at (or above) that depth for a certain time, and then ascend directly to the surface. This is the “no stop” or “no decompression” dive, as shown in **Table 1** below. The time allowed at the bottom depends on the depth of the dive. For example, the table says that you may stay (“stay” includes descent and ascent) at 70 ft for 50 min.

Table 1.
Diving table (from Hammes and Zimos [1988]).

Depth (ft)	40	50	60	70	80	90	100	110	120	130
Time (min)	200	100	60	50	40	30	25	20	15	10

A no-stop diving table can be produced from our model in the following manner.

We wish to model a situation in which a diver starts with an initial gas tissue pressure of 1 atm and wishes to stay at a depth d ft where the external pressure is $p_e = 1 + d/33$ (33 ft of water gives a pressure of 1 atm; the equation contains a 1 because there is already a pressure of 1 atm at the surface $d = 0$). We use (2) to tell us the tissue gas pressure after t min, which will be

$$p = 1 + \frac{d}{33} - \frac{d}{33} e^{-kt} \quad (k \text{ being known for the given tissue}).$$

Haldane's decompression experiment says that the diver may ascend directly to the surface where the pressure is 1 atm provided that the pressure p attained in the tissues is less than 2.15 atm. Thus the diver has a limiting dive time t_d given by

$$2.15 = 1 + \frac{d}{33} (1 - e^{-kt_d}),$$

$$\frac{d}{33} = \frac{1.15}{1 - e^{-kt_d}}.$$

This relation gives the time for the tissue as characterized by its value of k (equivalently, by its half-time $T = \ln 2/k$).

The allowable time t_d becomes longer as k becomes less, that is, as the half-time $T (= \ln 2/k)$ becomes greater. To be safe for all tissues, t_d is limited by the tissue with the shortest half-time, which is 5 min in Haldane's scheme. This would give the relation

$$d = \frac{38}{1 - \exp(-t_d \ln 2/5)}.$$

Tables are usually written with t_d as a function of depth d , which our model gives as

$$t_d = \frac{5 \ln \left(\frac{d}{d-38} \right)}{\ln 2}.$$

You will find that this relation gives qualitative agreement with published tables (see **Figure 4**); but the quantitative agreement is not very good, because of the conservative nature of Haldane's value of M and his tissue half-time of 5 min for short dives.

7. Dives with Decompression Stops

For dives that fall outside the no-stop dive range, a more complicated set of conditions must be satisfied. Again we follow Haldane's recipe.

The standard method to calculate a decompression routine is to consider a series of stops at depths that are multiples of 10 ft. The first stop must be such that the external pressure at that depth is not less than M times the pressure in each of the tissues that has been reached during the stay at the diving depth. The tissue pressures depend on the time spent at depth and on the tissue half-time. The greatest tissue pressure will be in the tissue with the shortest half-time. Consider the following three examples.

Example 1. Consider a one-hour dive at a depth of 66 ft, where the pressure is approximately 3 atm. To save some calculations, we assume

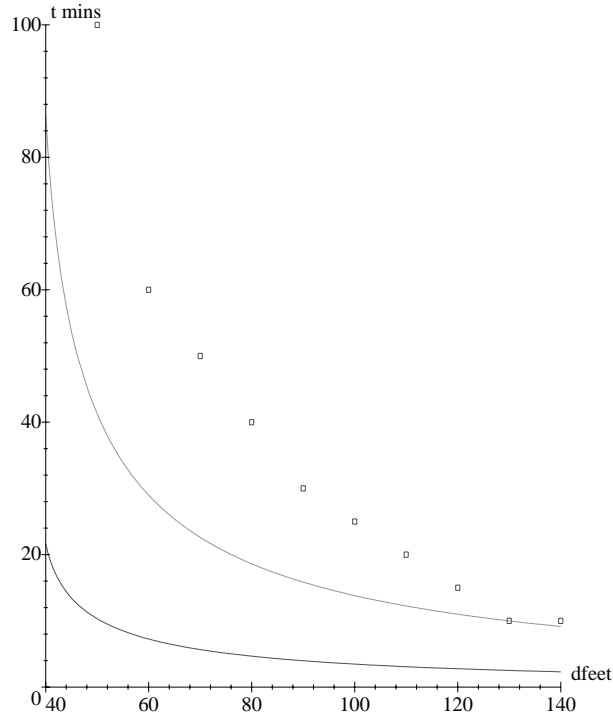


Figure 4. No-stop dive. Graph of $t_d = 5 \ln \left(\frac{d}{d-38} \right) / \ln 2$ and $t_d = 20 \ln \left(\frac{d}{d-38} \right) / \ln 2$, compared with points from the diving table of **Table 1**.

that there are three tissues (as opposed to Haldane’s five) with half-times 10, 20, and 40 min. From (4), the pressure of a tissue at an external pressure p_e is

$$p = p_e + (p_0 - p_e) \left(\frac{1}{2} \right)^{t/T},$$

where p_0 is the initial tissue pressure, T the tissue half-time, and t is the length of time at depth (in minutes). The pressure p_0 at the beginning of the dive is 1 atm. After one hour at 66 ft (3 atm for 60 min, $p_e = 3$ atm), tissue pressures are

$$\begin{aligned} T = 10\text{-min tissue} : \quad p &= 3 - 2 \left(\frac{1}{2} \right)^6 && \approx 2.97 \\ T = 20\text{-min tissue} : \quad p &= 3 - 2 \left(\frac{1}{2} \right)^3 && \approx 2.75 \\ T = 40\text{-min tissue} : \quad p &= 3 - 2 \left(\frac{1}{2} \right)^{3/2} && \approx 2.29. \end{aligned}$$

It is safe to ascend to an external pressure of $2.97/2.15 = 1.35$, or about 12.5 ft. To keep the ascent steps in multiples of 10 ft, the first ascent is made to 20 ft (1.60 atm).

At this point, the diver makes a stop. We have to decide how long this stop should be. To do this, we must decide the depth for the next stop. We choose 10 ft or 1.30 atm. The diver must remain at 20 ft until

all tissue pressures have declined to a value that will be safe when the diver ascends to 1.30 atm—that is, until all tissue pressures are reduced to $2.15 \times 1.30 = 2.795$ atm. The three tissue pressures at the beginning of the 20-ft stop are 2.97, 2.75, 2.29. The pressures of the 20-min and 40-min tissues are already low enough to ascend to 10 ft. For the 10-min tissue, t min will result in pressures

$$\text{10-min tissue:} \quad p = 1.6 + 1.37 \left(\frac{1}{2}\right)^{t/10}.$$

(Again we are using (4), with $p_e = 1.6$ and $p_0 = 2.97$ for $T = 10$.) The diver must remain at the 20-ft level until all tissue pressures are below the pressure 2.795 atm that is safe at the 10-ft stop (1.3 atm). For the 10-min tissue, this means t must be greater than the solution of

$$2.975 = 1.6 + 1.37 \left(\frac{1}{2}\right)^{t/10} \quad \text{or} \quad t = 10 \ln \left(\frac{1.37}{1.195}\right) / \ln 2 \approx 1.971.$$

Suppose that we make a 2-min stop at 20 ft. We must calculate the tissue pressures after 2 min at 20 ft:

$$\begin{aligned} \text{10-min:} \quad p &= 1.6 + 1.37 \left(\frac{1}{2}\right)^{.2} = 2.79 \\ \text{20-min:} \quad p &= 1.6 + 1.15 \left(\frac{1}{2}\right)^{.1} = 2.67 \\ \text{40-min:} \quad p &= 1.6 + .69 \left(\frac{1}{2}\right)^{.05} = 2.27. \end{aligned}$$

These are the initial pressures at the 10-ft (1.3-atm) stop. The next ascent will be to the surface (1 atm), where the safe pressure will be 2.15. The stop at 10 ft (1.3 atm) must be long enough that all three pressures will drop below 2.15. For a stop of t min, the pressures will be

$$\begin{aligned} \text{10-min:} \quad p &= 1.3 + 1.49 \left(\frac{1}{2}\right)^{t/10} \\ \text{20-min:} \quad p &= 1.3 + 1.37 \left(\frac{1}{2}\right)^{t/20} \\ \text{40-min:} \quad p &= 1.3 + .97 \left(\frac{1}{2}\right)^{t/40}, \end{aligned}$$

and t must be large enough that all three are less than 2.15. For the 10-min tissue, this requires 7.62 min, for the 20-min tissue 13.77 min, and for the 40-min tissue 8.09 min. The stop at 10 ft must be greater than 13.77 min—say 14 min. An appropriate decompression procedure for a one-hour dive at 66 ft would feature stops of

$$\begin{array}{ll} 2 \text{ min} & \text{at 20 ft} \\ 14 \text{ min} & \text{at 10 ft.} \end{array}$$

The ascent would also be lengthened by the time to ascend the 66 ft, about 1.5 min.

Example 2. We take an ascent as recommended in Haldane’s tables [Hempleman 1982, 330]. For a dive of 130 min at 90 ft, Haldane’s tables recommend stops of

5 min at 30 ft
 25 min at 20 ft
 30 min at 10 ft.

In this calculation, we will use all five of Haldane’s half-times of 5, 10, 20, 40, and 75 min.

First, we calculate the saturation levels for a dive of 130 min at 90 ft ≈ 3.73 atm. Then we calculate the pressures at the end of the period spent at each stopping point. Finally, we note the safe pressure to ascend to the next stop (see **Table 2**).

Table 2.
 Analysis of ascent recommended by Haldane for a 130-min dive at 90 ft.

Tissue half-time (min)	Pressure			
	90 ft = 3.73 atm	30 ft = 1.9 atm	20 ft = 1.6 atm	10 ft = 1.3 atm
5	3.73	2.82	1.64	1.30
10	3.73	3.19	1.88	1.37
20	3.70	3.41	2.36	1.67
40	3.44	3.31	2.71	2.14
75	2.91	2.86	2.60	2.285
Safe pressure at next stop	4.08	3.44	2.8	2.15

We see that at every stage except one, a safe pressure is attained in each tissue to allow the diver to ascend to the next stop. The exception is the last ascent to the surface for the 75-min tissue. Haldane allowed 2 min to move to and from the stops; if this time were included, the final pressures would be slightly reduced. This example, however, shows a problem with Haldane’s tables for long dives.

A much more recent U.S. Navy Table T-10 (reproduced in Hammes and Zimos [1988]) gives for this dive stopping times of

5 min at 30 ft
 36 min at 20 ft
 74 min at 10 ft.

This decompression procedure allows for even larger half-times than 75 min.

Figure 5 shows graphs of the tissue pressures for half-times of 5, 10, 20, 40, and 75 min, using the decompression scheme from Haldane’s tables. The piecewise “step” graph at the right indicates the safe pressure at the stops.

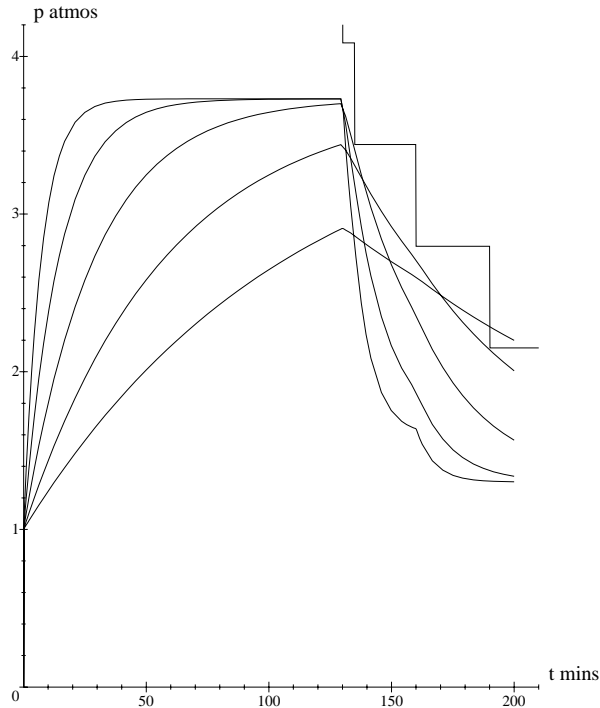


Figure 5. A 130-min dive to 90 ft followed by ascent with decompression stops as recommended by Haldane's tables. At left, from top to bottom, are tissue pressures at 90 ft for half-times of 5, 10, 20, 40, and 75 min. At right, from top to bottom, are the tissue pressures during ascent. The piecewise "step" graph at far right indicates the safe pressures at the stops.

Example 3. We consider a dive to 80 ft = 3.43 atm for one hour. Haldane's tables give stops of

$$\begin{array}{ll} 9 \text{ min} & \text{at } 20 \text{ ft} \\ 18 \text{ min} & \text{at } 10 \text{ ft}. \end{array}$$

Again we give the pressures as the diver *leaves* each level to proceed to the next (see **Table 3**).

In this case, a safe tissue pressure has been reached at all levels for all tissues before proceeding. This decompression procedure, however, is now considered to be rather conservative. The U.S. Navy table suggests 17 min at 10 ft as the only stop for this dive.

A procedure of this kind can be calculated with as many tissues as appropriate. (You might like to write a computer programme to carry out the steps.)

Table 3.

Analysis of ascent recommended by Haldane for a 60-min dive at 80 ft.

Tissue	Pressure		
	80 ft = 3.43 atm	20 ft = 1.6 atm	10 ft = 1.3 atm
5	3.43	2.13	1.37
10	3.39	2.56	1.66
20	3.17	2.75	2.08
40	2.57	2.43	2.13
75	2.03	2.00	1.89
Safe pressure at next stop	3.44	2.80	2.15

Exercises

In all exercises, assume that $M = 1/2.15$.

1. Find a decompression procedure for a dive of 40 min at 3.5 atm (80–85 ft) with stops at 1.7 atm (23 ft) and 1.3 atm (10 ft). (Consider only 10- and 20-min tissues.)
2. Find a decompression procedure for a 2-hr dive at 4.0 atm (100 ft) with stops at 1.9 atm (30 ft), 1.6 atm (20 ft), and 1.3 atm (10 ft). (Consider 10-, 20-, and 40-min tissues.)
3. Show that a slightly faster ascent for the dive of **Exercise 2** could be made if three stops of equal duration T_1 are made, the first at 1.9 atm (30 ft) and the second and third at depths to be determined. (As a first step, consider only the 40-min tissue; then verify that the steps are appropriate for the 10-min and 20-min tissues.)
4. Show that for a single tissue half-time T and an n -stop decompression schedule, the shortest total ascent time is achieved by using equal times at each step and determining the depths of each step according to the time. (The actual time at each step is determined by the number of steps.)
5. Show that for a single tissue, it is possible to have a continuous ascent in which the tissue pressure at time t is exactly 2.15 times the external pressure that the diver is experiencing at that time. Find the diver's depth at time t (pressure = $1 + d/33$ atm, where d is in feet). Using such a scheme, find how long it would take to ascend from a long dive at 4 atm. (Assume a single tissue of half-time 40 min and an instantaneous ascent from 4 atm to $1.86 = 4/2.15$ atm.)
6. If the nitrogen (partial) pressure in a tissue is 80% of the pressure, and the safe nitrogen pressure for a no-stop dive is 2.15 times that of the nitrogen partial pressure in the atmosphere (0.8 atm), show that the equation relating time and depth for no-stop dives is unaltered.

7. Check for safety the following recommendations from Haldane's tables for a dive of 45 min at 85 ft (3.58 atm). Stop 2 min at 30 ft, 7 min at 20 ft, 15 min at 10 ft. (U.S. Navy Table T-10 [Hammes and Zimos 1988] gives one stop of 17 min at 10 ft for this dive.)

8. Repetitive Dives

A major portion of the scuba diving tables is devoted to repetitive diving. The problem with repetitive diving is the fact that after one "no decompression" dive, the tissue pressure may be 2.15 times atmospheric pressure. An immediate dive back to a depth greater than 37 ft (external pressure greater than 2.15 atm) would raise the tissue pressure to above the limit that would allow a safe ascent to the surface. A break at the surface between dives lessens the pressure when the second dive is commenced, but it takes about twelve hours to restore all tissue pressures to 1 atm. The tissue pressure remaining after the first dive is known as the *residual nitrogen pressure* (RNP). We consider only a 20-min tissue in making our calculations, to keep things simple.

Example 4. Dive (1): 15 min at 80 ft. Dive (2) is to be to a depth of 100 ft after a one-hour break at the surface. We calculate the safe time for a "no decompression" second dive (20-min tissue only).

Tissue pressure p after 15 min at 80 ft ($p_e \approx 3.4$ atm):

$$p = 3.4 - 2.4 \left(\frac{1}{2} \right)^{3/4} = 1.97.$$

Since this is less than 2.15, it is safe to ascend to the surface.

Tissue pressure p after one hour at the surface ($p_e = 1$ atm):

$$p = 1 + .97 \left(\frac{1}{2} \right)^3 = 1.12.$$

Descent to 100 ft (4 atm):

$$p = 4 - 2.88 \left(\frac{1}{2} \right)^{t/20}.$$

The diver may remain until $p = 2.15$, that is, until

$$t = 20 \ln(2.88/1.85) / \ln 2 = 12.77 \text{ min.}$$

Figure 6 shows the pressure as a function of time for this example.

Actual scuba tables cover the large numbers of different calculations by classifying the residual nitrogen pressures into groups A, B, C, etc. The group

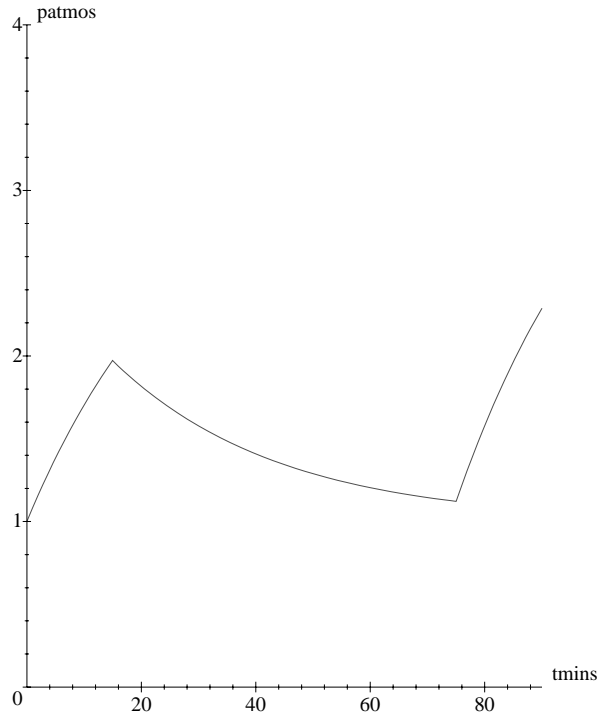


Figure 6. Repetitive dive.

is found after the first dive. The effect of remaining at the surface for a given time period is to change the group; the new group determines the safe time for the next dive. We give an example.

Example 5.

- Dive 1: 100 ft for 15 min
- Dive 2: 80 ft
- Time at surface between dives: 1 hr

We consult **Table 4**. First look at the row for a dive to 100 ft. Note that the no-stop time is 25 min. Our dive is for 15 min, so we go across the row until we reach 15. We then move down the corresponding column and find the repetitive group label “E”.

The stay at the surface is for 60 min. We continue along the column until we come to the two numbers that bracket 60 min:

$$\begin{aligned} &0 : 55 \\ &1 : 57. \end{aligned}$$

We now proceed left across this row until we find a new repetitive group label “D”.

For the second dive, at depth 80 ft, we use the label D. We continue across the row until we reach the column corresponding to 80 ft (at right).

The entry contains the numbers 18 (RNT) and 22 (TR). This means that because of the previous dive, it is as if we had already been at this depth for 18 min, and our time remaining is 22 min. We must be back at the surface within 22 min.

Exercises

Use (2) in the following exercises.

8. Consider the same sequence of dives as in **Example 4** but include a 40-min tissue. Does this make a difference for the second stop time?
9. Find the safe time for a second dive to 80 ft one hour after a first dive to 100 ft for 10 min. Consider tissue half-times of 20 min and 40 min.

9. Changes in Pressure During Descent and Ascent

To this point, we have assumed that the passage from one level to another is instantaneous. This is not possible; moreover, rapid motion is not recommended. A steady ascent or descent rate of about 60 ft/min is not unreasonable, and we will now examine the effect on tissue pressure of ascending at such a rate.

Our basic equation

$$\frac{dp}{dt} = k(p_e - p)$$

(where p_e is the external pressure) still holds, but p_e is no longer constant. For a descent at a constant rate of 60 ft/min, we have $p_e = 1 + 60t/33$ atm, and the differential equation becomes

$$\frac{dp}{dt} = k \left(1 + \frac{60t}{33} - p \right) \quad \text{or} \quad \frac{dp}{dt} + kp = k \left(1 + \frac{60t}{33} \right). \quad (5)$$

This is no longer a separable equation but a first-order linear equation, and it must be solved in a different manner. Here we describe one possible method.

First we try to guess a solution. After examining the equation, we feel that $p = A + Bt$, where A and B must be selected, seems a possible guess. If we substitute this into (5), we see that we get a solution if we can choose A , B so that

$$B + k(A + Bt) = k \left(1 + \frac{60t}{33} \right).$$

Table 4.
Diving table. ©NAUI 1987. Reproduced by permission.

NAUI® DIVE TABLES

TABLE 1 - END-OF-DIVE LETTER GROUP

ABT ACTUAL BOTTOM TIME
+RNT RESIDUAL NITROGEN TIME
TBT TOTAL BOTTOM TIME

(USE THIS FIGURE TO DETERMINE END-OF-DIVE LETTER GROUP)

START DEPTH M FEET	00 NO-DECOMPRESSION LIMIT (NDL)					00 BOTTOM TIME REQUIRING DECOMPRESSION NO. MINUTES REQUIRED AT 10' STOP (3m)								
	12 40 →	5	15	25	30	40	50	70	80	100	110	130	150	170
15 50 →		10	15	25	30	40	50	60	70	80	90	100	140	160
18 60 →		10	15	20	25	30	40	50	55	60	70	80	100	120
21 70 →		5	10	15	20	30	35	40	45	50	60	70	80	90
24 80 →		5	10	15	20	25	30	35	40	50	60	70	80	90
27 90 →		5	10	12	15	20	25	30	35	40	50	60	70	80
30 100 →		5	7	10	15	20	22	25	30	35	40	50	60	70
33 110 →			5	10	13	15	20	25	30	35	40	50	60	70
36 120 →			5	10	12	15	20	25	30	35	40	50	60	70
39 130 →			5	8	10	15	20	25	30	35	40	50	60	70
42 140 →			5	7	10	15	20	25	30	35	40	50	60	70

M	12	15	18	21	24	27	30	33	36	39	42	NEW GROUP		
FT.	40	50	60	70	80	90	100	110	120	130	140	A	B	
7	193	94	55	46	36	27	22	17	12	7	2	← A	1200 010	
17	13	11	9	8	7	7	6	6	6	6	5	← B	1200 211	
183	87	49	41	32	23	18	14	9	4	5	5	← C	1200 250	
25	21	17	15	13	11	10	10	9	8	7	7	← D	1200 549	
175	79	43	36	27	19	15	10	6	2	3	3	← E	1200 633	
37	29	24	20	18	16	14	13	12	11	10	10	← F	1200 706	
163	71	36	30	22	14	11	7	3				← G	1200 736	
49	38	30	26	23	20	18	16	15	13	12	12	← H	1200 800	
151	62	30	24	17	10	7	4					← I	1200 822	
61	47	36	31	28	24	22	20	18	16	15	15	← J	1200 841	
138	53	24	19	12	6	3						← K	1200 859	
73	56	44	37	32	29	26	24	21	19	18	18	← L	1200 913	
127	44	16	13	8	1							← M	1200 929	
87	66	52	43	38	33	30	27	25	22	20	20	← N	1200 944	
113	34	8	7	2										1200 988
101	76	61	50	43	38	34	31	28	25					1200 993
90	24													1200 998
116	87	70	57	48	43	38								1200 999
84	13													1200 999
138	99	79	64	54	47									1200 999
62	1													1200 999
161	111	88	72	61	53									1200 999
30														1200 999
187	124	97	80	68	58									1200 999
13														1200 999
213	142	107	87											1200 999

TABLE 2 - SURFACE INTERVAL TIME (SIT) TABLE

TIME RANGES IN HOURS MINUTES ©1987 NAUI

NEW GROUP	A	B	C	D	E	F	G	H	I	J	K	L	M	N
← A	1200 010	1200 211	1200 250	1200 549	1200 633	1200 706	1200 736	1200 800	1200 822	1200 841	1200 859	1200 913	1200 929	1200 944
← B	←	210 010	249 140	548 239	632 323	705 358	735 426	759 450	821 513	840 541	858 549	912 603	928 619	943 633
← C	←	←	139 010	238 110	322 158	357 229	425 259	449 321	512 344	540 403	548 420	602 436	618 450	632 504
← D	←	←	←	109 010	157 055	228 130	258 200	320 224	343 245	402 305	419 322	435 337	449 353	503 405
← E	←	←	←	←	054 010	129 046	159 116	223 142	244 203	304 221	321 239	336 254	352 309	404 323
← F	←	←	←	←	←	045 010	115 041	141 107	202 130	220 148	238 204	253 220	308 235	322 248
← G	←	←	←	←	←	←	040 010	106 037	129 100	147 120	203 136	219 150	234 206	247 219
← H	←	←	←	←	←	←	←	036 010	059 034	119 055	135 112	149 126	205 140	218 154
← I	←	←	←	←	←	←	←	←	033 010	054 032	111 050	125 105	139 119	153 131
← J	←	←	←	←	←	←	←	←	←	031 010	049 029	104 046	118 100	130 112
← K	←	←	←	←	←	←	←	←	←	←	028 010	045 027	059 043	111 055
← L	←	←	←	←	←	←	←	←	←	←	←	026 010	042 026	054 040
← M	←	←	←	←	←	←	←	←	←	←	←	←	025 010	039 025
← N	←	←	←	←	←	←	←	←	←	←	←	←	←	024 010

TABLE 3 - REPETITIVE DIVE TIMETABLE

00 LIGHT FACE NUMBERS ARE RESIDUAL NITROGEN TIMES (RNT)
00 BOLD FACE NUMBERS ARE ADJUSTED NO-DECOMPRESSION LIMITS (ANDL) • ACTUAL BOTTOM TIME SHOULD NOT EXCEED THIS NUMBER

The choice $B = 60/33$, with $kA + B = k$, hence $A = 1 - 60/33k$ gives a solution

$$p = 1 + \frac{60}{33} \left(t - \frac{1}{k} \right).$$

We call this a *particular integral*. If we then write

$$u = p - \left[1 + \frac{60}{33} \left(t - \frac{1}{k} \right) \right] = p - 1 - \frac{60}{33}t + \frac{60}{33k},$$

where p is any solution of the equation, it follows that

$$\frac{du}{dt} + ku = \frac{dp}{dt} + kp - \frac{60}{33} - k \left(1 + \frac{60t}{33} \right) + \frac{60}{33} = 0,$$

since p is a solution of (5).

If $du/dt + ku = 0$, then we can again use separation of variables to get

$$\int \frac{1}{u} \frac{du}{dt} dt = \int k dt$$

which implies that $-\ln|u| = kt + C$, or $u = Ae^{-kt}$, where A is an arbitrary constant. In this approach, u is usually called the *complementary function*. Thus, if p is any solution of (5), it can be written as

$$p = 1 + \frac{60}{33} \left(t - \frac{1}{k} \right) + u = 1 + \frac{60}{33} \left(t - \frac{1}{k} \right) + Ae^{-kt};$$

that is, any solution is the sum of a particular integral and a complementary function. The technique may be used on any first-order linear equation. To satisfy an initial condition $p(0) = p_0$, we get

$$1 - \frac{60}{33k} + A = p_0 \quad \text{or} \quad A = p_0 - 1 + \frac{60}{33k},$$

$$p = 1 + \frac{60t}{33} - \frac{60}{33k} + \left(p_0 - 1 + \frac{60}{33k} \right) e^{-kt} \quad (6)$$

$$= 1 + \frac{60t}{33} - \frac{60}{33k} + \left(p_0 - 1 + \frac{60}{33k} \right) \left(\frac{1}{2} \right)^{(t/T)}. \quad (7)$$

A similar solution could be obtained for an ascent from a given depth.

Example 6. Find the pressure in a 20-min tissue on arrival at a depth of 100 ft (4 atm) after a descent from the surface at a rate of 60 ft/min.

The time to descend 100 ft at 60 ft/min is $100/6 = 5/3$ min.

The initial pressure is $p_0 = 1$, and $k = \ln 2/T = .03466$.

Therefore,

$$p = 1 + \frac{60}{33} \cdot \frac{5}{3} - \frac{60}{33(.03466)} + \frac{60}{33(0.3466)} \left(\frac{1}{2}\right)^{1/12} = 1.086.$$

To do a complete dive, we would have to include these changes of pressure in the complete diving schedule. We will not do this, although it is merely tedious rather than difficult.

We note finally that if the descent had been considered instantaneously, the pressure after 5/3 min at a depth of 100 ft would be 1.17 atm.

Exercise

- Find the tissue pressure for a 20-min tissue at the end of an ascent from 100 ft to 10 ft at a speed of 60 ft/min, assuming that the pressure at the beginning of the ascent was 4 atm. Compare it with the pressure at 10 ft after an instantaneous ascent.

10. Conclusion

In this Module, we have discussed a simple technique for derivation of diving tables, which is based on a model proposed by Haldane. Although modern diving tables cannot be devised by means of such simple techniques, most of them have been developed by refinements to the simple model and methods proposed by Haldane, as tempered by experience (see, for example, Bornmann [1970]).

11. Solutions to the Exercises

All solutions use either (4) or its inverse:

$$p = p_e + (p_0 - p_e) \left(\frac{1}{2}\right)^{t/T} \quad \text{or} \quad t = T \ln \left(\frac{p_0 - p_e}{p - p_e}\right) / \ln 2.$$

- During the dive, $p_e = 3.5$, $t = 40$ min, $p_0 = 1$.
For $T = 10$, $p = 3.344$; for $T = 20$, $p = 2.875$. It is safe to ascend to 20 ft = 1.6 atm, because $2.15 \times 1.6 = 3.44$. The stop at 1.6 should be long enough that an ascent to 1.3 will be safe. This requires that p be reduced to $2.15 \times 1.3 = 2.795$.

For $T = 10$, this requires $t = 10 \ln \left(\frac{3.344 - 1.6}{2.795 - 1.6}\right) / \ln 2 \simeq 5.454$ min
($p_0 = 3.344$, $p_e = 1.6$).

For $T = 20$, this requires $t = 20 \ln \left(\frac{2.875 - 1.6}{2.795 - 1.6} \right) / \ln 2 \simeq 1.870$ min ($p_0 = 2.875$, $p_e = 1.6$).

Thus, a stop of 5.454 min is required. After 5.454 min, the pressure in the $T = 20$ tissue is 2.655, and that in the $T = 10$ tissue is 2.795.

The stop at 1.3 (10 ft) should be long enough that an ascent to the surface (1 atm) is safe. This requires that p be reduced to 2.15.

For $T = 10$, this requires $t = 10 \ln \left(\frac{2.795 - 1.3}{2.15 - 1.3} \right) / \ln 2 \simeq 8.146$ min.

For $T = 20$, this requires $t = 20 \ln \left(\frac{2.655 - 1.3}{2.15 - 1.3} \right) / \ln 2 \simeq 13.455$ min.

A safe schedule is then a 5.454-min stop at 1.6 (20 ft) and a 13.455-min stop at 10 ft. The total stopping time is 18.909 min.

2. By similar means as in **Exercise 1**, the pressures at the end of the dive where $p_e = 4$, $p_0 = 1$, $t = 120$ are: for $T = 10$, $p = 4$; for $T = 20$, $p = 3.953$; for $T = 40$, $p = 3.625$.

Stop 1 at 1.9 atm: (This is safe since since $1.9 \times 2.15 = 4.085$.) Times to reduce pressure to $1.6 \times 2.15 = 3.44$ are: for $T = 10$, 4.47 min; for $T = 20$, 8.296 min; for $T = 40$, 6.547 min.

A stop of 8.296 min is required. After this stop, the $T = 10$ tissue will have a pressure below that of the $T = 20$ tissue, and this will remain true for the rest of the dive. We need not consider the $T = 10$ tissue further.

After 8.296 min at 1.9, $T = 20$ has pressure 3.44 and $T = 40$ has pressure 3.39.

Stop 2 at 1.6 atm: Times to reduce pressure to $2.15 \times 1.3 = 2.795$ are: for $T = 20$, 12.5 min; for $T = 40$, 23.4 min. From this point on we need only consider the $T = 40$ tissue. After stop 2, its pressure is 2.795.

Stop 3 at 1.3 atm: Time to reduce pressure to 2.15 is 32.584 min for the $T = 40$ tissue.

The total time for all stops is 64.3 min.

3. The same dive as in **Exercise 2**. We consider the 40-min tissue only and make three stops of equal time. The first stop is at 1.9, but the depth of the remaining stops must be calculated from the condition of equal times.

After the dive, the pressure in the $T = 40$ tissue is 3.625. Ascent to 1.9 is certainly safe.

Suppose that the second and third stops are at pressures p_2 , p_3 . Then the diver must stay at 1.9 until $p = 2.15p_2$, must stay at p_2 until $p = 2.15p_3$, and must stay at p_3 until $p = 2.15$. From the inverse of (4), the equalization times are

$$\begin{aligned} t_1 &= \frac{40}{\ln 2} \ln \left(\frac{3.625 - 1.9}{2.15p_2 - 1.9} \right) = \frac{40}{\ln 2} \ln \left(\frac{2.15p_2 - p_2}{2.15p_3 - p_2} \right) \\ &= \frac{40}{\ln 2} \ln \left(\frac{2.15p_3 - p_3}{2.15 - p_3} \right). \end{aligned}$$

This gives

$$\frac{1.725}{2.15p_2 - 1.9} = \frac{1.15}{2.15\frac{p_3}{p_2} - 1} = \frac{1.15}{2.15\left(\frac{1}{p_3}\right) - 1}.$$

The last two equations give $p_3/p_2 = 1/p_3$, or $p_2 = p_3^2$. The first two then give $2.4725p_3^3 - 0.46p_3 - 3.70875 = 0$. The only real positive solution is $p_3 = 1.199$. Thus, $p_2 = p_3^2 = 1.438$ and $t_1 = 21.438$. The total stop time is $3t_1 = 64.314$, a very small improvement. We can verify that after the first stop, the $T = 20$ pressure is 2.877 and the $T = 10$ pressure is 2.375, both below the 3.092 ($= 2.15 \times 1.438$) of the $T = 40$.

4. We assume that the tissue pressure at the beginning of the ascent is p_0 , which is known. The three stops will be at pressures p_1, p_2, p_3 , where $p_1 = p_0/2.15$ and the pressures at the ends of the stops will be $2.15p_2, 2.15p_3, 2.15$. The times at each stop will then be

$$t_1 = \frac{T}{\ln 2} \ln \left(\frac{p_0 - p_1}{2.15p_2 - p_1} \right) = \frac{T}{\ln 2} \ln \left(\frac{1.15}{2.15\frac{p_2}{p_1} - 1} \right),$$

$$t_2 = \frac{T}{\ln 2} \ln \left(\frac{2.15p_2 - p_2}{2.15p_3 - p_2} \right) = \frac{T}{\ln 2} \ln \left(\frac{1.15}{2.15\frac{p_3}{p_2} - 1} \right),$$

$$t_3 = \frac{T}{\ln 2} \ln \left(\frac{2.15p_3 - p_3}{2.15 - p_3} \right) = \frac{T}{\ln 2} \ln \left(\frac{1.15}{2.15\left(\frac{1}{p_3}\right) - 1} \right).$$

We wish to minimize $t_1 + t_2 + t_3$ by choosing p_2 and p_3 . This is equivalent to maximizing

$$F(p_2, p_3) = \ln \left(\frac{p_2}{p_1} - M \right) + \ln \left(\frac{p_3}{p_2} - M \right) + \ln \left(\frac{1}{p_3} - M \right),$$

where $M = 1/2.15$ and p_1 is known. Using calculus, we find

$$\frac{\partial F}{\partial p_2} = \frac{1}{\left(\frac{p_2}{p_1} - M\right)} \frac{1}{p_1} + \frac{1}{\left(\frac{p_3}{p_2} - M\right)} \left(\frac{-p_3}{p_2^2}\right) = 0,$$

$$\frac{\partial F}{\partial p_3} = \frac{1}{\left(\frac{p_3}{p_2} - M\right)} \frac{1}{p_2} + \frac{1}{\left(\frac{1}{p_3} - M\right)} \left(\frac{-1}{p_3^2}\right) = 0.$$

This gives

$$\frac{p_2}{p_2 - Mp_1} = \frac{p_3}{p_3 - Mp_2} = \frac{1}{1 - Mp_3},$$

and hence $p_2^2 = p_1 p_3$, $p_2 = p_3^2$, and finally

$$p_3 = p_1^{1/3}, \quad p_2 = p_1^{2/3}.$$

This also gives

$$t_1 = t_2 = t_3 = \frac{T}{\ln 2} \ln \left(\frac{1.15}{2.15/p_1^{1/3} - 1} \right).$$

(For $p_0 = 3.625$, we have $p_1 = 1.686$, $p_2 = 1.417$, $p_3 = 1.190$, $t_1 = 20.482$ min, and the total time $3t_1 = 61.447$ min.)

5. For a safe continuous ascent, the external pressure should be the tissue pressure divided by 2.15. The differential equation for $p(t)$ then becomes

$$\frac{dp}{dt} = k(p_e - p) = k \left(\frac{p}{2.15} - p \right) = -k \frac{1.15}{2.15} p, \quad k = \ln 2/T$$

$$\frac{dp}{dt} = -.535kp.$$

The solution of this equation is $p = p(0)e^{-.535kt}$, where $p(0)$ is the pressure at time $t = 0$. The diver's depth at time t is related to $p_e(t)$ ($= p(t)/2.15$) by

$$1 + \frac{d}{33} = p_e(t) = p(0)e^{-.535kt}/2.15.$$

For a long dive at 4 atm and $T = 40$, we have

$$\begin{aligned} d &= 33 \times 1.86 \left(\frac{1}{2} \right)^{.535t/40} - 33 \\ &= 33 \left[1.86 \left(\frac{1}{2} \right)^{.0134t} - 1 \right]. \end{aligned}$$

The time to ascend to the surface is the value of t at which $d = 0$, that is,

$$t = \frac{1}{.0134} \frac{\ln 1.86}{\ln 2} \approx 66.81.$$

6. If the partial pressure of nitrogen is $0.8p$, where p is the tissue pressure, then the maximum safe pressure for the nitrogen is 0.8×2.15 , so that the condition $p < 2.15$ is retained. Moreover, if the external gas pressure is p_e , the external nitrogen pressure is $0.8p_e$, and the equation for absorption of nitrogen will be

$$\frac{d}{dt} (.8p) = k(.8p_e - .8p)$$

with initial nitrogen pressure $.8p_0$. Thus, the differential equation for the pressure is the same and the criterion for safe ascent is the same.

7. **Table 5** gives the pressures at the ends of the stops for the half-times 5, 10, 20, 40, and 75 min.

Table 5.
Pressures at the ends of the stops for the dive of **Exercise 7**.

	5	10	20	40	75	Safe pressure at next stop
45 min at 3.58	3.57	3.46	3.04	2.40	1.88	4.08
2 min at 1.9	3.17	3.26	2.96	2.38		3.44
7 min at 1.6		2.62	2.67	2.29		2.795
15 min at 1.3			2.11	2.06		2.15

From the table, we see that a safe pressure has been reached to ascend to the next stop in all cases. The blanks in the 75 column have not been calculated, since they will all be less than 1.88. In the 5 column, the blanks will be less than the corresponding entries in the 10 column, and the final entry in the 10 column will be less than that in the 20 column.

8. First dive at 3.4 atm for 15 min. Pressures will be for $T = 20$, 1.97; for $T = 40$, $p = 1.55$.

After 60 min at the surface, $p_e = 1$. For $T = 20$, $p = 1.12$; for $T = 40$,

Descent to 4 atm. Diver may remain until tissue pressure is 2.15. For $T = 20$, this requires 12.77 min; for $T = 40$, 24.12 min.

The diver must still return to the surface after 12.77 min.

9. First dive 4 atm for 10 min. Pressures: for $T = 20$, $p = 1.88$; for $T = 40$, $p = 1.48$.

After 60 min at 1 atm: for $T = 20$, $p = 1.11$; for $T = 40$, $p = 1.17$.

Second dive to 3.4 atm until $p = 2.15$. For $T = 20$, 17.47 min; for $T = 40$, 33.40 min.

The diver must ascend after 17.47 min.

10. We use (7). The time for ascent is $90/60 = 3/2$ min, so we have

$$p = 1 + \frac{60}{33} \cdot \frac{3}{2} - \frac{60}{33(\ln 2/20)} + \left(3 + \frac{60}{33(\ln 2/20)} \right) \left(\frac{1}{2} \right)^{3/40} = 3.92.$$

A stop of 1.5 min at 10 ft (1.3 atm) reduces the pressure to 3.86 atm.

References

- Bachrach, Arthur. 1982. A short history of man in the sea. In Bennett and Elliott [1982], 1–14.
- Barnes, Ron. 1987. Compartment models in biology. UMAP Modules in Undergraduate Mathematics and Its Applications: Module 676. *The UMAP Journal* 8 (2): 133–160. Reprinted in *UMAP Modules: Tools for Teaching 1987*, edited by Paul J. Campbell, 207–234. Arlington, MA: COMAP, 1988.
- Bennett, Peter B., and David H. Elliott, ed. 1982. *The Physiology and Medicine of Diving*. 3rd ed. London: Bailliere Tindall.
- Bornmann, Robert C. 1970. U.S. Navy experiences with decompression from deep helium oxygen saturation excursion diving. In *Human Performance and Scuba Diving*, Proceedings of the Symposium on Underwater Physiology at Scripps Institute of Oceanography, La Jolla, Calif. Chicago, IL: Athletic Institute.
- Hammes, Richard B., and Anthony G. Zimos. 1988. *Safe Scuba*. Long Beach, CA: The National Association of SCUBA Diving Schools. Chapter 9.
- Hempleman, Henry V. 1982. History of evolution of decompression procedures. In Bennett and Elliott [1982], 319–351.
- Hills, B.A. 1977. *Decompression Sickness*. New York: John Wiley & Sons.
- Moon, Richard E., Richard D. Vann, and Peter B. Bennett. The physiology of decompression illness. *Scientific American* 273 (2) (August 1995): 70–77.
- Vann, Richard D. 1982. Decompression theory and applications. In Bennett and Elliott [1982], 352–382.
- Walder, Dennis. 1982. The compressed air environment. In Bennett and Elliott [1982], 15–30.

About the Author

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DECOMPRESSION THEORY AND THE RDP

The Haldanean Decompression Model:

Virtually all dive tables and dive computers calculate no decompression limits and decompression stops (when needed) based on a *Haldanean decompression model*. This is named after John Scott Haldane who developed the first such mathematical decompression model and based on it the first dive tables in 1906.

Modern decompression models are based on the same ideas.

When the diver descends to a given depth, the nitrogen pressure in his breathing air is higher than the nitrogen *tissue pressure* in his body, so more nitrogen dissolves into the body tissues.

With enough time, the nitrogen pressure equalizes, and the body cannot take on any more nitrogen. This is called saturation.

When the diver ascends the nitrogen *tissue pressure* in the body becomes higher than the nitrogen pressure in his breathing air, causing the tissues to release nitrogen to equalize the nitrogen pressure again.

The difference between the dissolved nitrogen tissue pressure and the nitrogen pressure in the breathing air is called the *pressure gradient*. Whether the diver is descending or ascending.

When the diver ascends the tissues can tolerate some gradient of high tissue pressure without causing decompression sickness.

If the *pressure gradient* exceeds acceptable limits (supersaturation), bubbles may form and cause decompression sickness.

Decompression sickness can be avoided by keeping the gradient within acceptable limits.

This means the diver must stay within the limits dictated by his table or computer and maintain a slow ascent rate as indicated by his tables or computer.

Haldane discovered that different parts of the body absorb and release dissolved nitrogen at different rates – slow and fast compartments.

To account for these differences he constructed a model consisting of five theoretical tissues.

These theoretical tissues do not directly correspond to any particular body tissue so they are called compartments or tissue compartments. The RDP has 14 compartments.

Each compartment has a halftime for the rate at which it absorbs and releases nitrogen.

Halftime is the time, in minutes, for a compartment to go halfway from its beginning tissue pressure to complete saturation.

After one halftime the tissue would be 50% saturated

After two halftimes the tissue would be 75% saturated

After three halftimes the tissue would be 87.5% saturated

After four halftimes the tissue would be 93.75% saturated

After five halftimes the tissue would be 96.875% saturated

After six halftimes the tissue would be 98.4375% saturated

It would never reach 100% using the halftime concept, so after *six halftimes the tissue compartment is considered full or empty.*

Haldane's original halftimes ranged from 5 to 75 minutes.

The RDP's halftimes range from 5 to 480 minutes, split over 14 compartments.

They are 5, 10, 20, 30, 40, 60, 80, 100, 120, 200, 240, 300, 360 and 480 minutes.

Sometimes tissue pressure is expressed in metres of seawater (gauge) - msw.

Example: A 5 minute halftime compartment will have a tissue pressure of 9msw after 5 minutes in 18 metres of seawater.

Example: A 20 minute halftime compartment will have a tissue pressure of 18msw after 40 minutes in 24m of seawater.

Example: A 60 minute halftime compartment will take 360 minutes (6 hours) to saturate to a given depth. (60 x 6 halftimes).

Besides different halftimes each compartment has a different M-value.

The M-value is the maximum tissue pressure allowed in the compartment when surfacing to prevent exceeding the acceptable gradient.

There are actually different M-values for each compartment at different depths, these are used to calculate decompression schedules. *In no decompression diving we only use the one that applies to the surface*

The *slower* the compartment, the *lower* the M-value.

The *faster* the compartment, the *higher* the M-value.

The M-value is determined by test dives showing what does and what does not result in Doppler detectable bubbles.

Remember that the M-values are calculated for surfacing at sea level which is why you need to apply special procedures when diving at altitudes above 300m.

When any compartment reaches its M-value the dive ends or it becomes a decompression dive.

On deeper dives faster compartments will reach their M-values first, hence deeper dives have short no decompression limits.

On shallower dives, the depth is not enough for the faster compartments to reach their M-values. Therefore a slower compartment controls the dive and the model allows more no decompression time.

The compartment that reaches its M-value first is called the *controlling compartment*.

These models are mathematical extrapolations; there is no **direct** relationship between the decompression model and the human body. This is why divers learn that there is always some risk of DCS even within table/computer limits and are asked to dive conservatively within the limits.

US Navy tables:

The first dive tables to be widely used and adapted to recreational diving where the U.S.Navy tables designed in the 1950's.

Six compartments were used with a slowest halftime of 120 minutes.

While at the surface all compartments would lose nitrogen at a different rate depending on their halftime. Any compartment could control a repetitive dive, depending on the first dive, the surface interval and the second dive.

To solve this problem the U.S.Navy designed its surface interval credit on the worst case scenario, the slowest compartment (120 mins). This is why it takes 12 hours (720 mins. 6 x 120) to be "clean" when using their tables.

These tables were tested with US Navy divers, subjects were all male in their 20's and 30's and reasonably fit. The test criteria were bends/no bends.

The Recreational Dive Planner (RDP):

In the mid-1980's, Dr. Raymond Rogers recognized that the USN tables were not ideal for recreational diving.

The 120 minute half time used for surface interval credit, while appropriate for decompression diving, seemed excessively conservative for recreational divers making only no-decompression dives.

The test group the USN used didn't reflect recreational divers who include females and people of all ages.

New technology in the shape of Doppler ultrasound flowmeters had come into being ; these showed that silent bubbles often formed at USN table limits, suggesting lower M-values would be more appropriate for recreational divers.

With the help of DSAT (Diving Science & Technology), Rogers developed the RDP. It was tested in 1987/88 at the Institute of Applied Physiology & Medicine with Dr. Michael Powell as the principal investigator.

A 60 minute gas washout tissue was used. Multi-level diving was tested with a large range of test subjects - recreational divers. Limited to Doppler detectable bubbles instead of bends/no bends. Tested to the limits for 4 dives per day for 6 days. Though more conservative diving practices are recommended.

Dr. Rogers found that the old 120 minute gas washout tissue was too conservative for recreational diving and adopted a 60 minute gas washout tissue. This means you get twice as much credit for surface intervals and are clean in 6 hours. The WXYZ rules make sure the slower compartments stay within limits. Dr. Rogers also lowered the M-values to match recent Doppler data. These are sometimes called Spencer limits after the physician who first proposed them.

They produced different versions of the RDP. The table version, (because that's what divers were familiar with) and the multilevel electronic planner eRDPML version (originally the Wheel), to enable you to calculate multi level profiles. DSAT have also produced four tables for enriched air diving. Tables for using EANx32 and EANx36 an equivalent air depth table and an oxygen exposure table. The pressure groups from all versions of the RDP are interchangeable.

The RDP works on 14 compartments, instead of the 6 used in making the U.S. Navy tables.

You cannot use PADI RDP pressure groups with other agencies' tables.

Dive Computers:

Dive computers offer maximum bottom time by essentially writing a custom dive table for the dive undertaken – this eliminates unnecessary rounding and therefore gives more dive time.

There are essentially 5 different groups of models or algorithms used in the many computers available to the recreational diver. This will normally be described in the instruction book for the particular computer. They are being developed all the time with diver safety in mind as more research is done.

1. Spencer limits, EE washout

- Same M-values as RDP
 - All compartments release nitrogen at the surface at their underwater halftime rates.
 - Can permit dives that are beyond what is safe, i.e. short deep repetitive dives with short surface intervals.
2. Spencer limits, 60 minute washout
 - Based on data for the RDP
 - Dives similar to what the RDP allows.
 3. Buhlmann limits, EE washout
 - Further reduced M-values
 - All compartments release Nitrogen at the surface at their underwater halftime rates.
 - Because of the reduced M-values similar to what the RDP data supports despite the EE washout.
 4. RGBM (Reduced Gradient Bubble Model) and others.
 - Research is providing lots of new information on the behavior of divers and micro bubble build up.
 - Most dive computer models take this into account
 - If a diver exceeds a safe ascent rate on one dive he will be penalized on repetitive dives, the same with yoyo profiles.
 - Some take the water temperature into account and adjust accordingly.
 - Nearly all have altitude settings and settings for conservatism.
 - Some are integrated with air supply and take the divers breathing rate into account.
 - Nearly all models now support Nitrox diving.
 - Some support gas switch extended range and technical diving.
 - Some support trimix and CCR diving.

PADI Recommendation for Diving with Computers:

- Divers should not attempt to share a dive computer.
- Each diver must use the same computer through a series of dives.
- Each diver must have his own computer.
- Computers have the same theoretical basis as tables so one is neither better nor safer than the other.
- All standard guidelines apply, such as deepest dives first.
- Follow all manufacturers' recommendations.
- End the dive based on the most conservative computer of a buddy team (you're supposed to stick together anyway)!
- If a computer fails whilst diving, ascend slowly to 5m and make a long safety stop as long as your air supply permits. You should then remain out of the water for 12 – 24 hours, so you can start clear again with another table or computer.
- Make sure it is capable of altitude diving if diving at altitude.

- Do not lend you computer to another diver if either of you have been diving.
- Do not use a computer from another diver if either of you have been diving.
- Do not try to change the battery between dives or underwater.
- If it caters for mixed gas make sure it is set to the gas you are using.
- Do not use the computer if it is displaying any error or not functioning correctly
- When you turn your dive computer on, do not turn your brain off, after all the latter is a better computer!



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