The equivalent bulk-diffusion model of the pneumatic decompression computer*

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Abstract—*It is shown that the pneumatic-decompression computer is essentially an analogue of the bulk-diffusion equation into a rectangular slab of tissue. The empirical parameters of the instrument have been used to match the nearest equivalent diffusion time scale which appears longer than that determined by Hempleman (1969) and Hills (1969). The equivalent diffusion coefficient is a linear function of the pressure with consequent faster gas uptake than elimination on a dive.*

Keywords—*decompression formats, analogue simulation, during physiology*

Introduction

A METHOD of simulating in real time the decompression format of a diver has been developed by the Canadian Forces Medical Service (STUBBS and KIDD, 1965), and has been successfully used in multilevel exposures to 60 m depth.

The instrument consists of four equal resistors in series (originally different resistors in parallel), through which inert gas flows, supposedly matching gas uptake and elimination in living tissue. A mathematical analysis of the gas-flow characteristics of the resistors has been given by WEAVER and STUBBS (1968), and agrees well with experimental data.

Much emphasis has been attributed to the nonlinear behaviour of the computer, in its undoubted success in predicting safe dive formats, especially of a repetitive nature (KIDD *et al.*, 1969, 1970). However, the question arises whether the computer is at all matched with existing decompression models of gas uptake in tissue.

Thus, if one accepts that bulk diffusion into cellular material controls gas uptake in tissue necessary for predicting a decompression format (HILLS, 1969), it is of interest to know the relative importance of the nonlinear aspects of the instrument and its overall time scale.

The analogue

The gas mixture that is supplied to the diver is also fed to the analogue, and begins flowing through the first resistor into a chamber, and thence through a series of three more resistors and chambers. The pressure in each cavity is metered by a bourdon tube gauge.

A decompression format is achieved by the diver himself controlling the highest pressure of the four chambers in such a way as to ensure that this peak tension never exceeds a constant times the ambient pressure.

Gas flow in each resistor is essentially governed by 'slip flow', where the length of the free path of a molecule is comparable to the typical pore diameter of the resistor ($\simeq 50$ nm).

Thus the equations of flow take the form (WEAVER, KUEHN and STUBBS, 1968)

$$\frac{dp_i}{dt} = -A\{(p_i - p_{i-1})(p_{i-1} + p_i + B) - (p_{i+1} - p_i)(p_i + p_{i+1} + B)\} \quad (1)$$

$$i = 1, 2, 3, 4$$

where p_i is the pressure in the ith chamber, and $p_0 = p_A$, the external ambient pressure, and $p_5 = p_4$.

Typical values of A and B for air or nitrogen were determined empirically, in the region of

$$A = 1{\cdot}8 \times 10^{-4}\ \text{min}^{-1}$$

$$B = 8{\cdot}274 \times 10^{5}\ \text{N/m}^2$$

The above equations are nonlinear, and this effect will become increasingly noticeable the greater the pressure.

Now, to simulate a bulk-diffusion system, an analogue is required which obeys Fick's law, i.e. the mass flux is proportional to the diffusion coefficient (D) times the gradient of the gas density (or concentration). This law, in conjunction with a mass-balance equation, leads to the partial-differential equation (CRANK, 1956)

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial p}{\partial x}\right) \quad . \quad . \quad . \quad (2)$$

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where p is the pressure after time t at a point x, and x is measured in the direction of flow from one end of the slab of material.

Eqn. 2 may be split up into a finite difference scheme. If a 4-compartment array is employed, it is necessary to determine the form of D such that eqn. 2 reduces to eqn. 1.

It is easily verified below that the form of D is

$$D = \frac{AL^2}{16}(2p+B) \quad . \quad . \quad . \quad (3)$$

where L is the thickness of the material.

Using this form of D, eqn. 2 can now be rewritten in a more appropriate form for finite differences as

$$\frac{\partial p}{\partial t} = \frac{L^2A}{16}\frac{\partial^2 p^2}{\partial x^2} + \frac{L^2AB}{16}\frac{\partial^2 p}{\partial x^2}.$$

Finite differences on the above equation yield

$$\frac{dp}{dt} = A(p^2{}_{i+1} - 2p^2{}_i + p^2{}_{i-1}) + AB(p_{i+1} - 2p_i + p_{i-1})$$

and eqn. 1 is quickly recovered on rearrangement of the latter.

Thus the pneumatic computer is essentially an approximation of the bulk-diffusion equation into a slab of material, where the diffusion coefficient is a linear function of pressure, given by eqn. 3.

Discussion

Whether the gas diffusion coefficient in cellular material varies with pressure in a similar manner is at present unknown. It is possible that at very high pressures, the diffusion coefficient may be an increasing function of pressure, in which case gas uptake will be more rapid than elimination. This form of asymmetry has been noted by HEMPLEMAN (1957) and EATON (1970). Their explanation that bubble formation and expansion on ascent interfere with gas elimination is, if correct, a much more dominant reason for asymmetrical behaviour than the likely variation of the diffusion coefficient in the small range of pressures considered.

However, at ultrahigh pressures in the region of 40 atm, it is very likely that not only the diffusion coefficient but also the solubility of the inert gas will vary with pressure. The behaviour of the inert gas at these pressures will need to be known precisely in order to optimise decompression formats.

It is of interest to determine the numerical values of D for the pneumatic computer using the typical values of A and B. The computer has been thoroughly tested in the range 0–60 m, and the pressure p varies from $1{\cdot}034 \times 10^5$ to $6{\cdot}895 \times 10^5$ N/m^2; thus from eqn. 3 it is found that

$$0{\cdot}00169 \leqslant D/L^2 \leqslant 0{\cdot}00360 \text{ min}^{-1}$$

Comparing these values with HEMPLEMAN's (1969) linear-model value of 0·00321 min^{-1} and HILLS' (1969) averaged radial model value of 0·00516 min^{-1}, it is seen that the pneumatic computer's time scale is generally longer than that of straightforward diffusion models, and approaches current values only at the greater depth zones.

The value of L is unknown, but may be assumed to be of the order of the semidiameter of a typical tissue cell (15·4 μm); D then lies in the range

$$0{\cdot}67 \times 10^{-10} \leqslant D \leqslant 1{\cdot}4 \times 10^{-10} \text{ cm}^2/\text{s}$$

somewhat less than the value $2{\cdot}2 \times 10^{-10}$ cm^2/s for nitrogen in cytoplasm, as predicted by HILLS (1967b), based on acetylene uptake in excised rabbit tissue at atmospheric pressure. However, a revised value of $D = 1{\cdot}7 \times 10^{-10}$ cm^2/s has been obtained by HENNESSY (1971), using a more general mathematical analysis.

The fact that the pneumatic computer has successfully achieved a large variety of safe decompression formats, while using a longer diffusion time scale than current models, may be attributed to its inherent asymmetry.

Thus, since gas uptake is slower than in the constant-diffusion-coefficient models, and in turn uptake is faster than elimination, the combined effect appears to be such that it agrees well with linear models.

The fact that the linear bulk-diffusion equation appears to provide an incomplete model for decompression formats has been indicated by HEMPLEMAN (1969). Here, it was found necessary to lower the critical ratio on long deep dives.

This requirement, of course, may have a number of explanations, including bubble interaction.

However, the pneumatic computer uses a single ascent ratio, based on the peak tension of any one of its four chambers. While seeking the peak, the tension may be thought as effectively using a variable ratio, and it should be noted that the gas flow is markedly nonlinear on long deep dives (STUBBS and WEAVER 1968).

Thus the position appears confusing, especially as EATON (1970) has suggested that there is no unique ascent path to the surface.

In fact, it does not seem likely that either pneumatic flow through micropores, or bulk diffusion with D a linear function of pressure, are models representative of the real biological situation at low pressures. It is far more likely that ordinary linear diffusion with bubble growth on ascent is the main cause of the observed gas uptake–elimination asymmetry.

In any event, it may be supposed that the basic parameters and critical ratio used in the computer may be merely an optimum combination, approximating to a particular safe decompression format in the range 0–60 m.

It should be pointed out that the pneumatic computer uses only four compartments to what now

appears to be a numerical finite-difference approximation to the bulk-diffusion equation. Normally, at least eight compartments should be used for accuracy. HILLS (1967*a*), for example, takes 27 compartments to fit the radial diffusion equation used in his linear pneumatic analogue.

Thus it may be surmised that any extension of the present computer to as many compartments may be difficult because the nonlinear effects of the compressible gas may tend to dominate undesirably.

If, in fact, gas uptake in tissue is governed by a diffusion coefficient which is some function of pressure, any pneumatic analogue will clearly eventually deviate from the true situation, unless it matches faithfully the nonlinear diffusion law in tissue.

The important experiment of determining the precise dependence of the cellular-diffusion coefficient and solubility on pressure at ultradepths remains to be carried out.

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Modèle de diffusion en masse équivalent à l'ordinateur de décompression pneumatique

Sommaire—Il a été démontré que l'ordinateur de décompression pneumatique était essentiellement analogique à l'équation de diffusion en masse dans une plaque rectangulaire de tissu. On a utilisé les paramètres empiriques de l'instrument pour correspondre à l'échelle de temps de diffusion équivalente la plus proche qui semble plus longue que celle déterminée par Hempleman (1969) et Hills (1969). Le coefficient de diffusion équivalente est une fonction linéaire de la pression avec comme conséquence une prise de gaz plus rapide que l'élimination sur une plongée.

Das äquivalente Massendiffusionsmodell zum pneumatischen Dekompressionsrechner

Zusammenfassung—Es wird gezeigt, dass der pneumatische Dekompressionsrechner ein Analogon zu der Gleichung darstellt, welche die Massendiffusion in eine rechteckige Gewebescheibe beschreibt. Die empirischen Parameter des Gerätes wurden für eine Annäherung an die nächste äquivalente Diffusionszeit benutzt, welche länger erscheint als von Hempleman (1969) und Hills (1969) angegeben. Der äquivalente Diffusionskoeffizient ist eine lineare Funktion vom Druck mit daraus resultierender schnellerer Gasaufnahme als -abgabe beim Tauchen.