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The square-root principle in the calculation of one-stage (no-stop) decompression tables

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Behnke, A. R. 1979. The square-root principle in the calculation of one-stage (no-stop) decompression tables. Undersea Biomed. Res. 6(4): 357–365.—A square-root formulation, $\sqrt{\text{time}} \times$ excursion depth $= k$, developed at the Royal Navy Physiological Laboratory (RNPL) predicts the safe duration of air excursions at 1 ATA for periods up to 6 h (air) and up to 3 h (heliox atmosphere). At pressures above 1 ATA, the value of k was initially considered to be proportional to Haldanian multiples of ATA and not to the $\sqrt{\text{ATA}}$ used in this paper. Excursions of infinite (∞) duration can also be predicted from a square-root formula derived from the systematic heliox saturation excursions conducted at the Experimental Diving Unit in which net excursion depth $= k \sqrt{\text{P abs}}$. This paper presents a broad spectrum format that extends the square-root principle: 1) to values of k above 1 ATA in the RNPL formula and 2) to excursions for (∞) time from habitat depths to 820 ft; and 3) which demonstrates a relationship between fractional time excursions and excursions for infinite periods of time. Application of the generalization that net excursion depths are proportional to $\sqrt{\text{P abs}}$ would reduce to a minimum the number of programmed test dives and would, in addition, allow such dives to be conducted at relatively low pressures, with only occasional validating tests at deep depths.

decompression tables
decompression calculations
excursion diving
saturation diving
no-decompression dives

It is time consuming, laborious, and often unrewarding to conduct test excursion dives from various habitat or chamber depths under steady-state (saturation) conditions, even though the only requirement is the attainment of the safest type of decompression, namely a one-stage (no-stop) reduction in pressure. If the steady-state environment is limited to 1 ATA or relatively low pressures, with only an occasional foray at deep depths, progress in current (and often desultory) test programs would be expedited.

The objective of this paper is to outline the type of tabular format directed to reducing the number of excursion test dives to a minimum and allow them, for the most part, to be conducted at relatively low pressures. Sufficient data are available as a basis for the requisite format 1) from Barnard (1976), and from the classical investigations of Spaur, Thalmann, Flynn, Zumrick, Reedy, and Ringelberg (1978) in the development of unlimited-duration excursion tables for helium-oxygen saturation diving, and 2) from Hempleman's (1952) square-root formula for the prediction of the duration of one-stage (0-SD) decompression excursions from surface (1 ATA) steady-state conditions. Hennessy (1977) has formulated regression equations to convert standard air decompression tables to tables for no-stop diving from altitude or pressurized habitats and which also further the objectives outlined above.

GENESIS OF A SQUARE-ROOT FORMULA

Hempleman (1952) introduced a theory of decompression based on the assumption that if an individual is exposed to raised partial pressures of nitrogen, there is a single tissue into which the gas diffuses to produce bends. He stated, "In order to calculate an air diving table a single piece of information is necessary, e.g., the maximum safe exposure time at 100 ft of seawater followed by immediate surfacing, of 22 min. This fixes the volume of nitrogen, as measured at atmospheric pressure, which the susceptible tissue can retain without a bend being produced. In arbitrary feet of seawater units this will be $100\sqrt{22} = 475$ units." Subsequently, in the systematic exposition of principles underlying the development of this formula, Hempleman (1975) employed a value of 25 min for a 100-ft air dive from 1 ATA, which yields a value of 500 for the constant k.

Assumptions

1. The symptom-free volume of gas that the body can accommodate at 1 ATA after an excursion dive is constant, in accord with Hempleman's formula:

$$\sqrt{\text{time (min)}} \times \text{D} = k$$

2. At increased or decreased steady-state pressures either above or below 1 ATA, the volume of accommodated gas is proportional to the product of k and the square root (not a Haldanian multiple) of the absolute pressure for specified time periods:

$$\sqrt{\text{time (min)}} \times \text{D} = k \times \sqrt{\text{ATA}}$$

3. For excursions of infinite (∞) duration:

$$\Delta\text{D} = k\sqrt{\text{ATA}}$$

ANALYTICAL DATA

Restrictive considerations

The format data are in tabular form and are limited to single excursions from saturation levels. This restriction is mandated by the safety requirements of 0-SD excursions and by long-standing and disconcerting evidence that successive decompression stages are attended by gas phase separation, which provokes symptoms and is difficult to control except by long sojourn in hyperoxic atmospheres (Hills 1975).

A further consideration is that the analytical data apply only to the relatively circumscribed characteristics of a young adult "reference diver," body fat 12–14%, aerobic capacity 3.5 to

TABLE 1

DESCENDING EXCURSIONS FROM HELIOX SATURATION DEPTHS FOR UNLIMITED TIMES

Deeper than Saturation Depth		
Saturation Depth, fsw	Excursion Depth, ft Experimental	Calculated
Sea Level	-	32
150	75	75
200	83	85
250	91	93
300	99	101
350	106	109
400	114	115
450	122	122
500	130	128
550	138	134
600	146	140
650	153	145
700	161	150
750	169	155
820	180	162

Experimental data from Spaur et al. (1978); descending excursion depths calculated by $5.55\sqrt{\text{P abs (ft)}}$.

4 liters/min, and whose respiratory quotient reflects the resting state prior to descent to plateau pressure. This reference diver's slowest (fatty) tissue half time is of the order of 60 min (He-O_2), or 120 min (air).

Tabular data

Unlimited descending excursions from heliox saturation depths

In Table 1, the value of the constant (5.55) for the calculation of the net (ΔD) descending excursion depth in accord with the formula $\Delta D = 5.55\sqrt{\text{P abs(ft)}}$ was obtained from a single experimental datum from Spaur et al. (1978), namely, ΔD (122 ft), habitat depth 450 ft.

The extrapolated descending excursion at 1 ATA is 32 ft (from $5.55\sqrt{33}$) and is representative of values in the literature that are not too rigidly established. Agreement between experimental and calculated excursion depths is satisfactory to habitat depths of 600 ft. Deeper than 600 ft, the calculated excursions are lower than the test data, that is, more conservative, which suggests the need for a correction, perhaps in the direction of van der Waal's constants, referrable at high pressures to a decrease in molecular size and an increase in the intermolecular attraction forces of gases.

TABLE 2

CALCULATED Δ DEPTH EXCURSIONS FROM GRADED HELIOX SATURATION DEPTHS, FOLLOWED BY ONE-STAGE DECOMPRESSION

Pressure, ATA	Habitat Depth, ft	k*	Excursion Time, min 25	45	60	90	100	120	180	240	300	∞*
1.0	Sea Level	500	100	75	64	53	50	46	37	32	29	-
					64	53	50	46	37	34	33	32
5.54	150	1178	236	176	152	124	118	100	88	76	68	-
					150	125	117	107	86	80	77	75
10.09	300	1588	318	237	205	161	159	145	118	103	92	-
					202	168	158	144	115	108	104	101
16.15	500	1946	389	290	251	205	195	178	145	126	112	-
					256	207	194	177	142	132	129	128
22.21	700	2358	471	351	304	249	236	215	176	152	136	-
					300	232	219	200	171	160	155	150
25.85	820	2542	508	379	328	268	254	232	189	164	147	-
					324	251	236	216	185	173	167	162

Values are feet of depth; *k in Hempleman's formula (sea level), $\sqrt{\text{time (min)}} = k/\text{depth (ft)}$; at habitat depths, $k = 500\sqrt{\text{ATA}}$; **depth (duration unlimited) = $5.55\sqrt{\text{P abs (ft)}}$, and 0.5 (depth = $k\sqrt{60 \text{ min}}$). Calculation of underlined values: Δ depth ∞ time × 100/% sat. 60 min (T/2) tissue.

Calculated depth excursions for fractional periods of time from steady-state pressurized habitats

Table 2 shows depths for excursions of unlimited (∞) duration derived from Table 1; the underlined excursions for fractional time periods (to 60 min) are computed from:

$$\frac{(\infty)\text{ excursion depth} \times 100}{\%\text{ sat. 60 min (T/2) tissue}} \qquad (1)$$

where 60 min (T/2) is projected as the slowest uptake or clearance tissue in a heliox atmosphere for a reference diver.

Excursion depths for time periods of 25–300 min are estimated from

$$\sqrt{\text{time (min)}} = k/\text{depth (ft)}$$

where k is 500 (1 ATA), and at pressures above 1 ATA

$$k = \sqrt{+\text{ ATA}} \times 500 \qquad (2)$$

Nearly identical values accrue from Formulas *1* and *2* for times between 60 and 180 min.

Excursion depths for ∞ time also can be computed from Formula *2* as excursion depth (60 min)/2 = $31.88\sqrt{\text{ATA}}$ and $5.55\sqrt{\text{P abs (ft)}}$. The salient fact emerging from Table 2 is that the excursion (Δ) depths, in rounded values, from a habitat at 820 ft are square-root multiples of the excursions at 1 ATA.

TABLE 3

CALCULATED AIR EXCURSION Δ DEPTHS RELATIVE TO TIME FROM HABITAT PRESSURE LEVELS, FOLLOWED BY ONE-STAGE DECOMPRESSION

Pressure, ATA	Ft. Equiv.	k*	Duration, min 25	45	60	100	120	240	360	480	∞**
1.0	Sea Level	500	100	75	65	50	46	32	26	23	—
							46	31	26	25#	23
1.303	10	571	114	85	74	57	52	37	30	26	—
							52	35	30	28#	26
1.606	20	634	127	95	82	63	58	41	33	29	—
							58	39	33	31#	29
1.909	30	691	138	103	89	69	63	45	36	32	—
							64	43	37	34#	32
2.212	40	744	149	111	96	74	68	48	39	34	—
							68	45	39	36#	34
2.515	50	793	158	118	102	79	72	51	42	36	—
							72	48	41	38#	36

Values are ft of depth; *k (sea level) in Hempleman's formula, $\sqrt{\text{time (min)}} = k/\text{depth (ft)}$; value of 500 derived from experimental depth of 100 ft, 25 min. Increments of k proportional to $\sqrt{\text{ATA}}$; **depth (time unlimited) = $4\sqrt{\text{P abs (ft)}}$. #Calculation of underlined values: Δ D ∞ time × 100/% sat. 120 min (T/2) tissue.

Air excursion depths from habitat pressure levels

The net (∞) excursion depths for time (Table 3) are derived from $4\sqrt{\text{P abs (ft)}}$ and the underlined values (120–480 min) from

$$\frac{\text{depth} \propto \text{time} \times 100}{\%\ \text{sat. 120 min (T/2) tissue}}$$

The 120 min (T/2) tissue is projected as the slowest (fatty) tissue for a reference diver. At 120 min, the excursion depth is twice the depth for ∞ time. Hence, excursion depth (∞ time) = excursion depth 120 min/2.

Again, the excursions from habitats at higher pressures (+ ATA) are square-root multiples of the excursion depths from 1 ATA.

Air excursion depths after equilibration at altitude

For this condition, the same format (Table 4) has been followed as for calculating the excursions shown in Table 3. The tabular data are less conservative than the depth excursions at altitude in Cross's widely employed tables (1970). These tables have been shown to have an appreciable safety margin, since the Haldanian multiples of fractional atmospheres are less than the square root of altitude atmospheric fractions.

The tabular data are in accord with excerpts from the analysis of Hennessy (1977) and are in part confirmed by recent tests by Bell and Borgwardt at Lake Tahoe (unpublished data).

TABLE 4

AIR DEPTH EXCURSIONS CALCULATED BY THE SQUARE-ROOT PRINCIPLE, FOLLOWED BY ONE-STAGE (NO-STOP) DECOMPRESSION AFTER EQUILIBRATION AT ALTITUDE

Pressure, ATA	Altitude, ft	k*	Duration, min								
			25	45	60	100	120	180	240	360	∞**
1.0	Sea Level	500	100	75	64	50	46	37	32	26	—
							46	36	31	26	23
0.93	2000	482	96	72	62	48	44	36	31	25	—
							44	34	29	25	22
0.864	4000	465	93	69	60	46	42	35	30	25	—
							42	33	28	24	21
0.801	6000	448	90	67	58	45	41	33	29	24	—
							41	32	27	24	21
0.743	8000	431	86	64	56	43	39	32	28	23	—
							40	31	27	23	20
0.688	10000	415	83	62	54	42	38	31	27	22	—
							38	29	25	22	19
0.636	12000	399	80	59	52	40	36	30	26	21	—
							36	28	24	21	18
0.564	15000	375	75	56	48	38	34	28	24	20	—
							34	26	23	19	17

Values are ft of depth; *k in Hempleman's formula, $\sqrt{\text{time(min)}} = k/\text{depth (ft)}$; k is proportional to $\sqrt{\text{ATA}}$; **∞ time, depth = $4\sqrt{\text{P abs (ft)}}$. Underlined values: (∞ time) Δ depth × 100/% sat. 120 min (T/2) tissue.

TABLE 5

CALCULATED OPTIMAL EXCURSION BOTTOM TIMES FROM 30 AND 20-FT STAGES COMPARED WITH USN BOTTOM TIMES, FOLLOWED BY DECOMPRESSION TO FIRST STOPS OF 30 AND 20 FT

	Diving Depth, ft										
	80	90	100	110	120	130	140	150	160	170	180
From 30 ft:	75	59	48	39	33	28	24	21	18	16	15
USN Bottom Time (1st Stage 30 ft)	240	130	90	70	60	50	40	40	30	25	20
From 20 ft:	63	50	40	33	28	24	21	18	16	14	12
USN Bottom Time (1st Stage 20 ft)	150	120	80	60	50	40	30	30	25	20	15

Values are optimal bottom times, min; from 30 ft, time (min) = $(691/\text{depth (ft)})^2$; from 20 ft, time (min) = $(634/\text{depth (ft)})^2$. Note: from surface, time (min) = $(500/\text{depth (ft)})^2$ for no-stop decompression excursions. USN data from 1975 *Diving Manual*.

Comparison of excursion bottom times from 30- and 20-ft stages with USN bottom times, followed by first stops at 30 and 20 ft

Calculation (Table 5). Optimal excursion times from 30- and 20-ft stages can be calculated from formulas incorporating the square-root principle. Thus, at 30 ft, the calculated permissible stay = $(691/\text{depth})^2$, and at 20 ft, the calculated permissible stay = $(634/\text{depth})^2$, where the constants, 691 and 634, are multiples of $\sqrt{\text{ATA}} \times 500$ for 30- and 20-ft stages, respectively.

Examination of the U.S. Navy tables' bottom times in excess of the formula-derived values indicates that the initial stage of decompression is too shallow. This inference has been supported by the detection of bubbles by ultrasonic techniques on dive ascents to the first steps of the Navy tables (Spencer, Campbell, Sealey, Henry, and Lindbergh 1969).

DISCUSSION

The definitive physical basis for the proportionality between symptom-free ΔD and $\sqrt{\text{P abs}}$ over a wide range of pressure remains to be elucidated at pressures above 1 ATA, using such techniques as radioisotopes of gases and a gelatin model, continuing the investigations of Jones (1950). If blood flow and circulatory gas transport are unchanged at elevated pressure, the progressive increase of symptom-free ΔD or ΔP with progressive elevation of pressure suggests that diffusion of inert gas from the tissue matrix to the capillaries is inversely proportional to the square root of the mass concentration of gas, but this is speculative until quantitative data become available.

The time taken for a one-stage ascent from habitat pressure may be critical. In the author's experience, remarkably large quantities of helium were recovered in the brief time required to reach the first stop, using the Navy decompression table.

The purpose of this paper will have been achieved if the format can be used to extend safely the duration of excursions for fractional time periods. The following example of current requirements comes from Bornmann's (1970) data derived from the execution of over 1000 symptom-free excursion dives.

Excursion Time, min:	No Limit	270	150	60
Net Excursion Depth* from 150–300 ft:	25	50	75	100
Net Excursion Depth** from 150 ft:	75	78	91	150
from 300 ft:	101	106	111	202

*Bornmann data; **excursion depths calculated in accord with $\sqrt{\text{P abs}}$ principle. The question is: are the calculated excursions safe, compared to Bornmann's conservative values?

It is a serious omission in test procedure 1) not to know the physical characteristics of the test diver, specifically his body water and fat content, which now are readily determined, and 2) not to measure the clearance of inert gases from tissues after excursion dives, using the radio-krypton technique of a past generation (Tobias, Jones, Lawrence, and Hamilton 1949), and the altitude tolerance test for clinical indication of clearance of residual excess tissue gas employed at the Experimental Diving Unit about 1940. These quantitative and presumptive studies are feasible in excursions from normal pressure. To this end, this paper will demonstrate, by applying the square-root principle, that data obtained from excursions at sea level can be extrapolated accurately to calculate safe excursions from elevated steady-state pressures.

Last, a great deal of valuable data has accrued from animal experiments in test diving, e.g., using dogs (Reeves and Beckman 1966) and goats in the classical experiments throughout the era from Haldane to Hempleman. For example, at 1 ATA, an excursion to the 60-ft level and return to 1 ATA is well tolerated after an equilibration exposure of some 6 h. If the half time for the slowest tissue of the reference dog or goat is 60 min, then from our calculations, the tolerated excursion is to a depth of 120 ft (2 × ∞ depth) for 60 min. This gives a k value in Hempleman's formula of 930 ($\sqrt{ATA}$ × 500). With this value of k, the whole spectrum of excursions can be computed for the dog and the goat. If these tests are coupled with those using radioisotopes of inert gases and post-mortem whole body tissue analysis, we are in the modern era of test procedures, where the biometrician leaves the arm chair, dons gloves, and goes to work in the laboratory.

A. R. Behnke. Le principe de la racine carrée dans la calculation des tables de décompression d'une phase (sans arrêt). Undersea Biomed. Res. 6: 357–365, 1979. Une formulation de la racine carrée $\sqrt{\text{temps}}$ × la profondeur excursionaire = k, développée par Hempleman (1952), prédit la durée acceptable pour les excursions d'air à un ATA pendant les durées de jusque 6 h et jusque 3 h (atmosphère héliox). Aux pressions au-dessus d'un ATA la valeur de k était d'abord considerée d'être proportionnelle aux multiples Haldanian de l'ATA, et non proportionnelle à $\sqrt{ATA}$ comme il en est dans ce papier. Les excursions des durées infinies (∞) aussi peut être prédire de la formule de la racine carrée dérivé des excursions systematiques de la saturation d'héliox au Profondeur Nette de l'Excursion = $k \sqrt{P\ abs}$. La contribution générale de ce papier est un format de champ vaste qui s'étendent le principe de la racine carrée 1) aux valeurs de k au-dessous d'un ATA dans la formule Hempleman; 2) aux excursions pour (∞) temps des profondeurs de l'habitat à 250 mètres; et 3) qui montre une rélation entre les excursions du temps fractionnel et les excursions pour les durées infinies. L'application de la généralisation que les profondeurs excursionaires nettes sont proportionnelles à $\sqrt{P\ abs}$, sert à réduire au minimum le nombre des plongées d'essai qui de plus, peut être éffectuées aux pressions relativement basses avec les examens validisant aux profondeurs profondes.

racine carrée calculation
décompression hélium-oxygène

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