

## Converting standard air decompression tables for no-stop diving from altitude or habitat

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Hennessy, T.R. 1977. Converting standard air decompression tables for no-stop diving from altitude or habitat. *Undersea Biomed. Res.* 4(1):39-53.—Using the phase equilibration theory of Hills (1966), as modified by Hennessy and Hempleman (1977), it is possible to predict formulas for converting standard air decompression tables for no-stop diving at altitude or from a normoxic habitat, breathing air. For diving following equilibration at altitude, the Royal Navy, Royal Naval Physiological Laboratory, and Haldane-type rules appear to be too conservative, with the opposite result for diving after excursion to altitude. Predictions in the latter case are in fair agreement with the Swiss (Boni, Schibli, Nussberger, and Bühlmann 1976) no-stop altitude tables. In the case of habitats, close agreement is found between the Hamilton, Kenyon, Freitag, and Schreiner (1973) normoxic tables for no-stop downward excursions and indefinite dive upward-excursions on air. In the case of flying directly after no-stop diving, the US Navy rule of using repetitive group D appears to be conservative for dives less than 50 fsw, and possibly unsafe for dives over 50 fsw. It is concluded that for no-stop diving a single tissue and single safe ascent pressure formula are all that is necessary to generate equivalent air dives. This enforces the hypothesis that it is the volume of gas released on ascent that governs marginal type I bends, and that in a no-stop ascent, all excess dissolved gas is released in the worst case.

excursion diving

supersaturation ratio

Rules for converting standard air decompression tables for diving at altitude have been in use for a number of years (Cross 1967, 1970, Royal Navy (RN) Diving Manual, 1972). However, not much is known about the effectiveness of these rules or the frequency of their use. This is undoubtedly because of limited areas where frequent diving at altitude is possible.

On the other hand the converse situation, that of air excursion diving from a normoxic habitat, has been extensively examined (Edel 1969, Hamilton et al. 1973). There does not appear to have been any attempt to produce a method to convert the standard air tables in these cases. The nearest similar situation has been use of an "equivalent air depth" for closed-circuit mixture diving from the surface only.

The purpose of this paper is to examine the methods of converting the standard air tables, restricted to the case of nil-decompression diving. The latter is not a severe assumption, because nearly all excursion diving from a habitat falls into this category. Also, the great majority of amateur dives at altitude avoid decompression whenever possible.

In any event, much of the simplicity of the discussion would be lost without this assumption, since the correct mathematical model for the elimination of tissue inert gas during staged decompression is still undetermined.

### DIVING AT ALTITUDE

There are two cases which must be considered when diving at altitude: whether the diver is or is not equilibrated at that altitude before diving. It is tempting to incorporate the first case into the second as a safety factor, and to limit discussion to diving following excursion from sea level to altitude. This is indeed the basis of the RN rules and the Swiss tables (Bühlmann 1975; Boni, Schibli, Nussberger, and Bühlmann, 1976). However, the basis of the method described by Cross (1967, 1970) presupposes that the diver is already equilibrated at altitude, although this is not stated explicitly. The NOAA Diving Manual (1975) refers to diving at altitude and the need for correcting the standard tables, but gives no details.

There is a large difference between the two cases. If the dive is made following excursion from sea level to altitude, the diver carries nitrogen with him from sea level. If the diver first equilibrates at altitude, the quantity of dissolved nitrogen will be correspondingly lower and much less decompression would be required. The difference in decompression would become larger as altitude increases. This effect was noticed in the Transvaal, South Africa, where all inland diving is carried out at an altitude of between 1300 and 1800 m above sea level. Depths of between 50 and 60 m are regularly attained. Initially, RN rules were applied, which required increasing the dive depth by 30% for purposes of calculating the equivalent air depth. Later, the Cross method was used, requiring a depth increase of 25%. Thus, a 10-min dive to 60 m (involving 5 min of decompression if carried out at sea level) would require 12 min of decompression (the Swiss tables would require 25 min of decompression). This becomes a heavy penalty for repetitive diving, and it prompted considerable experimentation based on trial and niggles. The net result is that the correction factor has been abandoned.

There are several reasons why the present altitude correction rules may be considered too conservative. The main one must be that the above dives were carried out after equilibration at altitude, in calm, nonarduous conditions. An additional possibility which may account for the large safety factor in the altitude correction rule is that it may be based upon an incorrect hypothesis.

### Haldane altitude conversion method

The method described by Cross (1970) is actually based on Haldane (1908) principles, and will be called the Haldane method. The formula simply reduces to dividing the actual depth,  $D_A$ , of water dived by the atmospheric pressure,  $P_A$ , (in atmospheres) at that altitude to obtain the theoretical depth,  $D_T$ , (or equivalent air depth). Thus

$$D_T = \frac{D_A}{P_A} \quad (1)$$

For example, a dive to 60 m at an altitude of 1800 m (atmospheric pressure, 0.8 atm) requires that the standard air tables be entered for a theoretical depth of  $D_T = 60/0.8 = 75$  m. (Obviously, the standard air tables must extend well beyond 50 m to be of any use for 50 m dives at high altitude).

The derivation of Eq. 1 would probably be along the following lines:

The Haldane hypothesis supposes that a particular body tissue may retain its dissolved gas provided that a critical ratio of dissolved nitrogen partial pressure to ambient pressures

is not exceeded. Thus for a given dive at altitude, the equivalent air dive from sea level can be worked out by ensuring that the ratio of tissue nitrogen partial pressure to ambient pressure is the same for each dive, the duration of the dives being the same in each case. Since the dive from sea level is known to be safe, so must the equivalent dive from altitude. The result is given by formula 1.

However, there are a number of points omitted in the above argument. First, the reasoning is only correct if the two dives commence from a state of equilibrium with the surrounding air at sea level and altitude. The formula fails if it is applied to excursion from sea level to altitude diving, since the dive begins with the diver preloaded with sea level nitrogen. This important fact is not referred to in the original articles. A correct version is given later. Second, the formula assumes that the supersaturation ratio is a constant for all depth zones as implied by the use of Eq. 1 for a given altitude. This is known to be incorrect; the ratio appears to decrease with increasing pressure (see, for example, Hempleman 1975). In fact it can be said that if Eq. 1 is correct, it could just as well be applied to the converse, that of diving from an air habitat. Hennessy (1976) has shown that this produces absurd predictions for the equivalent air depth. On the other hand, the formula proposed in this paper describes both situations. The final point against Eq. 1 is that the Haldane approach requires that no bubbles form on a safe decompression. It is now believed that this is also incorrect; showers of bubbles exist on safe decompression, as revealed by ultrasonic monitoring (Evans 1975). Given this case, one can only compare two dives if the volume of released gas per unit volume of tissue is the same in each case.

#### **A modified safe ascent criterion**

Even though supersaturation ratios will probably continue to be used because of their convenience conceptually as well as computationally, it is doubtful that such a ratio exists in tissue. Hills (1966) proposes essentially that no critical supersaturation is possible. A gas phase forms almost immediately in a given tissue on decompression, with the released nitrogen in equilibrium with dissolved nitrogen. The vital hypothesis is that it is the volume of released gas that is responsible for bends symptoms. Although this hypothesis has not yet been proven, it is remarkable that Haldane (1908) appears to have actually used this concept of volume of released gas to develop the critical supersaturation approach.

However, when using the volume concept two aspects must be considered. The first is that the influence of surface tension, tissue elasticity, and the presence of  $O_2$ ,  $CO_2$  and  $H_2O$  must be taken into account (Nims 1951). The second is that the process of uptake of inert gas is quite different from that of elimination if accompanied by a drop in ambient pressure.

Thus while it may be supposed that inert gas uptake is a linear process (controlled by perfusion and/or diffusion) it is suggested that elimination is an overall nonlinear process in decompression, by virtue of nonlinear boundary conditions caused by the presence of a gas phase. If this is the case, dives from two different starting points and/or gas mixtures can only be usefully compared if no decompression at all is involved. By considering only no-stop dives, we avoid having to put forward an undoubtedly controversial mathematical model to predict elimination on ascent.

Instead, we assume that a no-stop decompression involves the same end point, namely that in the worst possible case, all excess dissolved inert gas in physical solution is dumped on ascent, and that no gas is eliminated via the circulation during this process. To compare two dives, the total quantity of gas per unit volume of tissue released on ascent back to the surface (or habitat) must be the same in each case, and exactly equal to a critical quantity just below

the threshold for marginal symptoms of type I decompression sickness. Providing that no gas is eliminated via the circulation during ascent, no account need be taken of the rate of ascent. This assumption will begin to fail for deeper excursions where the descent/ascent time forms a significant part of the dive. The hypothesis agrees with the qualitative conclusions of Kindwall, Baz, Lightfoot, Lanphier, and Seireg (1975) who compared whole body nitrogen elimination of gas at various depths after a 40-min air dive to 100 ft. They suggested that  $N_2$  elimination at the shallowest stop would be very likely to cause capillary occlusion and form a sink of extravascular gas. Obviously, the worst case would be where no gas can escape during the ascent in a particular (critical) tissue, in which case the volume of this gas becomes the most outstanding parameter to choose.

The Haldane hypothesis requires that for safe ascent

$$P_1/P_2 \leq R \quad (2a)$$

where  $R$  is the critical ratio,  $P_1$  is the pressure at which saturation of a given tissue occurred, followed by a rapid (but safe) ascent to a pressure,  $P_2$ .

In a simple analysis, the Appendix demonstrates that the Hills hypothesis leads to a similar condition

$$P_1 \leq A P_2 + B \quad (2b)$$

where  $A$  and  $B$  are constants. Indeed, the Haldane (1908) hypothesis can be interpreted to lead to this same result by simply including a term which allows for a possible excess pressure in the gas phase if it does form after exceeding the critical ratio. And, as Hills (1966) points out in his derivation of Eq. 2b, by using Eq. 2a, the ratio  $R$  will appear to drop with increasing pressure (for then  $R = P_1/P_2 = A + B/P_2$ ).

Recently, Hennessy and Hempleman (1977) derived, using the Hills hypothesis, a formula for air diving by analyzing helium dive data of Barnard (1976)

$$P_1 \leq 1.361 \bar{P}_2 + 3.4 \text{ msw} \quad (3)$$

where  $P_1$  and  $P_2$  are measured in meters of seawater absolute.

This paper presents an additional set of six equivalent empirical formulas used by the Canadian computer, RNPL, caisson workers' formula, the Workman  $M_{240}$  value relation, and formulas derived from an analysis of the supersaturation ratio curves of the USN Standard Air Table and the Swiss tables, in the range of air diving to 60 m. All agree to a remarkable degree.

The implications of Eq. 3 are fully exploited in the Appendix to produce a set of formulas to calculate the equivalent air depth for diving on any nitrox mixture from any initial starting point and initial tissue inert gas partial pressure.

### Diving following excursion to altitude

Assume that the diver is equilibrated at sea level and ascends rapidly to altitude (pressure,  $P_A$ , atm) and carries out a dive to a depth,  $D_A$ , immediately thereafter. The relevant formula ( $A_6$  in the Appendix) for the theoretical depth,  $D_T$ , for entering the standard air tables is

$$D_T = \frac{D_A - 10(1 - P_A)}{1.94 P_A - 0.94} \quad \text{msw} \quad (4)$$

$D_A$  and  $D_T$  are measured in meters of seawater gauge, even though the dive is likely to be made in fresh water (the small error of about 2% is on the side of safety).

The equivalent Haldane formula (see Appendix) is

$$D_T = \frac{D_A - 10(1 - P_A)}{\frac{R}{R-1} P_A - \frac{1}{R-1}} \quad \text{msw} \quad (5)$$

where  $R$  is the critical supersaturation ratio. If the classic supersaturation ratio,  $R = 2$ , is substituted into Eq. 5, an almost identical formula to Eq. 4 is obtained

$$D_T = \frac{D_A - 10(1 - P_A)}{2.00 P_A - 1.00} \quad \text{msw} \quad (6)$$

This result hints at the great difficulty in distinguishing between the two decompression hypotheses; in the region of one atmosphere, the two safe ascent criteria, 2a and 2b, are of similar order. Clearly, in the case of habitats, the essential differences will begin to increase.

The results predicted by Eq. 4 may now be compared with the RN rules and the Swiss Tables (Boni et al. 1976). Table 1 shows that the RN/RNPL rules are a poor approximation with gross underestimation at higher altitudes. Although the instructions to these tables do not explicitly state this, they assume that the rules will cover excursion to altitude diving. If, on the other hand, the RN/RNPL rules are only to be used following equilibration at altitude, the next section will demonstrate that they are too conservative.

TABLE 1  
Depth increase at altitude (rapid excursion from sea level)

Altitude, m	Pressure, atm	RN and RNPL, %	Prediction, %
0- 100	0.99	Nil	2
101- 300	0.97	25	6
301-1000	0.89	30 and 33½	25
1001-1500	0.84		41
1501-2000	0.79		62
2001-2500	0.74		92
2501-3000	0.69	50	136

Prediction is for Eq. 4 evaluated at depth of 50 m.

It is difficult to make a direct comparison with the Swiss tables because they do not provide an equivalent sea level air depth. In addition, there are a number of departures from RN/USN decompression procedures (such as an ascent rate of 10 m/min and a compulsory 3-min stop at 3 m even for no-stop dives). The main difference, however, (Bühlmann 1975) is that the surfacing ratio has been adjusted to allow for an ascent to a higher altitude after a dive at altitude (such as flying from sea level to a mountain lake by helicopter, diving, and then returning by air). This would have the effect of shortening permissible no-stop times. Table 2 shows a comparison between the no-stop times and those predicted by Eq. 4 when applied to the USN tables. Agreement is reasonable, with the Swiss times more conservative, as expected. Correlation would be considerably improved in the shallower dives in the table if interpolation of the USN tables had been possible.

TABLE 2  
No-stop decompression times

Altitude, m	Depth, m							
	12	15	18	20	25	30	35	40
0-700	200	75	50	30	25	20	15	10
	100	60	50	40	25	15	10	5
701-1500	90	30	20	15	10	5	4	—
	100	50	40	30	15	10	5	5
1501-2000	50	25	15	10	6	4	—	—
	60	40	30	25	10	5	5	—
2001-2500	40	15	7	5	—	—	—	—
	50	30	20	15	5	5	—	—
2501-3200	30	10	5	—	—	—	—	—
	40	20	10	5	5	—	—	—

Upper row, each entry, is Swiss value; lower row, values are minutes predicted by Eq. 4.

The Swiss no-stop times seem to deviate from a smooth curve. This is probably caused by their computer program, which rounds off in multiples of 5 min and adds to the difficulty of making a comparison. Finally, better correlation would have been obtained if the more conservative RN no-stop tables in Eq. 4 rather than those of the USN had been used. The converse habitat situation would be equilibration at sea level followed by an air dive and return to a habitat (the depth,  $D_A$ , must now be taken as being measured from the habitat). Albano and Columba (1971) actually carried out a simulation of this type of dive. They exposed 8 subjects to air at 4.5 ATA for different periods of time followed by direct ascent (in 2 min) to 1.635 atm. They found bend-free dives of up to 63-min duration. Equations 4 and 6 both predict a theoretical depth of about 15.5 m (51 ft). The next deeper entry in the tables is 60 ft, which indicates a 60-min no-stop dive.

#### Diving following equilibration at altitude

The relevant formula (A7 in the Appendix) is

$$D_T = \frac{D_A}{0.51 P_A + 0.49} \text{ msw} \quad (6)$$

This formula can be compared with the Haldane-type formula in Eq. 1. Note that both formulas give the same value only at sea level,  $P_A = 1$ .

It is simple to compare the effects of these formulas against the RN/RNPL rules (1972), as shown in Table 3. The latter are evidently far too conservative in this situation (and not conservative enough for excursion to altitude). Obviously, if Eq. 6 is correct, it explains why divers equilibrated at altitudes up to 2000 m are able virtually to ignore the current correction methods.



TABLE 3  
Depth increase after equilibration at altitude

Altitude, m	RN and RNPL, %	Haldane*, %	Prediction*, %
101- 300	25	3	2
301-2000	30 and 33½	27	12
2001-3000	50	45	19

Haldane data based on Cross (1970). \*Maximum value for each altitude range.

Table 4 shows no-stop durations for various dives at 1800 m altitude calculated by the various methods. The longer that can be spent equilibrating at altitude before diving, the smaller the correction factor; very much longer dives become possible. A single day spent at 1800 m enables a no-stop dive to 30 m to be extended from 5 to 20 min.

TABLE 4  
Dive at 1800 m altitude

Depth, m	Swiss	No-stop time, min		Prediction
		RN	Haldane	
15	30	40	60	60
20	15	25	30	40
25	10	17	20	25
30	5	9	10	20
50	—	—	—	5

Swiss, Haldane and prediction data based on equilibration at altitude before diving.

## AIR DIVING FROM A HABITAT

Assume that a diver is saturated with a suitable normoxic atmosphere in the habitat and wishes to make a no-stop excursion, breathing air, to a greater depth. It is also possible to examine the case of upward excursion, but only to that depth at which the diver may stay indefinitely (the equivalent of a no-stop saturation dive to the habitat).

### Downward excursion

The relevant formula (A8 in the Appendix) is

$$D_T = \frac{7(D_A + 10 - \frac{x}{0.8}(D_H + 10))}{(1.361 - \frac{x}{0.8})(D_H + 10) + 3.4} \quad \text{msw} \quad (7)$$

where  $D_A$  is the actual depth of the dive (measured from the surface),  $D_H$  is the depth of the habitat (measured from the water level in the hatchway),  $x$  is the proportion of nitrogen in the

habitat, and  $D_T$  is the equivalent depth on air, starting equilibrated with air from the surface. Equation 7 may be used to generate a set of no-stop tables for air diving from a habitat by calculating the equivalent air depth from the surface and using the USN standard air table for no-stop dives. The proportion of nitrogen in the mixture is usually based on maintaining a normoxic environment in the habitat. By assuming an oxygen partial pressure of 0.21 atm, Table 5 has been calculated. It is compared directly to Table 12-2 in the NOAA Diving Manual (1975), computed by Hamilton et al. (1973), using a spectrum of half times up to 480 min and a habitat  $P_{O_2}$  of 0.21 atm. The latter table has been partially tested in sea and dry chamber.

Agreement is very good. Had the USN no-stop times been interpolated for intermediate depths, agreement would in most cases have been considerably increased. Nevertheless, there are a number of anomalous points: saturation at 15 and 24 m with a no-stop dive to 39 and 51 m, respectively, for example. These points appear to occur primarily near the limits for very long no-stop dives.

The impression gained from examining the two sets of tables is that the Hamilton et al. table is not smooth. There are inconsistencies, probably occurring at switchover points between different tissues and M values. In some parts of the table, values are more conservative than those predicted by Eq. 7, and vice versa in other parts. It is possible that some of the longer dives of the former table may be too long, for example in the first three entries at a saturation depth of 110 ft.

The maximum excursion depth from a habitat for any given duration may also be derived from Eq. 7 (or directly). The equivalent air dive is entered into the formula together with the nitrogen fraction in the habitat. Three given durations may be identified: an indefinite dive, a 6-h dive, and a 4-h dive. From the USN standard tables, a 4-h dive appears to be close to about 12 m. A 6-h dive at 9 m was chosen (agreeing with values quoted by the RN). The only controversial depth is that of an indefinite dive. Haldane originally supposed 33 ft (10.1 m). However, bends have been found in caisson workers decompressed from 12 psi gauge (8.2 m). The value predicted by Eq. 3 is 7 m, and this was chosen to represent an indefinite dive on air followed by immediate safe return to the surface. The formula for a maximum excursion depth from a habitat located at a depth,  $D_H$ , for each of these three cases are

$$D_{\text{Max}} = \begin{array}{ll} 1.443 D_H + 12.0, & \text{for a 4-h dive;} \\ 1.390 D_H + 9.0, & \text{for a 6-h dive; and} \\ 1.361 D_H + 7.0, & \text{for an indefinite dive} \end{array} \quad (8)$$

These simple formulas can be used to predict the maximum safe depth of a habitat at 40, 50 and 100 fsw, as used in the Tektite I and II projects. Table 6 shows such a comparison. All the results are in very good agreement for a 4-h dive. It is possible to show that had the old Haldane formula with a 2:1 ratio been used for these calculations, gross inaccuracies would have resulted.

### Upward excursion

An upward excursion from a habitat will eventually produce decompression sickness in divers if the minimum depth is exceeded. Any shallower dive will release a volume of gas which will either grow or coalesce to a pain-provoking volume. The transport of inert gas after the gas phase has formed is believed to be nonlinear. A simple equivalent air depth does not generally exist once gas has formed, if it is accompanied by change in ambient pressure.



TABLE 5  
Downward no-stop air excursion from normoxic saturation.

Saturation depth, fsw	Time, min																	
	Excursion depth, fsw																	
	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
30	350 200*	156 100*	91 60*	68 50*	55 50	45 40	36 30*	28 25	22 20	15 10	12 10	10 10*	8 10*	7 5*	6 5	5 5	5 5	— 5
40			240 200	147 100*	92 60*	70 50*	50 40*	39 30*	32 30	28 25*	23 20*	17 15	13 10	11 10	9 10	8 5	7 5	6 5
50				267 200*	187 100*	140 60	64 50*	45 50	36 40	30 30	26 25	22 25	20 20	18 15*	14 10*	12 10	10 10	8 5*
60					224 200	161 100*	127 100	63 60	45 50	35 40	29 40	25 30	22 25	19 20	17 20	15 15*	12 10	10 10
70				279	174 200*	129 200	103 100	62 60	44 60	35 50	29 40	25 30	21 25	19 20	17 25	15 20	13 15	15 15
80						317 200*	215 100*	70 100	47 60	36 50	29 40	25 30	22 25	19 20	17 25	15 20	13 15	15 20
90								339 200	168 100*	68 60*	47 60	37 50	26 40	23 30	20 40	17 30	15 25	13 20
100										281 100*	135 100	93 60*	59 50*	44 50	36 40	31 30	28 25	25 20
110											334 200	257 100	132 60	65 60*	48 50	39 50	36 40	31 25
120														291 100*	237 100*	72 60*	51 50	46 40

Upper row in each entry based on data from Hamilton et al. (1973); bottom row in each entry based on predictions according to Eq. 7.

\*In most cases, agreement would have been closer had the USN no-stop times been interpolated for in-between equivalent air depths (they were always rounded to the next deeper depth).

TABLE 6  
Maximum safe descent excursion depth on air from a normoxic habitat

Habitat depth, fsw	Indefinite, fsw	6-h dive, fsw	4-h dive, fsw	Tektite, fsw (4 h)
40	77	85	97	100
50	91	99	111	—
100	159	169	183	175

Tektite data from Beckman and Smith (1972).

All that can be done is to calculate the minimum safe ascent depth (minimum excursion depth). The relevant formula can easily be derived using information in the Appendix. Instead, the formula

$$P_1 = 1.089 P_2 + 4.9 \text{ msw} \quad (9)$$

is used (Hennessy and Hempleman 1977). This was derived by assuming an oxygen partial pressure of 0.22 bar, very slightly different from a normoxic  $P_{O_2}$  of 0.21 atm.  $P_1$  is the saturation pressure (msw absolute,  $10 + D_H$ ) and  $P_2$  is that pressure (msw absolute,  $10 + D_{Min}$ ) to which a safe ascent can just be made. The equivalent formula is thus

$$D_{Min} = 0.918 D_H - 5.3 \text{ msw} \quad (10)$$

This formula predicts the results in Table 7 which agree closely with known values used in Tektite and those derived from Table 12-3 of the NOAA Diving Manual, computed by Hamilton et al. (1973).

TABLE 7  
Indefinite dive upward excursion from normoxic habitat

Saturation depth, fsw	Prediction	Depth, fsw Hamilton et al. (1973)	Tektite
30	11	15	
40	20	25	25
50	29	30	(32)
60	38	40	
70	47	50	
80	57	60	
90	66	70	
100	75	80	75
110	84	85	
120	93	95	

Tektite data from Beckman and Smith (1972).

It is interesting to compare Eq. 10 with that obtained using a supersaturation ratio,  $R$ , on air. On air,  $P_1 \leq R P_2$ , and on a normoxic mixture,  $P_1 \leq \frac{0.8}{x} R P_2$ , where  $x$  is the inert fraction. It is easily shown that in this case the classic Haldane approach gives

$$D_{\text{Min}} = \frac{1.25}{R} D_H - 10 \frac{(R-1)}{R} \text{ msw} \quad (11)$$

choosing  $R = 2$ ,  $D_{\text{Min}} = 0.625 D_H - 5$  msw. This predicts the absurd minimum safe excursion depth of 15 fsw from a normoxic habitat at 50 fsw. On the other hand, taking  $R = 1.5$ , the currently accepted supersaturation ratio for these dives is  $D_{\text{Min}} = 0.833 D_H - 3.3$  msw. This conveniently predicts a correct minimum safe excursion depth of 30 fsw. But by reversing the argument, a supersaturation ratio of 1.5 indicates a maximum saturation depth of only 16.5 fsw on air, followed by direct ascent to the surface. This value is now too low, and is believed to lie in the range from 23–28 ft (Eq. 3 predicts 23 ft). Finally, if a value of 1.5 is chosen, it will still be too large for deeper dives, because in fact the correct formula is believed to be Eq. 3,  $P_1 = 1.361 P_2 + 3.4$  msw. Thus, for large  $P_1, P_2$  values,  $P_1/P_2$  will approach 1.361.

This shows clearly that the value,  $B$ , cannot be ignored in Eq. 3. Its role is to create the impression of a large  $R$  for shallow depth, decreasing to low values for greater depths. For normoxic deep environments the ratio will be even lower, approaching unity (Eq. 9).

## FLYING AFTER DIVING

It is possible to apply the results of the Appendix to flying after a no-stop air dive back to the surface, and then a direct ascent to altitude. The interval spent on the surface before ascent to altitude is assumed to be very short, because otherwise it must be taken as a decompression stop, and would thus be beyond the scope of this paper.

Assume that the flight is conducted at a cabin altitude of 8000 ft (0.74 atm). The relevant formula (A10 in Appendix) is then

$$D_T = 2 D_A \quad (12)$$

where, as before,  $D_A$  is the actual depth of the dive, and  $D_T$  the theoretical depth. This formula may be compared with the Haldane-type formula,

$$D_T = \frac{(R-1)}{0.74 R-1} D_A \quad (13)$$

Thus,  $D_T = 2.1 D_A$ , for  $R = 2$ , and  $D_T = 4.6 D_A$ , for  $R = 1.5$ .

The latter formula leads to absurdly conservative predictions of no-stop dives followed by ascent to 8000 ft. Once again, however, the 2:1 ratio is a good approximation in the region near 1 atm absolute.

Equation 12 may be compared with the US Navy rule for flying after diving. The latter states that the diver should not ascend to 8000 ft altitude until he has attained repetitive group D. Thus, if he terminates the dive at group D, he should be able to ascend directly to this altitude. Table 8 has been prepared from the USN no-decompression limits table at group D, and gives the maximum bottom time for a given depth, such that direct ascent to 8000 ft altitude is possible. Equation 12 is used to predict a theoretical no-stop depth for a given dive, which in turn predicts a maximum bottom time using the USN and RN no-stop tables.

The group D method appears to be too conservative for shallow dives and is perhaps unsafe for deeper dives. For no-stop dives over 50 fsw, we propose that the use of group C or B may be safer. Below 50 fsw, group E gives the best match. Unfortunately, it is not possible to compare Eq. 12 with the RNPL rules because the latter only allow for direct ascent to an altitude of 300 m (983 ft).

It is also possible to examine the case of no-stop diving at altitude followed directly by a flight to a higher altitude (for example, excursion by helicopter to dive in a mountain lake), but we have omitted this example in our study.

## DISCUSSION

It has been shown that it is not necessary to use multiple tissue half times and supersaturation ratios to calculate no-stop decompression from altitude or habitat, by simply converting a standard table. It now appears doubtful whether it is necessary to use as many parameters to compute tables for dives involving decompression. Only one tissue and one pressure equation, 3, are needed to generate various equivalent and in some cases validated dives, using only standard air decompression tables. The USN standard air tables are themselves generated by a 12-parameter mathematical model (Hennessy, unpublished observation); this too is questionable, since, for example, RNPL and Canadian tables depend on only four parameters.

TABLE 8  
No-stop dives followed by flight at 8000-ft cabin altitude

Depth, fsw	USN Group D	No-stop time, min	
		Prediction, USN no-stop	Prediction, RN no-stop
10	300	$\infty$	$\infty$
15	160	$\infty$	$\infty$
20	100	200	135
25	75	100	85
30	60	60	60
35	40	50	40
40	30	40	30
50	25	25	20
60	20	15	14
70	15	10	9
80	15	5	—
90	12	5	—
100	10	—	—

Such doubt can conveniently be removed in the case of no-stop diving using the conversion formulas presented in this paper. Other situations are equally amenable to this type of analysis, but are not discussed here. These include a nonnormoxic habitat or nonair excursion diving. It is also simple to convert to helium-oxygen mixtures. In fact, Eq. 3 was derived by converting the equivalent helium formula, using a fatty tissue solubility partition coefficient (Hennessy and Hempleman 1977).

## APPENDIX

### Analysis of uptake of inert gas

Suppose that the tissues of the diver are saturated with nitrogen at a pressure  $P_{1N_2}$ . The diver now dives to a depth where the nitrogen partial pressure breathed is  $P_{2N_2}$ . At some subsequent time,  $T$ , suppose that the critical tissue nitrogen partial pressure will have changed to that given by

$$P_{N_2}(T) = P_{1N_2} + (P_{2N_2} - P_{1N_2})\phi(T) \quad (A1)$$

where  $\phi(T)$  is a function depending on the actual mechanics of uptake. This relationship requires a linear uptake process controlled by perfusion or diffusion (or both). The function  $\phi(0) = 0$ ,  $\phi(\infty) = 1$ , but is otherwise unknown. On the other hand, if elimination is a nonlinear process due to bubble interaction, it will not be possible to use an equation of the above form for the ascent.

Assume at this stage that there is only one type of tissue responsible for marginal type I bends; there is thus only one uptake function,  $\phi$ .

After the duration of the dive,  $T$ , the diver ascends at a steady rate to the surface or habitat located at a pressure,  $P_2$  (atm. absolute). Suppose that a critical volume,  $V_c$ , of gas per unit volume of tissue is then released into that tissue responsible for marginal type I bends on air diving. The gas is assumed to form rapidly so that none is lost to the circulation during its formation during the ascent. It is not necessary to represent the elimination pressure mathematically, since the worst case is assumed in each dive: all excess dissolved gas is liberated. Only in a staged ascent or a long direct ascent from an excessively deep excursion would it be necessary to consider elimination dynamics.

The nitrogen partial pressure in the gas phase (not necessarily a bubble) will obviously be at ambient pressure, plus a small constant,  $k$ , (atm) (to allow for the presence of other gases, surface tension and tissue elasticity, as in Hills (1966) and Nims (1951)).

A simple mass balance in unit volume of tissue gives

$$\alpha P_{N_2}(T) = \alpha(P_2 + k) + V_c(P_2 + k) \quad (A2)$$

where  $\alpha$  is the solubility of nitrogen in the tissue, ml gas/ml tissue per atm at 37°C. The above equation simply states that the total quantity of nitrogen in a unit volume of tissue before decompression must be the same as that remaining in the unit volume plus that in the cavity after a rapid no-stop ascent.

Note that if  $k$  is zero, Eq. A2 simply states that  $P_1/P_2$  is a constant that is identical with the Haldane critical ratio hypothesis. This feature is used later to establish equivalent Haldane-type formulas.

It is convenient to define the equivalent saturation depth,  $P_1$ , that would give rise to  $P_{N_2}(T)$  in the tissues. This is evidently given by  $P_1 = P_{N_2}(T)/0.8$  in the case of air, and  $P_{N_2}(T) + 0.21$  for a normoxic mixture.

Equation A2 may then be rewritten, in either case, as

$$P_1 = A P_2 + B \quad (A3)$$

The values of  $A$  and  $B$  have been derived by Hennessy and Hempleman (1977) by analyzing the helium saturation dive data of Barnard (1976):  $A = 1.361$ ,  $B = 0.34$  atm in the case of air, and  $A = 1.089$ ,  $B = 0.49$  atm in the case of a normoxic gas.

Equation A1 may be written

$$0.8P_1 = P_{1N_2} + (P_{2N_2} - P_{1N_2})\phi(T)$$

Eliminating  $P_1$  using Eq. A3 gives a value for the unknown function  $\phi$  at a point  $T$

$$\phi(T) = \frac{0.8AP_2 + 0.8B - P_{1N_2}}{P_{2N_2} - P_{1N_2}} \quad (A4)$$

An equivalent air dive is based on the simple concept that the same volume of gas must be released as in the dive from the habitat or surface. This means that  $A$  and  $B$  are the same for each dive. Since the duration of the dives is also equal in each case, Eq. A4 is rewritten so that  $P_{2N_2}$  now becomes  $0.8(D_T + 10)/10$ , where  $D_T$  is the theoretical air depth from the surface,  $P_2$  becomes 1 atm, and  $P_{1N_2} = 0.8$  atm. Thus

$$\phi(T) = \frac{10(A + B - 1)}{D_T}$$

Eliminating  $\phi(T)$  using Eq. A4 gives the main result:

$$D_T = \frac{12.5(A + B - 1)(P_{2N_2} - P_{1N_2})}{(AP_2 + B - 1.25P_{1N_2})} \quad (A5)$$

The following cases discuss various situations referred to in the text, and are not intended to cover all possibilities (the remainder of situations may be derived by the correct use of Eq. A5).

#### Case 1, equilibration at sea level, followed by dive at altitude

A dive, with duration  $T$ , is made to a depth of  $D_A$  in meters measured from the surface, where the pressure is  $P_A$ , atm. Assume that the diver is saturated with air breathed at sea level. This case covers an excursion to a mountain lake when normal residence is at sea level. Here  $P_{1N_2} = 0.8$  atm, and  $P_{2N_2} = 0.8(P_A + D_A/10)$  and  $P_2 = P_A$ .

Substituting into A5 gives

$$D_T = (A+B-1) (D_A - 10(1 - P_A)) / (A P_A + B - 1) \\ = \frac{D_A - 10(1 - P_A)}{1.94P_A - 0.94} \text{ meters} \quad (A6)$$

The appropriate converse habitat situation would be equilibration at sea level followed by an air dive and return to habitat. In this case,  $D_A$  is measured from the habitat.

#### Case 2, equilibration at altitude, followed by dive at altitude

This is exactly the same as case 1 except that  $P_{1N_2} = 0.8P_A$ , with the result  $D_T = (A+B-1) D_A / ((A-1)P_A + B)$ ,

Thus

$$= \frac{D_A}{0.51P_A + 0.49} \approx \frac{2D_A}{P_A + 1} \text{ meters} \quad (A7)$$

Here the converse habitat dive would be equilibration in an air habitat followed by an air dive and return to habitat.  $D_A$  is measured from the habitat.

#### Case 3, downward air excursion from a normoxic habitat

The habitat is located at a depth  $D_H$  meters below the surface (taken as 1 atm). The fraction of inert gas is  $x$ . Thus,  $P_{1N_2} = x(D_H + 10)/10$ ,  $P_{2N_2} = 0.8(D_H + 10)/10$  and  $P_2 = (D_H + 10)/10$ , with the result

$$D_T = \frac{7(D_A + 10 - \frac{x}{0.8}(D_H + 10))}{(1.361 - \frac{x}{0.8}(D_H + 10) + 3.4)} \text{ meters} \quad (A8)$$

#### Case 4, indefinite upward excursion from a normoxic habitat

It is not necessary to use Eq. A5 in this case. Equation A3 for a normoxic gas is  $P_1 = 1.089 P_2 + 0.49$  atm. Thus, saturation at a depth  $D_H$  gives  $P_1 = (D_H + 10)/10$  and an upward ascent for an indefinite period to a marginal depth  $D_{Min}$  gives  $P_2 = (D_{Min} + 10)/10$ , with the result

$$D_{Min} = 0.918 D_H - 5.3 \text{ msw} \quad (A9)$$

#### Case 5, diving followed by direct ascent to 8000 ft

This is equivalent to an ordinary air dive, except that the end point is  $P_2 = 0.74$  atm. In this case Eq. A5 reduces to the simple relation

$$D_T = 2D_A \quad (A10)$$

#### Haldane formulas

As noted before, Hills' critical volume hypothesis and Haldane's critical ratio hypotheses are essentially identical if the constant  $k$  is taken as zero. Thus, all Haldane-type formulas, Eqs. 1, 5, 11, and 13 used in the body of this paper may be derived from Eq. A3 or A5 by simply setting  $A = R$  or  $0.8R$ , and  $B = 0$  or  $0.2$ , depending on whether air or a normoxic gas is used. (Very little error is introduced using  $0.2$  instead of  $0.21$  for  $P_{O_2}$  of a normoxic gas).

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Hennessy, T.R. 1977. La conversion des tables standard de décompression en air pour des plongées sans arrêt de l'altitude ou à partir d'un habitat. Undersea Biomed. Res. 4(1):39-53.—On peut prédire des formules pour la conversion des tables standard de décompression en air pour des plongées sans arrêt de l'altitude ou à partir d'un habitat à air normoxique, en se servant de la théorie d'équilibration en phases de Hills (1966) modifiée par Hennessy et Hempleman (1977). Pour une plongée après équilibration à l'altitude, il paraît que les règles type RN/RNPL et Haldane sont trop conservatrices, tandis que pour les plongées après séjour à une altitude, elles ne le semblent pas assez. Dans ce cas-ci, les prédictions s'accordent bien avec les tables "sans arrêt" pour altitude de (Boni, Schibli, Nussberger, and Bühlmann 1976). Dans le cas des habitats, on trouve un accord étroit entre les tables normoxiques de Hamilton, Freitag, and Schreiner (1973) pour des excursions descendentes sans arrêts et celles pour des excursions montantes de plongée indéfinie en air. Dans le cas du vol immédiatement après une plongée sans arrêt, la règle de la Marine



américaine d'appliquer le groupe D réitératif paraît conservatrice pour des plongées de moins de 50 fsw, et peut-être dangereuse pour des plongées de plus de 50 fsw. On peut conclure que pour les plongées sans arrêt, on n'a besoin que d'un seul tissu et d'une seule formule sûre de pression pour la montée pour calculer les plongées équivalentes en air. Ce serait donc le volume des gaz libérés pendant la montée qui déterminerait les bends type I, et tout le gaz superflu dissous serait libéré dans le pire cas.

rapport de sursaturation

plongée à excursion

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